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## MAT 1339, Fall 2013 Assignment 2

Due Oct 25<sup>th</sup> 11:59 AM.

Late assignments will **NOT** be accepted. An assignment drop-off box is assigned for this course and is located at Math department. (KED 585)

Professor: Termeh Kousha

Student Name \_\_\_\_\_ Student Number \_\_\_\_\_

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Signature \_\_\_\_\_

12 QUESTION 1. Find the first and second derivative of the following functions.

⑥ a)  $f(x) = \frac{-4x}{x^2-4}$

③  $f'(x) = \frac{-4(x^2-4) - (-4x)(2x)}{(x^2-4)^2} = \frac{-4x^2+16+8x^2}{(x^2-4)^2} = \frac{4x^2+16}{(x^2-4)^2}$

③  $f''(x) = \frac{8x(x^2-4)^2 - (4x^2+16)(2(x^2-4)2x)}{(x^2-4)^4}$

6 b)  $g(x) = \sqrt{x^2-1} + \frac{2}{x} + \pi$

③  $g'(x) = \frac{2x \cdot (x^2-1)^{-1/2}}{2} - \frac{2}{x^2} = \frac{x}{\sqrt{x^2-1}} - \frac{2}{x^2}$

③  $g''(x) = \frac{\sqrt{x^2-1} - x \cdot \frac{1}{2} \cdot 2x (x^2-1)^{-3/2}}{(x^2-1)} + \frac{4}{x^3}$

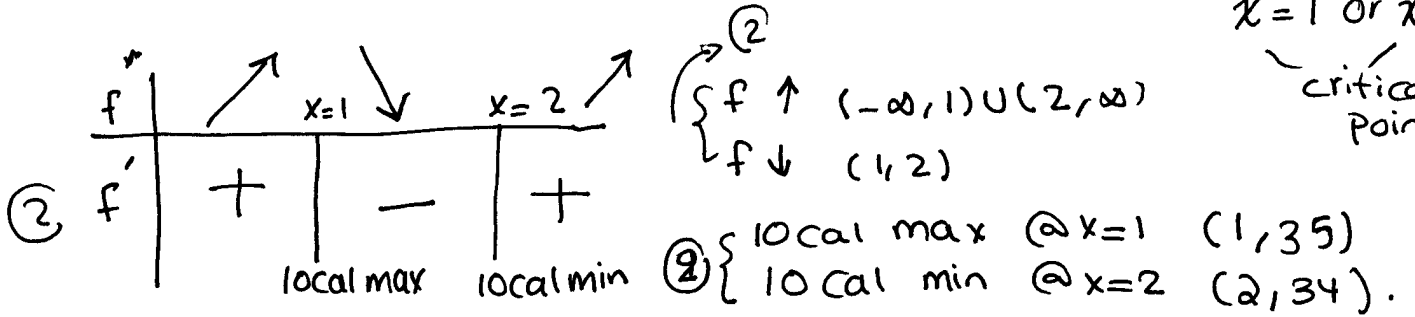
or

$$= \frac{\sqrt{x^2-1} - \frac{x^2}{\sqrt{x^2-1}}}{x^2-1} + \frac{4}{x^3}$$

18 QUESTION 2. For the following functions find the intervals on which the function is increasing or decreasing. Moreover, find all local (relative) extremas of the function if exists.

8 a)  $f(x) = 2x^3 - 9x^2 + 12x + 30$   $D_f = \mathbb{R}$ .

(2)  $f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 0 \Rightarrow (x-1)(x-2) = 0$   
 $x = 1$  or  $x = 2$   
 critical points.

