MA370 - Summer 2013
Assignment #2

- Please work in groups of two.
- Due Monday, June 24 at beginning of lecture. You may hand in your assignment at any time prior to the due date. My office is BA-546 - slip your assignment under the door if I am not there.
- You are strongly encouraged to use a computer for the computational problems, and required to use a computer to generate plots/graphs. I don't care what software you use. I will be in the computer lab on the fifth floor of Bricker from 1-3 in the afternoon on Friday June 21 in case you need last minute help. Don't hesitate to get in touch with me if you need assistance before then.

1. In lecture on June 5 we considered an instance of the binomial model where the expected return from buying a put option was negative. In this problem we prove that this will be the case for *any* derivative that pays off more in the downstate than it does in the upstate (actually we prove that the excess return, or expected return on the derivative minus the risk-free rate, is negative). So for the purposes of this problem assume we are in the binomial world, let p denote the probability that the upstate is realized and consider a derivative with payoffs f_d and f_u.

   (a) Prove that $pf_u + (1-p)f_d < f_0(1+r)$ if and only if $f_u < f_d$, where f_0 is the fair price of the derivative.

   Hint: Begin by writing $(1+r)f_0$ in terms of the risk-neutral probability q and the derivative payoffs f_u, f_d.

(b) Use (a) to show that if $f_d > f_u$ then the expected return on the derivative is less than r.

(c) Provide a brief intuitive explanation for the result in (b).

2. The current price of a stock is \$100. The stock will either be worth \$110 or \$95 in one month. The yield on risk-free bonds maturing in one month is 1%.

   (a) Determine the fair price of (i) a call option struck at \$105 and (ii) a put option struck at \$100.

   (b) How would you exploit a call struck at \$105 that was trading at \$4?

   (c) How would you exploit a put struck at \$100 that was trading at \$1?

3. Consider a binomial model with $\mu = 0.12$, $r = 0.05$ and $p = 0.5$. Further suppose that the stock is currently trading at $S_0 = 50$.

   (a) Plot the value of a call option on this stock as a function of σ for $K = 30$, $K = 50$ and $K = 70$. Comment on any differences and/or similarities between the plots. Please put all three plots on the same set of axes and use at least one hundred values of σ in the interval $(.05, .80)$.

   (b) Plot the delta of a call option on this stock as a function of σ for $K = 45$ (in the money option) and $K = 70$ (out of the money option). Comment on the shape of each graph individually, as well as any differences between the two. Please put each plot on a separate set of axes and, for each plot, use at least one hundred values of σ in the interval $(.05, .80)$.

4. Consider the trinomial example discussed in lecture over June 10 and June 12. Instead of the call struck at \$60, suppose that I am interested in introducing a put struck at \$48.

Note: When you encounter numbers of the form $\frac{x}{1.02}$ in this problem I would *much* prefer that you leave it in that form instead of carrying out the division.

(a) Write the price of the put in terms of the middle-state price Ψ_m.

(b) Determine the (i) no-arbitrage and (ii) economically reasonable no-arbitrage intervals for the price of the put.

(c) Suppose now that I am interested in introducing a call option struck at \$60 *and* a put option struck at \$48 (if it helps assume that I'm going to introduce these derivatives simultaneously and sell them to different clients). If my boss instructs me to price the call at c_0, then how should I price the put? In particular, am I still free to pick any price in the interval from (b)?

5. Consider a trinomial model with $u = 1.55$, $m = 1.10$ and $d = 0.65$. Further assume that $r = 0.04$, $p_u = p_d = 0.2$ and $p_m = 0.6$.

(a) Calculate the expected value and standard deviation of the stock's return.

(b) Find the general form of a state price vector in this market, parametrized by the price of a middle-state dollar.

(c) What range of middle-state prices is consistent with no arbitrage?

(d) What range of middle-state prices is consistent with no arbitrage *and* risk-aversion?

(e) Determine the arbitrage-free price range for a put struck at \$25, assuming the stock is currently trading at \$20, as well as the "economically reasonable" arbitrage-free price range.

(f) Suppose that you sell the call to a client for \$5.60 and hedge your exposure by purchasing δ shares (recall that if $\delta < 0$ the "purchase" is actually a short sale). On the same set of axes plot your profit-and-loss in each state as a function of δ, and explain why it is not possible to eliminate all of the risk associated with selling the option. For the plots please use at least 100 points in the range $(-1, 0)$.

(g) Determine the hedge ratio (i.e. value of δ) that minimizes the volatility (i.e. standard deviation) of your profit-and-loss. How much does hedging reduce volatility relative to selling a naked (i.e. unhedged) put.