MAT 1339, Fall 2013 Assignment 3

Due NOV8th11:59 AM.

Late assignments will **NOT** be accepted. An assignment drop-off box is assigned for this course and is located at Math department. (KED 585)

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Student Name ______________________ Student Number ______________________

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature Marking Scheme and Solutions.

15 points

QUESTION 1. Find and classify all the local (relative) extrema of the following functions using the Second Derivative test.

6

a) $f(x) = -3x^5 + 5x^3$.

1 $f'(x) = -15x^4 + 15x^2$ $\quad D_{f'} = \mathbb{R}$

$= -15x^2(x^2 - 1) = 0$

1 $x = 0, \quad x = \pm 1$ critical points

1 $f''(x) = -60x^3 + 30x$

1 $f''(0) = 0 \Rightarrow$ The second derivative fails

1 $f''(-1) = +30 > 0 \rightarrow (-1, -2)$ is a local min

1 $f''(1) = -30 < 0 \rightarrow (1, 2)$ is a local max

5 points

b) $g(x) = \sqrt{4 - x^2}$

(1) $g'(x) = \dfrac{-2x}{2\sqrt{4-x^2}} = \dfrac{-x}{\sqrt{4-x^2}} = 0 \Rightarrow x = 0$

(1) ($4 - x^2 = 0 \Rightarrow x^2 = \pm 2, \quad D_{g'} = (-2, 2)$

The extreme points are part of domain, so we do not check)

(2) $g''(x) = \dfrac{-\sqrt{4-x^2} + x \cdot \frac{1}{2}(4-x^2)^{-1/2}(-2x)}{(4-x^2)}$

(1) $g''(0) = \dfrac{-2}{4} < 0 \Rightarrow (0, 2)$ is local max.

4 points

c) $h(x) = \frac{8}{x^2+2}$

$D_h = \mathbb{R}$.

(1) $h'(x) = \dfrac{-16x}{(x^2+2)^2} = 0 \Rightarrow x = 0$ (1)

(1) $h''(x) = \dfrac{-16(x^2+2)^2 + 16x(2(x^2+2))2x}{(x^2+2)^4}$

(1) $h''(0) = \dfrac{-64}{16} = -4 < 0 \rightarrow (0, 4)$ is a local max.

16 points.

QUESTION 2. For the following function find the appropriate information, if it exists, (listed next page) to sketch the graph and sketch the graph.

$$f(x) = \frac{x^2+2}{x^2-4}$$

(1) $D_f = \mathbb{R} - \{-2, 2\}$

(0.5)
x-intercept $\to$ $y \neq 0$
(No x-intercept)

(0.5)
$x = 0 \Rightarrow y = -1/2$ $\quad (0, -1/2)$
y-intercept

(1) $x = \pm 2$ V.As

(1) $\underbrace{\lim_{x \to \infty} f(x) = 1}_{0.5\text{ point}} \Rightarrow \underbrace{y = 1 \quad \text{H.A}}_{0.5\text{ point}}$

(1) $f'(x) = \dfrac{2x(x^2-4) - 2x(x^2+2)}{(x^2-4)^2} = \dfrac{-8x - 4x}{(x^2-4)^2} = \dfrac{-12x}{(x^2-4)^2} = 0$

$D_{f'} = \mathbb{R} - \{\pm 2\}$

$\underbrace{x = 0 \quad , \quad x = \pm 2}_{\text{critical points}}$ (1)

$x = \pm 2$: f' Not defined

(1)

		-2		0		2	
f	↗		↗		↘		↘
f'	+		+		−		−

$(0, -1/2) \to$ local max $\to$ (1)

(1) $f''(x) = \dfrac{-12(x^2-4)^2 + 12x \cdot 2(x^2-4) \cdot 2x}{(x^2-4)^4} = \dfrac{-12(x^2-4)[x^2-4-4x^2]}{(x^2-4)^4}$

