Solutions

Part A (50 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET. FOR SAFETY, ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

(2 marks) A1. The points $(0, 2)$ and $(1, -1)$ are on the graph of $y = a + b \cdot 2^x$. Find the values of a and b.

Because the points $(0,2)$ and $(1,-1)$ are on the graph, then they satisfy the equation $y = a + b \cdot 2^x$

$(0,2) \Rightarrow 2 = a + b \cdot 2^0 \Rightarrow a + b = 2$ $\quad a = 2 - b = 5$

$(1,-1) \Rightarrow -1 = a + b \cdot 2^1 \quad a + 2b = -1$

$-b = 3 \Rightarrow b = -3$

A: $a = 0, b = 2$	B: $a = 5, b = 3$	C: $a = 3, b = 5$	D: $a = 2, b = -1$	E: $a = 5, b = -3$

(2 marks) A2. Evaluate $\ln\left(e^{\sqrt{2}\ln 2}\right)$.

$\ln\left(e^{\sqrt{2}\ln 2}\right) = \sqrt{2}\ln 2$ (using the cancellation equation)

A: $\ln 2$	B: $\frac{\sqrt{2}}{2}$	C: $\sqrt{2}\ln 2$	D: $2^{\sqrt{2}}$	E: 2

(2 marks) A3. Solve for x : $\ln(2\ln x) = 1$.

$e^{\ln(2\ln x)} = e^1 \Rightarrow 2\ln x = e$, or $\ln x = \frac{e}{2}$

$e^{\ln x} = e^{e/2} \Rightarrow x = e^{e/2} = \sqrt{e^e}$

A: e^e	B: e^{2e}	C: $2e^e$	D: $\sqrt{e^e}$	E: $e^{e/\ln 2}$

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(2 marks) A4. Evaluate $\dfrac{\log_4 2 + \log_4 8}{\log_2 4}$. $=$

$$= \frac{\log_4(2\cdot 8)}{\log_2 4} = \frac{\log_4 4^2}{\log_2 2^2} = \frac{2}{2} = 1$$

A: 2	B: 1	C: 4	D: $\log_4 2$	E: $\log_2 4$

(2 marks) A5. Find the inverse function $f^{-1}(x)$ for $y = e^{2x} + 1$.

1) Solve for x: $e^{2x} = y-1$, $\ln(e^{2x}) = \ln(y-1)$,
$2x = \ln(y-1)$, $x = \frac{1}{2}\ln(y-1)$.

2) $y \leftrightarrow x \Rightarrow y = \frac{1}{2}\ln(x-1)$.

A: $y = \frac{1}{2}\ln(x-1)$	B: $y = \frac{1}{2}\ln x - 1$	C: $y = \ln(e^{2x}+1)$
D: $y = \ln\left(\frac{1}{2}(x-1)\right)$	E: $y = \frac{1}{2}\ln x + 1$	

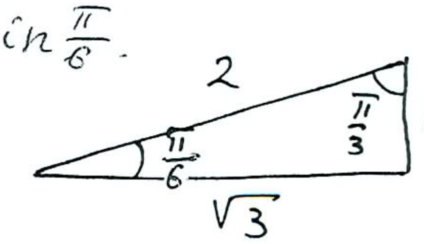
(2 marks) A6. Find the exact value of $\sin^{-1}\left(\cos\frac{\pi}{3}\right)$.

$\cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right)$ or $\sin\theta = \frac{1}{2} \Rightarrow$

$\theta = \frac{\pi}{6}$ (it is in $[-\frac{\pi}{2}, \frac{\pi}{2}]$).

Or from the triangle: $\cos\frac{\pi}{3} = \frac{1}{2} = \sin\frac{\pi}{6}$.

A: $\frac{\pi}{6}$	B: $\frac{\pi}{3}$	C: $-\frac{\pi}{3}$	D: $\frac{2\pi}{3}$	E: $-\frac{5\pi}{6}$



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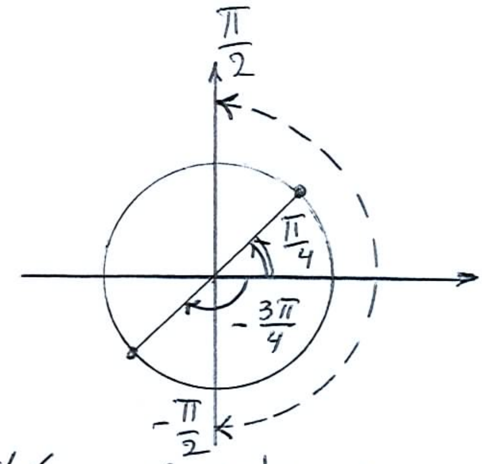
(2 marks) A7. Find the exact value of $\tan^{-1}\left(\tan\left(-\frac{3\pi}{4}\right)\right)$.



