ECON 395 Winter 2012

UNIVERSITY OF CALGARY
DEPARTMENT OF ECONOMICS
MIDTERM EXAMINATION 1
ECON 395 (L01)

October 23, 2013 **Marian Miles**

NAME:____________________________________

INSTRUCTIONS:

The exam will last for **50 minutes**. **Please allocate your time wisely**. You can obtain a total of **50 points**. **Show your formula and some steps in your calculations to receive points.** You may use non-programmable calculators during the exam.

1. **Fill in the blanks in the following six statements to make the statements true:** **[18]**

a) The probability of incorrectly rejecting the null hypothesis when it is true is called a(n) **___TYPE I___** error and has a(n) **___INVERSE___** relationship to the probability of Type II error.

b) Under Gauss Markov, a linear estimator is assumed to be **___BLUE____** if the sample estimate of the parameter has the **_smallest_** variance of all linear estimators under repeated sampling.

c) We reject the null if the p–value is **__Smaller__** than the **__level of significance (α)__.**

d) The Central Limit Theorem tells us that if we take a random sample of size n, where n is sufficiently **__large_** ,for a variable with any distribution, the means of each sample will distribute normally, with mean equal to the population mean and standard deviation approaching the standard deviation of the population divided by **_the square root of the sample size (n)_**.

e) One of the assumptions of the simple linear regression model is that **_error term (e)_** is a random variable in the model.

f) There are three desirable properties in an **_estimator**. The property **_unbiasedness** means that if we obtained many samples of size *n*, and calculated their average values for an unknown population parameter, and then we calculated the average of all those average values, this would then equal the true population parameter.

2. The joint probability distribution of being an immigrant (I) and having a university degree (D) is given in the accompanying table, where I indicates whether a person is an immigrant (I=1) or Canadian born (I=0), and D denotes whether a person is a university graduate (D=1) or not (D=0).

		I	
		0	1
D	0	0.4	0.1
	1	0.2	0.3

a) Determine the marginal probability distribution of being an immigrant and the marginal probability distribution of having a degree. **[3]**

I	0	1
p(I)	0.6	0.4

D	0	1
p(D)	0.5	0.5

b) What is the probability of having a university degree, if the person is Canadian born? Show the formula. **[3]**

P(D=1 | I = 0) = P(D=1 and I=0) / P(I=0) = 0.2/0.6 = 1/3
The probability of having a university degree, if the person is Canadian born is 1/3 or 0.33.

c) Are *I* and *D* independent? Prove your answer. **[4]**

One test is $P(D=1 | I=0) = P(D=1)$ but $0.33 = P(D=1 | I=0) \neq P(D=1) = 0.5$

OR

$P(D=1, I=0) = P(D=1)P(I=0)$ but
$P(D=1, I=0) = 0.2 \neq 0.5 * 0.6 = 0.3 = P(D=1)P(I=0)$
Being an immigrant and having a degree (I and *D)* are not independent.

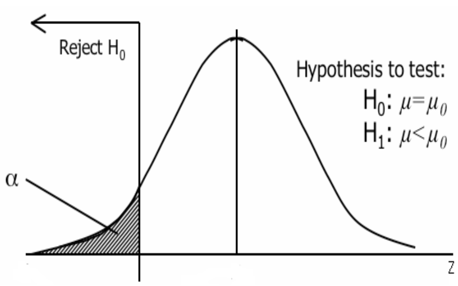
3. Suppose the Gap is considering putting a factory outlet store near the campus but they need to be sure that the mean student expenditure will not be lower than $220 per year. They hire you to conduct some required research. You implement a survey and ask 225 students how much they believe they would spend at the store each year. The student responses have a mean of $200 and a standard deviation of $150. Assume that the spending habits of the student population are normally distributed. Test the hypothesis, at a 5% significance level, that the (true) mean per-student expenditure is less than $220.

   a) State the null and alternative hypothesis **[2]**

   $H_0: \mu = \$220$
   $H_A: \mu < \$220$

   b) Draw the distribution and shade the rejection region. Clearly label your drawing. **[3]**

   $\underline{\boldsymbol{\alpha = .05}}$, Left Tail Test df=n-1=225-1=224



   t = -1.645

   c) Calculate the test statistic. **[3]**

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{200 - 220}{150/\sqrt{225}} = -2$$

   d) What is your decision rule? State the degrees of freedom, if appropriate. **[2]**

   Degrees of freedom = 225-1=224
   Reject H_0 if test statistic is less than the t critical value of -1.645
   Not needed [Test is <u>-2< -1.645 Reject H_0]</u>

   e) Is the (true) mean per-student expenditure less than $220? (State your conclusion) **[2]**
   There is sufficient evidence to reject the hypothesis that the mean per student expenditure is $220 at a significance level of 5%. The evidence supports the hypothesis that spending is less than $220. The Gap should not put a factory outlet store near campus.

4. A professor of economics wants to study the relationship between income (*y* in \$1000s) and education (*x* in years). A random sample of eight individuals is taken and the results are shown below.

Education	16	11	15	8	12	10	13	14
Income	58	40	55	35	43	41	52	49

Suppose the econometric model proposed is of the following form,

$$Y_i = \beta_0 + \beta_1 X_i + e_i,$$

where Y_i is income (\$1000s) and X_i is amount of education (in years).

You are given the following information:

$$\sum_{i=1}^{N}(Y_i - \overline{Y})(X_i - \overline{X}) = 145.25; \quad \sum_{i=1}^{N}(X_i - \overline{X})^2 = 49.875; \quad \sum_{i=1}^{N}(Y_i - \overline{Y})^2 = 457.875$$

$$\sum_{i=1}^{N} Y_i = 373; \quad \sum_{i=1}^{N} X_i = 99; \quad \sum_{i=1}^{N} X_i^{\ 2} = 1275 : \quad \sum_{i=1}^{N}(\hat{Y}_i - \overline{Y})^2 = 422.281$$

a) Determine the ordinary least squares regression line. **[5]**

$$\mathrm{b}_2 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \frac{145.25}{49.875} = 2.912, \quad b_1 = \bar{Y} - b_2\bar{X} = 46.625 - (2.912 * 12.375) = 10.589$$

$\underline{\hat{Y}_i} =$ <u>**10.589 + 2.912*X*$_i$**</u>

b) Interpret the coefficients of the regression line. **[5]**

The intercept coefficient suggests that on average, with no education, income would be \$10,589. As this is well beyond the scope of this data set, I would be NOT use this interpretation.
The slope coefficient is interpreted as for each additional year of education, the average increase in income is \$2,912.

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