$= \dfrac{-12[-3x^2-4]}{(x^2-4)^3} \neq 0$

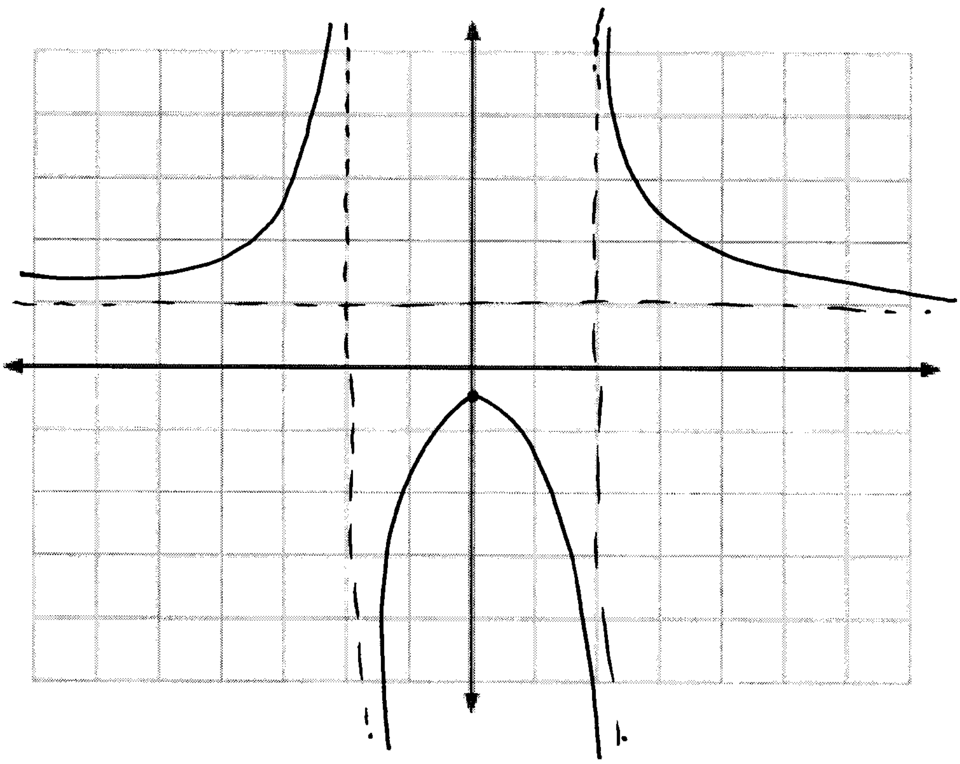
(1) No point of inflection
f'' Not defined @ $x = \pm 2$

(1)

		-2		2	
f	∪		∩		∪
f''	+		−		+

5points for the graph (2 points for asymptotes + local min).

Graph of $f(x)$



a) Find the domain of the function

b) Find the y-intercept and plot it

c) Find the x-intercepts and plot them

d) Find the horizontal asymptotes and plot them

e) Find the vertical asymptotes and plot them

f) Find the critical numbers

g) Find the intervals of increase and decrease

h) Identify the relative extrema and plot them

i) Find the intervals of concave up and concave down

j) Identify the points of inflection and plot them

k) Fill in the rest of the graph using (7) and (9)

15 points.

QUESTION 3. Find the derivative 0f the following functions.

5 points

a) $f(x) = 3\tan(2x) - \cos(3x+2) + \sin^4(x) + \sin(4-x^2)$

$f'(x) = \underbrace{6\sec^2(2x)}_{2} + \underbrace{3\sin(3x+2)}_{1} + \underbrace{4\sin^3 x\cos x}_{1} \underbrace{-2x\cos(4-x^2)}_{1}$

3 points

b) $g(x) = \frac{x^2}{1+\tan x}$

$$g'(x) = \frac{2x(1+\tan x) - x^2\sec^2 x}{(1+\tan x)^2}.$$

3

c) $h(x) = x^3\cos(3x+3)$

$h'(x) = 3x^2\cos(3x+3) - x^3 . 3\sin(3x+3).$

4

d) $l(x) = \frac{\sin(3x)}{4+5\cos(2x)}$

$$l'(x) = \frac{3\cos(3x)(4+5\cos 2x) - \sin(3x)[-10\sin(2x)]}{(4+5\cos(2x))^2}$$

$-10\sin(2x)\sin(3x)$

4 points

QUESTION 4. Find the equation of tangent to the curve $f(x) = \frac{1}{\sin x + \cos x}$ at the point $x = 0$.

Note $\sin 0 = 0$ and $\cos 0 = 1$.

$f(0) = \frac{1}{0+1} = 1$ 1

2 $f'(x) = \frac{-\cos x + \sin x}{(\sin x + \cos x)^2}\Big|_{x=0} = \frac{-1}{1} = -1.$

1 $m = -1$

$(0,1)$

$y - 1 = (-)(x - 0) \Rightarrow y = -x + 1$