MA370 - Summer 2013
Assignment #1

- Work in groups of two.
- Due Friday, May 31 by 4:00 PM. You may hand in your assignment at any time prior to the due date. My office is BA-546 - slip your assignment under the door if I am not there.
- You are strongly encouraged to use a computer for the computational problems. I don't care what software you use. I will be in the computer lab on the fifth floor of Bricker from 1-3 in the afternoon on Thursday May 30 in case you need last minute help. Don't hesitate to get in touch with me if you need assistance before then.

1. Consider the markets defined by the payoff matrices

$$D_1 = \begin{bmatrix} 100 & 100 & 100 \\ 70 & 110 & 150 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 100 & 100 & 100 \\ 70 & 110 & 150 \\ 10 & 50 & 90 \end{bmatrix}.$$

   (a) Explain why the first market is incomplete. Are there any redundant assets in this market?

   (b) Explain why the second market is incomplete. Determine the number of redundant assets, and for each redundant asset that you identify describe how its payoff can be replicated by trading in the other securities.

   (c) For each market, find all portfolios that have a terminal value of $\mathbf{v} = [20 \quad 10 \quad 0]$.

   (d) What does your answer from (c) tell you about the presence of redundant assets and uniqueness of replicating portfolios?

2. Refer to the market defined by the payoff matrix D_1 in Question 1.

   (a) Find a basis for the set of attainable payoffs.

   (b) Give an example of a payoff that is not attainable in this market.

   (c) Suppose your client is interested in a portfolio that is worth v_1 in state 1 and v_2 in state 2. Does your client have any freedom to choose the value of the portfolio in state 3? If so, explain how. If not, determine the implied state 3 payoff as a function of v_1 and v_2.

   (d) Now suppose your client is interested in a portfolio that is worth v_1 in state 1 and v_3 in state 3. Does your client have any freedom to choose the value of the portfolio in state 2? If so, explain how. If not, determine the implied portfolio value in state 2 in terms of v_1 and v_3.

   (e) What do your answers from (c) and (d) tell you about the relation between the rank of the payoff matrix and the number of states that you can "choose your payoff" in.

   (f) Find a basis for the set of attainable payoffs in the market defined by the payoff matrix D_2. Compare this basis with the one obtained in (a), commenting on any similarities or differences.

3. Consider a market defined by the payoff matrix and initial price vectors

$$D = \begin{bmatrix} 100 & 100 & 100 \\ 70 & 110 & 150 \\ 150 & 100 & 80 \end{bmatrix}, \qquad S_0 = \begin{bmatrix} 95 \\ 84.5 \\ 123 \end{bmatrix}$$

   (a) Show that the market is complete and free of arbitrage.

   (b) Find the portfolio that has terminal value $\mathbf{v} = \begin{bmatrix} 150 & 100 & 150 \end{bmatrix}$.

   (c) What is the cost of the portfolio from (b)?

   (d) Calculate $\mathbf{v}\Psi$, where Ψ is the unique state price vector for this market.

(e) Comment on any similarities between your answers from (b)/(c) and (d).

4. In a complete market with no redundant assets the payoff matrix is guaranteed to be invertible. Prove that in such a market the cost of a portfolio with a terminal value of $\mathbf{v}$ is equal to $\mathbf{v}\Psi$, where Ψ is the market's unique state price vector.

5. Consider the market defined by the payoff matrix

$$D = \begin{bmatrix} 100 & 100 & 100 \\ 70 & 110 & 150 \end{bmatrix}$$

(a) Show that if $S_0 = \begin{bmatrix} 50 \\ 90 \end{bmatrix}$ the market admits arbitrage.

(b) Find the portfolio that has a terminal value of $\mathbf{v} = \begin{bmatrix} 1.375 & 0.875 & 0.375 \end{bmatrix}$ and show that it is an arbitrage portfolio. What do you think would happen to the prices of the two assets in a well-functioning market?

(c) Show that if $S_0 = \begin{bmatrix} 90 \\ 75 \end{bmatrix}$ then the market is free of arbitrage.

(d) If prices are as in part (c), determine upper and lower bounds on state prices for each state.

6. Consider the market defined by the payoff matrix and initial price vector

$$D = \begin{bmatrix} 100 & 100 & 100 \\ 70 & 110 & 150 \\ 150 & 100 & 80 \end{bmatrix}, \qquad S_0 = \begin{bmatrix} 95.0 \\ 84.5 \\ 123 \end{bmatrix}.$$

(a) Show that the market is complete and free of arbitrage.

(b) Use the state prices found in (a) to determine (i) the implied risk-free rate of interest and (ii) the risk-neutral probabilities of each state.

(c) If the risk-neutral probabilities are used, calculate the expected return on the second and third assets.

(d) If the actual probabilities of each state are $p_1 = .1$, $p_2 = .8$ and $p_3 = .1$, determine the expected return on all three assets. The return you get for the third asset should snap your head back. What is going on there?