Solutions.

MAT 1339, Fall 2013 Assignment 1

Due Sep 27th11:59 AM.

Late assignments will **NOT** be accepted. An assignment drop-off box is assigned for this course and is located at Math department. (KED 585)

Student Name ______________________ Student Number ______________________

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ______________________

Total 50 points

7 points QUESTION 1. Let $f(x) = \dfrac{1}{x-1}$.

a) Determine the average rate of change of f over the interval $[3, 4]$.

2 points

$$\frac{f(4)-f(3)}{4-3} = \frac{\frac{1}{4-1} - \frac{1}{3-1}}{1} = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$

b) Determine the difference equation at the point $x = 3$ and simplify it.

3 points

$$\frac{\Delta f}{\Delta x}(3) = \frac{f(3+h) - f(3)}{h} = \frac{\frac{1}{3+h-1} - \frac{1}{3-1}}{h}$$

$$= \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \frac{2-2-h}{2h(2+h)} = \frac{-h}{2h(2+h)} = \frac{-1}{2(2+h)}$$

c) Estimate the slope of the tangent line at the point $x = 3$ with $h = 1/10$.

2 points

$$h = \frac{1}{10} \Rightarrow \frac{-1}{2(2+0.1)} = \frac{-1}{2(2.1)} = -\frac{1}{4.2}$$

25 total

QUESTION 2. Evaluate the limits:

3 points

a) $\lim_{x\to 3} \frac{x-3}{x+3} = \frac{3-3}{6} = 0$

4 points

b) $\lim_{x\to -2} \frac{4x^2+2x-12}{x+2} = \frac{0}{0}$ indeterminate

$= \lim_{x\to -2} \frac{\cancel{(x+2)}(4x-6)}{\cancel{(x+2)}} = \lim_{x\to -2} 4x-6 = -8-6=-14$

4 points

c) $\lim_{x\to 0} \frac{3-\sqrt{x+9}}{x} = \frac{3-\sqrt{0+9}}{0} = \frac{0}{0}$ indeterminate

$= \lim_{x\to 0} \frac{3-\sqrt{x+9}}{x} \cdot \frac{3+\sqrt{x+9}}{3+\sqrt{x+9}} = \lim_{x\to 0} \frac{9-(x+9)}{x(3+\sqrt{x+9})} = \lim_{x\to 0} \frac{-\cancel{x}}{\cancel{x}(3+\sqrt{x+9})}$

$= \lim_{x\to 0} \frac{-1}{3+\sqrt{x+9}} = \frac{-1}{3+\sqrt{0+9}} = \frac{-1}{3+3} = -\frac{1}{6}$

4 points

d) $\lim_{x\to 4} \frac{2-\sqrt{x}}{4-x} = \frac{0}{0}$ indeterminate

$= \lim_{x\to 4} \frac{2-\sqrt{x}}{4-x} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} = \lim_{x\to 4} \frac{\cancel{(4-x)}}{\cancel{(4-x)}(2+\sqrt{x})} = \lim_{x\to 4} \frac{1}{2+\sqrt{x}} = \frac{1}{2+2} = \frac{1}{4}$

2 points

e) $\lim_{x\to +\infty} \frac{9x^2+3x+1}{-2x^2+5} = \lim_{x\to +\infty} \frac{9\cancel{x^2}}{-2\cancel{x^2}} = -\frac{9}{2}$

2 points f) $\lim_{x\to+\infty} \frac{10000}{x+1} = \lim_{x\to+\infty} \frac{10000}{x} = 0$

6 points g) $\lim_{x\to-4} \frac{|x+4|}{x^2-16} =$

$$|x+4| = \begin{cases} x+4 & x+4 \geq 0 \\ -(x+4) & x+4<0 \end{cases} = \begin{cases} x+4 & x\geq -4 \\ -(x+4) & x<-4 \end{cases}$$

$$\Rightarrow \quad f(x) = \begin{cases} \frac{x+4}{x^2-16} & x \geq -4 \\ \frac{-(x+4)}{x^2-16} & x<-4 \end{cases}$$

2 points $\lim_{x\to-4^-} f(x) = \lim_{x\to-4^-} \frac{x+4}{x^2-16} = \frac{0}{0} = \lim_{x\to-4} \frac{(\cancel{x+4})}{(x-4)(\cancel{x+4})} = \lim_{x\to-4} \frac{1}{x-4}$

$= \frac{1}{-4-4} = \frac{-1}{8}$ 1

2 points $\lim_{x\to-4^+} f(x) = \lim_{x\to-4^+} \frac{-(x+4)}{(x^2-16)} = \lim_{x\to-4^+} \frac{-(\cancel{x+4})}{(x-4)(\cancel{x+4})} = \lim_{x\to-4^+} \frac{-1}{x-4}$

$= \frac{-1}{-4-4} = +\frac{1}{8}$ 2

2 points 2 $\neq$ 1 $\Rightarrow$ $\lim_{x\to-4} f(x)$ does not exist.

8 points

QUESTION 3. Find the values of a and b in such a way that f is a continuous function.

$$f(x) = \begin{cases} a\sqrt{2-x} & \text{if } x \leq -2, \\ 3x & \text{if } -2 < x < 1, \\ (x+b)^2 & \text{if } x \geq 1. \end{cases}$$

$\mathrm{Dom}(f) = \mathbb{R}$

we have to check the boundry points for continuity.

4 points

$$\left\{\begin{aligned} &\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} a\sqrt{2-x} = a\sqrt{4} = 2a \\ &\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} 3x = -6 \end{aligned}\right\} \Rightarrow 2a = -6 \Rightarrow \boxed{a = -3}$$

4 points

$$\left\{\begin{aligned} &\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 3x = 3 \\ &\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x+b)^2 = (1+b)^2 \end{aligned}\right\} \Rightarrow (1+b)^2 = 3 \Rightarrow 1+b = \pm\sqrt{3} \Rightarrow \boxed{b = \pm\sqrt{3} - 1}$$

10 points

QUESTION 4. Find the set of points that the following function is continuous at them.

$x+3 \neq 0$

$x \neq -3$

$-3 < 4$

$$f(x) = \begin{cases} \dfrac{2x}{x+3} & \text{if } x < 4, \\ \dfrac{\sqrt{x-4}+2}{x^2+4x-12} & \text{if } x \geq 4 \end{cases}$$

$x^2+4x-12 \neq 0$

$\Downarrow$

$x = 2, -6$

but both 2 and -6 are less Than 4.

2 points: $\mathrm{Dom}(f) = \mathbb{R} \setminus \{-3\}$

Now we check The boundary point for Continuity.

4 points

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{2x}{x+3} = \frac{8}{4+3} = \frac{8}{7}$$

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} \frac{\sqrt{x-4}+2}{x^2+4x-12} = \frac{0+2}{16+16-12} = \frac{2}{20} = \frac{1}{10}$$

2 points: $\Rightarrow$ $\lim f(x)$ does not exists at $x=4$.

and $x=4$ is a dis Continuity point.

set of Continuity points $= \mathbb{R} \setminus \{-3, 4\}$.

2points.

$= \mathrm{Dom}(f) \setminus \{4\}$.