Mat 2377 S12
Midterm exam sol.

Student # ____________________

Only answers recorded in the following table will be marked.

Question	Answer
1	A
2	C
3	B
4	D
5	A
6	D
7	B
8	B
9	C
10	D
11	A
12	D

Midterm Solutions

Q1

$X = 0, 1, 2, 3$

$$\sum_{k=0}^{3} f(k) = 1 \Rightarrow \quad c(0.9 + 0.7 + 0.5 + 0.3) = 1 \Rightarrow 2.4C = 1$$

$$C = \frac{1}{2.4} = 0.417$$

$$\mu = \sum_{k=0}^{3} k f(k) = 0.417\,(0 + 0.7 + 2 \times 0.5 + 3 \times 0.3) = 1.0842$$

(A)

Q2 Let X be the # of defective item. $X \sim B(3, 0.1)$

$$P(X \leq 1) = P(X=0) + P(X=1) = \binom{3}{0}(0.1)^0(0.9)^3 + \binom{3}{1}(0.1)^1(0.9)^2$$

$$= 0.729 + 0.243 = 0.972$$

(C)

Q3 X_n is a Poisson process with rate $(5n)$

Find n s.t. $P(X_n \geq 1) > 0.9 \Rightarrow P(X_n = 0) < 0.1$

$$\Rightarrow e^{-5n}\frac{(5n)^0}{0!} < 0.1 \Rightarrow -5n < \ln 0.1$$

$$\Rightarrow n > \frac{\ln 0.1}{-5} = 0.46 \text{ hr} = 27.6 \text{ min.}$$

B

Q4 $P(\text{top operates}) = P(\text{top left operates and top right op.})$

$= P(\text{top left op.})\,P(\text{top right op.}) = 0.95 \times 0.9 = 0.855$

$P(\text{Circuit operates}) = P(\text{top op. or bottom op.})$

$= P(\text{top}) + P(\text{bottom}) - P(\text{top})P(\text{bottom})$

$= 0.855 + 0.7 - (0.855)(0.7) = 0.9565$

D

Q5 D: diabetes D2: diabetes type 2 O: overweigh

Given

$$P(D) = \frac{3.3}{33} = 0.1, \quad P(D2|D) = 0.9, \quad P(O|D2) = 0.9$$

Therefore $P(D2 \cap O) = P(O|D2)P(D2) = 0.9 \times 0.09 = 0.081$

$= 8.1\%$

because

$$P(D2) = P(D2|D)P(D) + P(D2|D^c)P(D^c)$$

$$= (0.9)(0.1) + 0 = 0.09.$$

Another way, $(3.3)(0.9) = 2.97$ million have diabetes type 2

and $P(D2) = \frac{2.97}{33} = 0.09.$

A

Q6 Events: M: infected by Malware

I: information lost.

$P(M)=0.17$, $P(I|M)=0.86$, $P(I|M^c)=0.15$

Determine $P(M|I)$. Using Bayes' formula

$$P(M|I)=\frac{P(I|M)P(M)}{P(I|M)P(M)+P(I|M^c)P(M^c)}=$$

$$\frac{(0.86)(0.17)}{(0.86)(0.17)+(0.15)(0.83)}=\frac{0.1462}{0.2707}=0.54$$

D

Q7 Events: M: prefer MC exam

G: intend to pursue G.S.

$P(M\cap G)=\frac{66}{119}=0.55$ $P(M\cap G^c)=\frac{10}{119}=0.08$

$P(M^c\cap G)=0.24$ $P(M^c\cap G^c)=0.13$

B

First, $P(G)=P(M\cap G)+P(M^c\cap G)=0.79$, then $P(M|G)=\frac{P(M\cap G)}{P(G)}=\frac{0.55}{0.79}\approx 0.7$

Alternative solution by sample reduction

Q8 $P(G) = \frac{94}{119} = 0.79$

$P(M) = \frac{76}{119} = 0.64$ $\qquad P(M \cap G) = \frac{66}{119} = 0.64$

$P(M \cap G) \neq P(G)P(M) = 0.51 \implies$ M & G are dependent

B

Q9 Let X be the number of packets received in error for one email

$X \sim B(10, 0.1)$

$P(\text{email is corrupted}) = P(X \geq 1) = 1 - P(X = 0) = 1 - (0.9)^{10}$

$= 0.651$

Let Y be the number of emails until the first corrupted one. Y is geometric with $p = 0.651$

$E(Y) = \frac{1}{0.651} \approx 1.54$

C

Q10 X: number of patients in a day

$X \sim \text{Poisson}(7)$

$$P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= e^{-7}\left(\frac{7^0}{0!} + \frac{7^1}{1!} + \frac{7^2}{2!} + \frac{7^3}{3!}\right) = 0.082$$

$E(X) = 7$

D

Q11

a) p(at least one fuse is not acceptable) =

1 − p(all fuses are acceptable) =

$$1 - \frac{\binom{20}{6}}{\binom{30}{6}} = 1 - \frac{38760}{593775} \approx 0.935$$

b) Prob. of defective fuse in each draw is $\frac{10}{30} = \frac{1}{3}$

p(at least one defective) = 1 − p(all are acceptable) =

$$1 - \left(\frac{2}{3}\right)^6 = 0.912$$

A

Q12

$$P(1 < X \leq 3) = \int_1^3 \frac{x}{12.5}\,dx = \frac{1}{25}\,x^2\Big]_1^3 = \frac{1}{25}\,(9-1) = \frac{8}{25}.$$

D