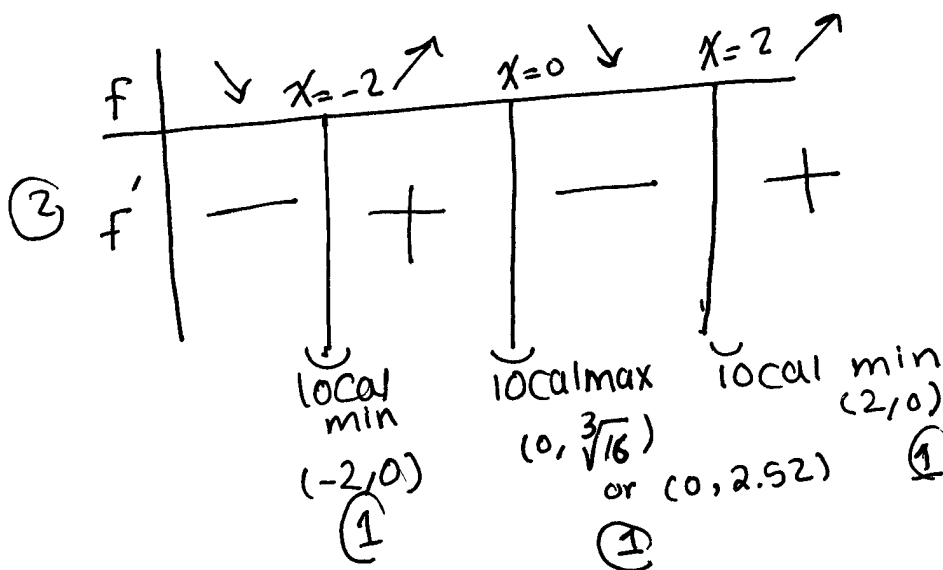


10 b)  $f(x) = (x^2 - 4)^{2/3}$

(1)  $f'(x) = \frac{2}{3}(x^2 - 4)^{-1/3} \cdot 2x = \frac{4x}{3(x^2 - 4)^{1/3}} = 0$

$\Rightarrow x=0$  (1)  $D_{f'} = \mathbb{R} - \{-2, 2\}$   
 $f'$  not defined @  $x = -2, 2$

(1)  $x = 0, -2, 2$  critical points



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QUESTION 3. Find the absolute maximum and minimum on the indicated interval of the following functions.

a)  $f(x) = 2x^3 - 12x^2, \quad x \in [-1, 2].$

(4)

$$f'(x) = 6x^2 - 24x$$

$$= 6x(x-4) = 0$$

$x=0$  ✓,  $x=4$  NOT in  $[-1, 2]$

$$f(-1) = -2 - 12 = -14$$

$$f(0) = 0$$

$$f(2) = 16 - 48 = -32$$

(-1) point if they check  $f(4)$

Answer : abs. min. = -32 at  $x = 2$

abs. max. = 0 at  $x = 0$

b)  $f(t) = \frac{t^2}{t^2+3}, \quad t \in [-1, 1].$

(4)

$$f'(t) = \frac{2t(t^2+3) - t^2(2t)}{(t^2+3)^2} = \frac{2t^3 + 6t - 2t^3}{(t^2+3)^2} = \frac{6t}{(t^2+3)^2} = 0$$

$\Rightarrow t=0 \in [-1, 1]$

$$f(-1) = \frac{1}{4}$$

$$f(1) = \frac{1}{4}$$

$$f(0) = 0$$

Answer : abs. min. = 0 at  $t = 0$

abs. max. =  $\frac{1}{4}$  at  $t = \pm 1$

QUESTION 4. Determine the intervals on which the graph of the following functions is concave upward or concave downward. Find all points of inflections if any.

a)  $f(x) = \frac{4}{x^2+3} = 4(x^2+3)^{-1}$

(1)  $f'(x) = -4(x^2+3)^{-2} \cdot 2x = -8x(x^2+3)^{-2}$

(2)  $f''(x) = -8(x^2+3)^{-2} - 8x(-2)2x(x^2+3)^{-3}$

$$= \frac{-8}{(x^2+3)^2} + \frac{32x^2}{(x^2+3)^3} = \frac{-8(x^2+3) + 32x^2}{(x^2+3)^3} = \frac{24x^2 - 24}{(x^2+3)^3}$$

$$= 0 \Rightarrow 24(x^2 - 1) = 0 \quad x = \pm 1, \quad D_{f''} = \mathbb{R}$$

(2)	$f$	$\cup$	$-$	$\cap$	$\cup$	
	$f''$	$+$	$-$	$+$		(2) $f \cup (-\infty, -1) \cup (1, \infty)$ $f \cap (-1, 1)$
			$(-1, 1)$		$(1, 1)$	
				P.O.I		

b)  $g(x) = x(6-x)^2$

$$g'(x) = (6-x)^2 + x \cdot 2(6-x)(-1)$$

$$= (6-x)(6-x-2x) = (6-x)(6-3x)$$

$$g''(x) = (-1)(6-3x) + (-3)(6-x)$$

$$= -6 + 3x - 18 + 3x = -24 + 6x$$

$$= 0 \Rightarrow x = 4$$

$$D_{g''} = \mathbb{R}$$

$g$	$\cap$	$x=4$	$\cup$
$g''$	$-$	$+$	
		$(4, 16)$	P.O.I

$$g \cup \text{ on } (4, \infty)$$

$$g \cap \text{ on } (-\infty, 4)$$