MATH 209/2 all sections except EC: - Fundamental Mathematics II

Midterm - Sunday, October 20, 2013, 2pm (1h30min)
Only approved calculators are permitted.

MARKS

[7] **1.** (a) Find $\lim_{x\to -2} \dfrac{x^2-x-6}{x^2+x-2}$.

[7] (b) Give examples of functions $g(x)$ and $h(x)$ with the following properties:

(i) $\lim_{x\to 5} g(x) = 0$ (ii) $\lim_{x\to 5} h(x) = 0$ (iii) $\lim_{x\to 5} \dfrac{h(x)}{g(x)} = 0$

[7] **2.** Let $h(x) = 7 - x^3$. Work out the following in detail:

$$\lim_{t\to 0} \frac{h(x+t)-h(x)}{t}$$

[12] **3.** (a) If $f(x) = 3\sqrt[4]{x^3} - \frac{1}{x^6}$, find $f'(1)$. You need not simplify.

(b) If $g(x) = [2 + ln(x^2)][3x^3 - 5]$, find $g'(2)$. You need not simplify.

(c) Find $h'(x)$ if $h(x) = \frac{x^3-3}{x^2+7e^x}$. You need not simplify.

(d) Find the value of dy if $y = x^3 - 2$, $x = 3$, and the change in x is 0.1.

[7] **4.** A stock portfolio grows from ten thousand dollars to forty thousand dollars in eight years. Find the associated annual rate of growth assuming that it is compounded continuously.

[10] **5.** Consider the cost function for the production of headphone sets $C(x) = 7,000 + 2x$.

(a) Find $\overline{C}(x)$ and $\overline{C}'(x)$.

(b) Find $\overline{C}(100)$ and $\overline{C}'(100)$, and interpret these quantities.

(c) Use the results in part (B) to estimate the average cost per headphone set at a production level of 101 headphone sets.

[10] **6.** Find x' for the function $x(t)$ defined implicitly below. Evaluate x' at the indicated point.

$$x^2 - tx^2 - 16 = 0;\ (-3, -2).$$

[10] **7.** Helium is pumped into a spherical balloon at a constant rate of 4 cubic feet per second. How fast is the radius increasing after 1 minute?

4. $A = Pe^{rt}$, $A = \$40000$, $P = \$10000$, $t = 8$, $r = ?$

$$\frac{A}{P} = e^{rt} \longrightarrow \ln\frac{A}{P} = rt \longrightarrow \boxed{\frac{1}{t}\ln\frac{A}{P} = r}$$

$$r = \frac{\ln 4}{8} = 0.1732 \quad \text{or} \quad \boxed{17.32\%}$$

5. a) $C(x) = 7000 + 2x$

$$\bar{C}(x) = \frac{7000 + 2x}{x} = \frac{7000}{x} + 2.$$

$$\bar{C}'(x) = -\frac{7000}{x^2}$$

b) $\bar{C}(100) = \frac{7000}{100} + 2 = 72$, $\bar{C}'(100) = -\frac{7000}{(100)^2} = -0.7$.

c) $\bar{C}(101) = \bar{C}(100) + \bar{C}'(100) = 72 - 0.7 = \boxed{\$71.03}$

6. $x^2 - tx^2 - 16 = 0$ $\quad (-3, -2)$.

$$2xx' - (x^2 + 2tx x') = 0$$

$$2xx' - x^2 - 2txx' = 0$$

$$x'(2x - 2xt) = x^2$$

$$x' = \frac{x^{\cancel{2}}}{2\cancel{x}(1-t)} = \frac{x}{2(1-t)}$$

$$x'\Big|_{(-3,-2)} = \frac{-3}{2(1+2)} = \frac{-3}{6} = \boxed{-\frac{1}{2}}$$

7. $V = \frac{4}{3}\pi r^3$, $V(t)$, $r(t)$, $r(t) = ?$

(1) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$, $\frac{dV}{dt} = 4$, $t = 60$, $V = 4 \times 60$

$\boxed{V = 240}$

(2) $V = \frac{4}{3}\pi r^3 \rightarrow r^3 = \frac{3}{4\pi} V = \frac{3 \times \overset{60}{\cancel{240}}}{\underset{1}{\cancel{4\pi}}} = \frac{180}{\pi}$

$$r = \sqrt[3]{\frac{180}{\pi}} = \boxed{3.85}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \rightarrow \cancel{4} = \cancel{4}\pi (3.85)^2 \frac{dr}{dt} \rightarrow 1 = \pi (3.85)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{\pi (3.85)^2} = \boxed{0.0215}$$

1. a) $\lim_{x \to -2} \frac{x^2 - x - 6}{x^2 + x - 2} = \lim_{x \to -2} \frac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)} = \lim_{x \to -2} \frac{x-3}{x-1} = \frac{-5}{-3} = \boxed{\frac{5}{3}}$

b) $g(x) = x - 5$, $h(x) = (x-5)^2$.