$-\frac{3\pi}{4}$ is not in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so we cannot use the cancellation equation, But $\tan\left(-\frac{3\pi}{4}\right)=1$, and also $\tan\left(\frac{\pi}{4}\right)=1$. $\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow$

$\tan^{-1}\left(\tan\left(-\frac{3\pi}{4}\right)\right) = \frac{\pi}{4}$

A: $-\frac{3\pi}{4}$	B: $\frac{\pi}{4}$	C: $-\frac{\pi}{4}$	D: $\frac{3\pi}{4}$	E: 0

(2 marks) A8. Determine $\lim_{x \to 3^-} \frac{3x}{x-3}$.

$x \to 3^- \Rightarrow x-3<0$ and also $3x>0$ when $x \to 3^-$

So $\lim_{x \to 3^-} \frac{3x}{x-3} = -\infty$

A: 9	B: 3	C: 0	D: ∞	E: $-\infty$

(2 marks) A9. Determine $\lim_{x \to 0} \frac{x^2-4}{|2-x|}$.

$|2-x|$ is continuous when x is close to 0 (it has discontinuity only when $x=2$). We do not need to check left-hand and right-hand limits.

$\lim_{x \to 0} \frac{x^2-4}{|2-x|} = \lim_{x \to 0} \frac{(x-2)(x+2)}{(2-x)} = \lim_{x \to 0}[-(x+2)] = -2$

A: -4	B: 4	C: 2	D: -2	E: does not exist

($|2-x| = 2-x$ when x is close to 0)

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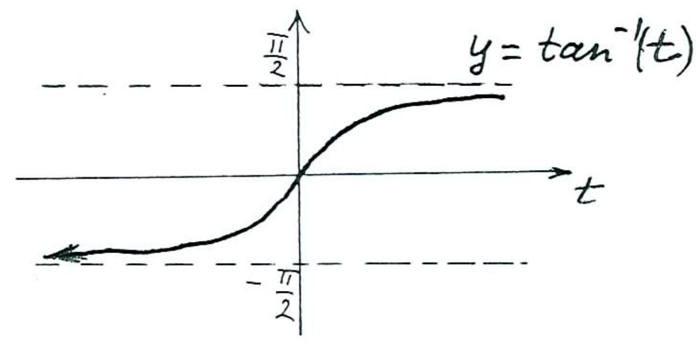
(2 marks) A10. Determine $\lim_{x\to-\infty} \arctan\left(x^3 - 1\right)$.

Let $t = x^3 - 1$. When $x \to -\infty$: $t \to -\infty$.

$\lim_{t\to-\infty} \arctan t = -\frac{\pi}{2}$

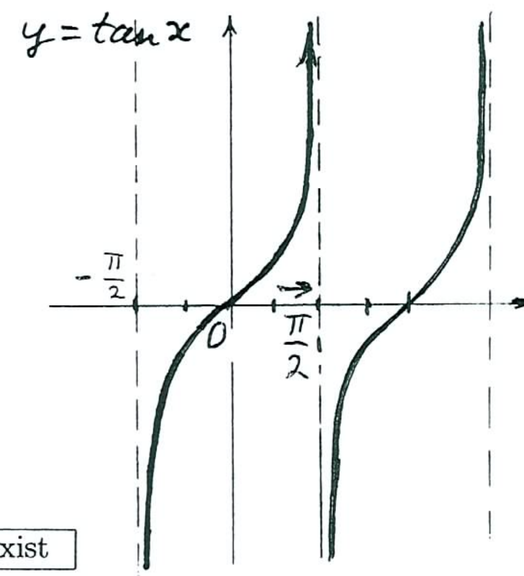


A: $-\infty$	B: ∞	C: $-\frac{\pi}{2}$	D: $\frac{\pi}{2}$	E: 0

(2 marks) A11. Determine $\lim_{x\to\frac{\pi}{2}^-} e^{\tan x}$.

