MAT 1339, Fall 2013 Assignment 1

Due Sep 27^{th} 11:59 AM.

Late assignments will **NOT** be accepted. An assignment drop-off box is assigned for this course and is located at Math department. (KED 585)

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Student Name ______________________ Student Number ______________________

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Total 50 points

Signature ______________________

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QUESTION 1. Let $f(x) = \dfrac{1}{x-1}$.

a) Determine the average rate of change of f over the interval $[3, 4]$.

(2)

$$\frac{f(4) - f(3)}{4 - 3} = \frac{\frac{1}{4-1} - \frac{1}{3-1}}{1} = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$

b) Determine the difference equation at the point $x = 3$ and simplify it.

(3)

$$\frac{f(a+h) - f(a)}{h} = \frac{f(3+h) - f(3)}{h} = \frac{\frac{1}{3+h-1} - \frac{1}{3-1}}{h} = \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$$

$$= \frac{\frac{2-2-h}{(2+h)(2)}}{h} = \frac{-h}{h(2+h)2} = \frac{-1}{2(2+h)}$$

c) Estimate the slope of the tangent line at the point $x = 3$ with $h = 1/10$.

(2)

$$h = 1/10 \implies \frac{-1}{2(2+0.1)} = \frac{-1}{2(2.1)} = \frac{-1}{4.2}$$

25 total

QUESTION 2. Evaluate the limits:

3 points
1 for domain
2 " rest

a) $\lim_{x \to 3} \frac{x-3}{x+3} = \frac{3-3}{6} = 0.$

$D_f = \mathbb{R} - \{-3\}$

or

$= \{x \in \mathbb{R} \mid x \neq -3\}$

$x = 3$ is in the D_f

$\Rightarrow$ you only need to plug in $x = 3$

4 points
1 for domain
3 " rest

b) $\lim_{x \to -2} \frac{4x^2 + 2x - 12}{x+2} = \lim_{x \mapsto -2} \frac{(x+2)(4x-6)}{x+2}$ $\quad x \neq -2$

$D_f = \mathbb{R} - \{-2\}$

$= \lim_{x \mapsto -2} (4x - 6) = -8 - 6 = -14$

4 points
(the same as b)

c) $\lim_{x \to 0} \frac{3 - \sqrt{x+9}}{x}$ $\xrightarrow{\text{Rationalize}}$ $\lim_{x \mapsto 0} \frac{3 - \sqrt{x+9}}{x} \times \frac{3 + \sqrt{x+9}}{3 + \sqrt{x+9}}$

$x \neq 0$ + $x + 9 \geq 0$, $x \geq -9$

$D_f = \{x \in \mathbb{R} \mid x \geq -9, x \neq 0\}$

or $[-9, \infty) - \{0\}$

or $[-9, 0) \cup (0, \infty)$

$= \lim_{x \mapsto 0} \frac{9 - (x+9)}{x(3 + \sqrt{x+9})}$

$= \lim_{x \mapsto 0} \frac{-x}{\cancel{x}(3 + \sqrt{x+9})} = \lim_{x \mapsto 0} \frac{-1}{3 + \sqrt{x+9}}$

$= \frac{-1}{6}.$

4 points
(same as b)

d) $\lim_{x \to 4} \frac{2 - \sqrt{x}}{4 - x} \times \frac{2 + \sqrt{x}}{2 + \sqrt{x}} =$ $\quad x \neq 4$

$D_f = \mathbb{R} - \{4\}$

$\lim_{x \mapsto 4} \frac{4 - x}{(4-x)(2 + \sqrt{x})}$

$= \lim_{x \mapsto 4} \frac{1}{2 + \sqrt{x}} = \frac{1}{4}$)

2 points

e) $\lim_{x \to +\infty} \frac{9x^2 + 3x + 1}{-2x^2 + 5} = \frac{9}{-2} = -\frac{9}{2}.$

$n = 2$
$m = 2$ $\Rightarrow n = m$

2 points

f) $\lim_{x \to +\infty} \frac{10000}{x+1} =$ 0

$n = 0$
$m = 1$

6 points

g) $\lim_{x \to -4} \frac{|x+4|}{x^2 - 16} =$

$$|x+4| = \begin{cases} x+4 & x \geq -4 \\ -(x+4) & x < -4 \end{cases}$$

1 $D_f = \mathbb{R} - \{-4, 4\}$ or $\{x \in \mathbb{R} \mid x \neq \pm 4\}$ or $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$.

2 $$\lim_{x \to -4^+} \frac{|x+4|}{x^2-16} = \lim_{x \mapsto -4^+} \frac{(x+4)}{(x+4)(x-4)} = \lim_{x \mapsto -4} \frac{1}{x-4} = \frac{-1}{8}.$$

2 $$\lim_{x \mapsto -4^-} \frac{|x+4|}{x^2-16} = \lim_{x \mapsto -4^-} \frac{-(x+4)}{(x+4)(x-4)} = \lim_{x \mapsto -4} \frac{-1}{(x-4)}$$

$$= \frac{-1}{-8} = 1/8$$

1 $\lim_{x \mapsto 4^-} f(x) \neq \lim_{x \mapsto 4^+} f(x) \Rightarrow$ The limit does NOT Exist!

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QUESTION 3. Find the values of a and b in such a way that f is a continuous function.

$$f(x) = \begin{cases} a\sqrt{2-x} & \text{if } x \le -2, \\ 3x+a & \text{if } -2 < x < 1, \\ (x+b)^2 & \text{if } x \ge 1. \end{cases}$$

Piecewise function. : We have to check @ boundry points $(-2, 1)$.

4

$\lim_{x \mapsto -2^-} f(x) = a\sqrt{4} = 2a$

2

$\lim_{x \mapsto -2^+} f(x) = -6 + a$

$\Rightarrow 2a = a - 6$

$a = -6$

2

4

$\lim_{x \mapsto 1^-} f(x) = 3 + a = 3 - 6 = -3$

2

$\lim_{x \mapsto 1^+} f(x) = (1+b)^2$

$\Rightarrow (1+b)^2 = -3$

$\rightarrow \leftarrow$

impossible

$\Downarrow$

This function is not continuous @ $x = 1$ for any value of b!

2

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QUESTION 4. Find the set of points that the following function is continuous at them.

$$f(x) = \begin{cases} \dfrac{2x}{x+3} & \text{if } x < 4, \\ \\ \dfrac{\sqrt{x-4}+2}{x^2+4x-12} & \text{if } x \geq 4 \end{cases}$$

$\Rightarrow x+3 \neq 0$ (1)
$x \neq -3$
$(-3<4)$
$\rightarrow f(x)$ Not Continuous @ $x=-3$

$\hookrightarrow x^2+4x-12 \neq 0 \quad (x-2)(x+6) \neq 0$ (2)
$x \neq 2$
$x \neq -6$
$2,-6$ are both less than $4 \rightarrow$ so f is cont every where for $x \geq 4$

@ $x=4$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4} \frac{2x}{x+3} = \frac{8}{7}$$

(2)

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4} \frac{\sqrt{x-4}+2}{x^2+4x-12} =$$

$$\frac{2}{16+16-12} = \frac{2}{20} = \frac{1}{10}$$

(2) $\lim_{x \to 4^-} f(x) \neq \lim_{x \to 4^+} f(x) \rightarrow f(x)$ is Not continuous @ $x=4$.

(2) f is continuous everywhere except $\{-3,4\}$

or $\mathbb{R} - \{-3,4\}$

on $(-\infty,-3) \cup (-3,4) \cup (4,\infty)$