(1) (2 pts) In the matrix below replace β with the **second last digit of your student number** and calculate the determinant:

$$\begin{vmatrix} 1 & 2 & \beta \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

(2) (4 pts) Let A be an $n \times n$ matrix with $\det(A) = 0$. For each of the following statements determine if it is true or false. Answer with T (for true) and F (for false)

(a) A is invertible.

(b) 0 is an eigenvalue.

(c) The columns of A span $\mathbb{R}^n$.

(d) The rows of A are linearly dependent.

(3) (2 pts) Write the complex number below in the form $a + bi$ for $a, b, \in \mathbb{R}$:

$$\frac{11 - 2i}{1 - 2i}.$$

(4) (4 pts) Determine if the statements below are true or false. Answer with T (for true) or F (for false). No justification required.

(a) $U = \{[\begin{array}{ccc} 2 & 3s & 4t \end{array}]^T : s, t \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.

(b) $U = \{[\begin{array}{ccc} s + 3t & 4r + 5s & r - 2s + 4t \end{array}]^T : r, s, t \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.

(c) If U is a subspace of $\mathbb{R}^n$ and $rX \in U$ for all $r \in \mathbb{R}$, then $X \in U$.

(d) If $U = \text{Span}\{X, Y\}$ and $Z \in U$, then also $U = \text{Span}\{X, Y, Z\}$.

(5) (3 pts) Find the area of the triangle OAB where $O(0,0,0)$, $A(-4,1,1)$ and $B(-4,2,3)$.

(6) (4 pts) Let A be a 2×2 matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$ and corresponding eigenvectors $X_1 = \begin{bmatrix} 2 & 1 \end{bmatrix}^T$ and $X_2 = \begin{bmatrix} 3 & 2 \end{bmatrix}^T$.

(a) What is the general solution of the linear system of differential equations $f' = Af$?

(b) Find the solution of $f' = Af$ satisfying the boundary conditions $f_1(0) = 3$, $f_2(0) = 1$.

(7) (8 pts) (a) Find all eigenvalues of the matrix $\begin{bmatrix} 4 & 8 \\ 3 & 2 \end{bmatrix}$.

(b) The eigenvalues of the matrix $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ are $\lambda_1 = 5$ and $\lambda_2 = -1$ (you do not have to show this!). For each eigenvalue of B find all eigenvectors.

(c) Decide if B is diagonalizable or not. If yes, give an invertible matrix P and a diagonal matrix D such that $P^{-1}BP = D$. Justify your answer, both for yes and no!

(8) (6 pts) Are the following sets of vectors linearly independent in $\mathbb{R}^3$?

(a) $\{ [\ 1\ \ -4\ \ 7\ ]^T, [\ -3\ \ 5\ \ 6\ ]^T, [\ 9\ \ 12\ \ 4\ ]^T, [\ 0\ \ 3\ \ -2\ ]^T \}$

(b) $\{ [\ 1\ \ 3\ \ -4\ ]^T, [\ 1\ \ 1\ \ -1\ ]^T, [\ -1\ \ 3\ \ -5\ ]^T \}$

(9) (4 bonus points) (a) Let $X_1, \ldots, X_k$ be vectors in $\mathbb{R}^n$. Define Span $\{X_1, X_2, \ldots, X_k\}$.
(b) Show that Span $\{X_1, X_2\}$ is a subspace of $\mathbb{R}^n$.