When $x \to \frac{\pi}{2}^-$: $\tan x \to +\infty$ and $e^{\tan x} \to \infty$, so

$\lim_{x\to\frac{\pi}{2}^-} e^{\tan x} = \infty$



A: $-\infty$	B: ∞	C: e	D: 0	E: does not exist

(2 marks) A12. Determine $\lim_{x\to 0} \ln\left(\sec x\right)$.

$\sec(0) = 1$ and $\ln(1) = 0$,

so $\lim_{x\to 0} \ln(\sec x) = 0$

A: $-\infty$	B: ∞	C: 1	D: 0	E: does not exist

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(2 marks) A13. Determine $\lim\limits_{x \to -\infty} \dfrac{\sqrt{4x^2+1}}{2-x}$. $=$ $\left[\sqrt{x^2} = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases} \right]$

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2}\sqrt{4+\frac{1}{x^2}}}{x\left(\frac{2}{x}-1\right)} = \lim_{\substack{x \to -\infty \\ (x<0)}} \frac{-x\sqrt{4+\frac{1}{x^2}}}{x\left(\frac{2}{x}-1\right)} =$$

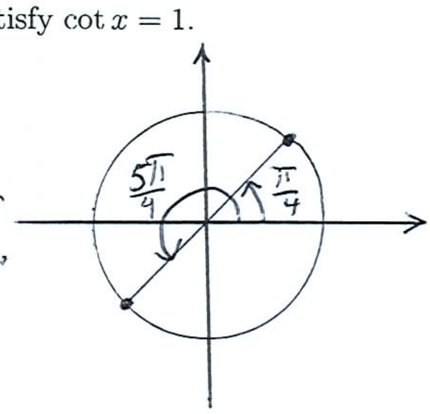
$$= \lim_{x \to -\infty} \frac{\sqrt{4+\frac{1}{x^2}}}{1-\frac{2}{x}} = \frac{\sqrt{4}}{1} = 2$$

$\frac{1}{x^2} \to 0$, $\frac{2}{x} \to 0$

A: $-\infty$	B: ∞	C: 2	D: -2	E: does not exist

(2 marks) A14. Find all values of x in the interval $[0, 2\pi]$ that satisfy $\cot x = 1$.

There are two values of x in $[0, 2\pi]$ that satisfy $\cot x = 1$: $\frac{\pi}{4}$ and $\frac{5\pi}{4}$.



A: $\frac{\pi}{4}, \frac{5\pi}{4}$	B: $\frac{\pi}{4}, -\frac{3\pi}{4}$	C: $-\frac{\pi}{4}, \frac{\pi}{4}$	D: only $\frac{\pi}{4}$	E: $-\frac{\pi}{4}, \frac{3\pi}{4}$

(2 marks) A15. If $f(x) = \sqrt{x^2+1}$, find $f'(x)$. Let $u = x^2+1$

$$f'(x) = \frac{1}{2\sqrt{u}} \cdot 2x = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}.$$

A: $\frac{1}{2\sqrt{x^2+1}}$	B: $\frac{x}{2\sqrt{x^2+1}}$	C: $\frac{x}{\sqrt{x^2+1}}$	D: $\frac{2x}{\sqrt{x^2+1}}$	E: $\frac{3}{2}\left(\sqrt{x^2+1}\right)^3$

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(2 marks) A16. If $f(x) = e^{\tan(x^3)}$, find $f'(x)$.

Let $u = \tan(x^3) = \tan v$ and $v = x^3$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = \frac{d(e^u)}{du} \cdot \frac{d(\tan v)}{dv} \cdot \frac{d(x^3)}{dx} =$$

$$= e^u \cdot \sec^2 v \cdot 3x^2 = e^{\tan(x^3)} \cdot \sec^2(x^3) \cdot 3x^2$$

A: $e^{\tan(x^3)}$	B: $e^{\tan(x^3)} 3x^2$	C: $e^{\tan(x^3)} \sec^2(x^3)$	D: $e^{\tan(x^3)} \tan(x^3) \sec(x^3) 3x^2$
E: $e^{\tan(x^3)} \sec^2(x^3) 3x^2$			

(2 marks) A17. If $f(x) = 2^{\cos x}$, find $f'(x)$.

$$f' = 2^{\cos x} \ln 2 \cdot \frac{d(\cos x)}{dx} = -2^{\cos x} \ln 2 \cdot \sin x$$

A: $-2^{\cos x} \cos x \sin x \ln 2$	B: $-2^{\cos x} \ln(\cos x)$	C: $\cos x \; 2^{\cos x - 1}$
D: $2^{\cos x} \ln(\cos x) \ln 2$	E: $-2^{\cos x} \sin x \ln 2$	

(2 marks) A18. Find the slope of the tangent line to the graph of $y = \sin x + \sin^2 x$ at the point $(0, 0)$.

$y' = \cos x + 2 \sin x \cdot \cos x$

$y'(0) = \cos 0 + 2 \sin 0 \cdot \cos 0 = 1$

A: 1	B: -1	C: 0	D: 2	E: 3

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(2 marks) A19. Determine $\lim_{x\to\frac{\pi}{2}} \frac{\sin 4x}{\sin 3x}$. $=$

$$= \frac{\sin\left(4\cdot\frac{\pi}{2}\right)}{\sin\left(3\cdot\frac{\pi}{2}\right)} = \frac{\sin(2\pi)}{\sin\left(\frac{3\pi}{2}\right)} = \frac{0}{-1} = 0$$

A: $\frac{4}{3}$	B: $\frac{3}{4}$	C: 1	D: 0	E: does not exist

(2 marks) A20. Suppose $F(x) = f(g(x))$ and $f(3) = 6$, $f'(2) = 3$, $f'(3) = 4$, $g(3) = 2$, $g'(3) = 5$ and $g(6) = 10$. Find $F'(3)$.

$$F'(x) = f'(g(x)) \cdot g'(x)$$

$$F'(3) = f'(g(3)) \cdot g'(3) = f'(2) \cdot g'(3) = 3 \cdot 5 = 15$$

A: 15	B: 20	C: 40	D: 30	E: 50

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1. A21 - A25. For each of the following, choose the letter which labels the graph below **and transfer each of your answers to the scantron sheet.**

(2 marks) A21. The graph of $y = \sin^{-1} x$ is C

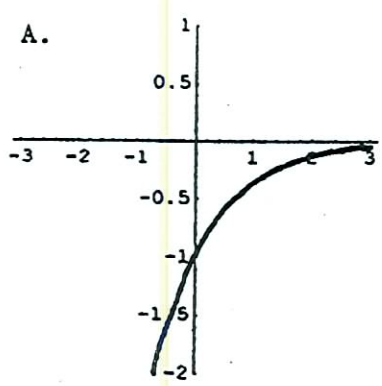
(2 marks) A22. The graph of $y = -e^{-x}$ is A

(2 marks) A23. The graph of $y = \frac{\pi}{2} + \tan^{-1} x$ is B

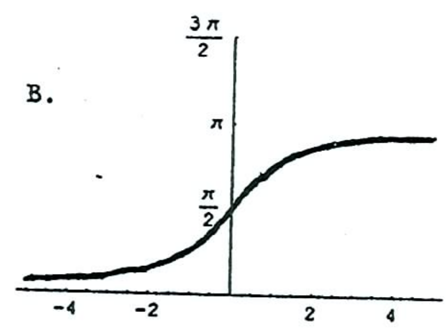
(2 marks) A24. The graph of $y = x^3$ is E

(2 marks) A25. The graph of $y = |\ln x|$ is D

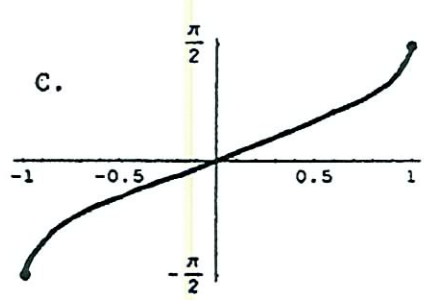
A.



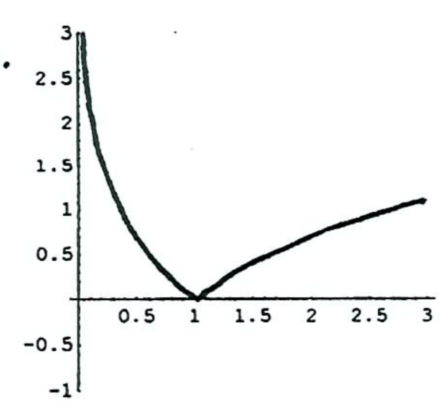
B.



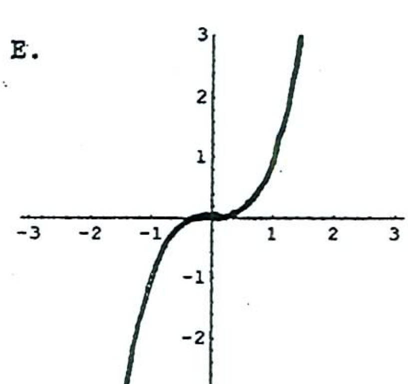
C.



D.



E.



Part B (50 marks)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(8 marks) B26. Let $f(x)$ be the function given by

$$f(x) = \begin{cases} x^2, & x \le 0 \\ e^x - 1, & 0 < x \le 1 \\ \frac{e}{x}, & x > 1 \end{cases}$$

State the value of the indicated limit, if it exists, in the space provided. Write ∞ or $-\infty$ if appropriate. If a limit does not exist, write DNE.

(a)

(i) $\lim_{x \to 0^-} f(x)$ 0

(ii) $\lim_{x \to 0^+} f(x)$ 0

(iii) $\lim_{x \to 0} f(x)$ 0

(iv) $\lim_{x \to 1^-} f(x)$ $e-1$

(v) $\lim_{x \to 1^+} f(x)$ e

(vi) $\lim_{x \to 1} f(x)$ DNE

(b) Is f continuous at 0 ? Justify your answer.

yes, because $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = \underline{\underline{f(0)}}$!

(c) Is f continuous at 1 ? Justify your answer.

No, because $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(4 marks) B27. The function

$$f(x) = \begin{cases} x + c, & x < 2 \\ -x^2 - cx, & x \geq 2 \end{cases}$$

is continuous at 2. What is the value of c ? Justify your answer.

The function is continuous at a, if

$\lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to a^-} f(x) = f(a)$. (1)

$f_1(x) = x + c$ is continuous on $(-\infty, 2)$ and

$f_2(x) = -x^2 - cx$ is continuous on $[2, +\infty)$.

Let us find c such that eq. (1) is satisfied:

$$\left.\begin{aligned} \lim_{x \to 2^-} f(x) &= \lim_{x \to 2^-}(x + c) = 2 + c \\ \lim_{x \to 2^+} f(x) &= \lim_{x \to 2^-}(-x^2 - cx) = -4 - 2c \\ f(2) &= -4 - 2c \end{aligned}\right\} \underset{\text{from (1)}}{\Rightarrow}$$

$2 + c = -4 - 2c \Rightarrow 3c = -6 \Rightarrow \underline{c = -2}$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(4 marks) B28. The limit

$$\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}$$

is the derivative of some function f at some number a.

(i) What are $f(x)$ and a ?

Answers: $f(x) =$ $\cos x$ or $\cos(x+\pi)$

$a =$ π 0

The answer to (i) is not unique.

(ii) Compute $\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}$ by finding $f'(x)$ and then evaluating $f'(a)$, where f and a are the answers found in part (i).

$f'(x) = (\cos x)' = -\sin x$

$f'(\pi) = -\sin \pi = 0$.

Or $f'(x) = (\cos(x+\pi))' = -\sin(x+\pi)$

$f'(0) = -\sin \pi = 0$.

Note: the answer to (ii) is 0, regardless what answer was obtained in (i) (of course, if (i) is correct).

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(4 marks) B29. Use the Intermediate Value Theorem to show that there is at least one root (i.e., a solution) of the equation $\sin x + \cos x = x^2$ in the interval $(0, \pi)$.

Let $f(x) = \sin x + \cos x - x^2$.

Because the function $f(x)$ is continuous on $[0, \pi]$, then the Intermediate Value Theorem can be applied.

Check: 1) $x = 0 \Rightarrow f(0) = 0 + 1 - 0 = 1 > 0$

2) $x = \pi \Rightarrow f(\pi) = -1 + 0 - \pi^2 = -1 - \pi^2 < 0$.

Because $f(0) > 0$ and $f(\pi) < 0$, then according to IVT, there is a number c in $(0, \pi)$ such that $\sin c + \cos c - c^2 = 0$, or the equation $\sin x + \cos x = x^2$ has at least one root on $(0, \pi)$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(5 marks) B30. Find an equation of the tangent line to the graph of $y = \dfrac{2}{1+e^{-x}}$ at the point $(0, 1)$.

$$y'(x) = \frac{-2}{(1+e^{-x})^2}(-e^{-x}) = \frac{2e^{-x}}{(1+e^{-x})^2}$$

[Or, writing the Chain Rule in steps:

$y = \frac{2}{u} = 2u^{-1}$, where $u = 1+e^{-x} = 1+e^{v}$,

where $v = -x$.

$$\frac{dy}{dx} = \frac{d(2u^{-1})}{du} \cdot \frac{d(1+e^{v})}{dv} \cdot \frac{d(-x)}{dx} = -2u^{-2} \cdot e^{v} \cdot (-1) =$$

$$= -2(1+e^{-x})^{-2} \cdot e^{-x} \cdot (-1) = \frac{2e^{-x}}{(1+e^{-x})^2}\ ].$$

$$y'(0) = \frac{2e^{0}}{(1+e^{0})^2} = \frac{2}{2^2} = \frac{2}{4} = \frac{1}{2}.$$

The equation of the tangent line is

$y - 1 = \frac{1}{2}(x - 0)$ or $\underline{y = \frac{1}{2}x + 1}$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(6 marks) B31. Find all vertical and horizontal asymptotes of the function $y = \dfrac{1}{x^2+x-2}$.

The horizontal asymptote: $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$.

$\lim_{x\to\infty} \frac{1}{x^2+x-2} = 0$ and $\lim_{x\to-\infty} \frac{1}{x^2+x-2} = 0$, so there is one horizontal asymptote: $y = 0$.

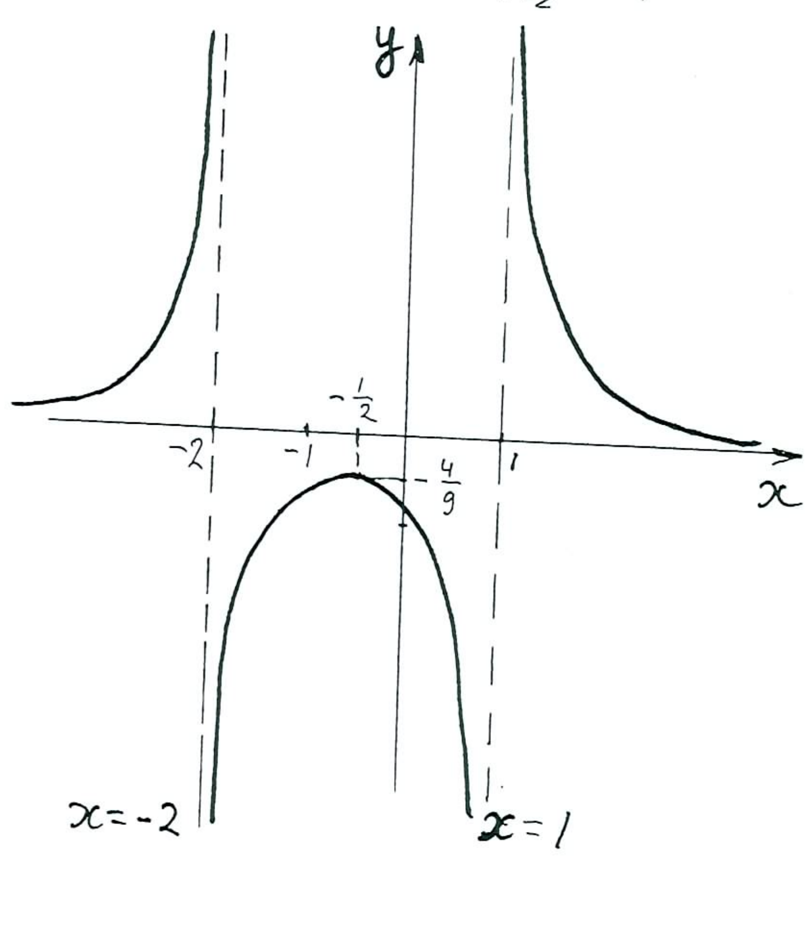
The vertical asymptote: $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$.

For a given function, $y \to \pm\infty$ when the denominator approaches zero.

$x^2+x-2=0 \Rightarrow x = \frac{-1\pm\sqrt{1+8}}{2} = \frac{-1\pm 3}{2} \Rightarrow \begin{array}{l} x_1=-2 \\ x_2=-1 \end{array}$

There are two vertical asymptotes: $x=-2$ and $x=1$.

(The graph was not required to get the full mark.)



NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(5 marks) B32. Find all values of x in the interval $[0, 2\pi]$ that satisfy

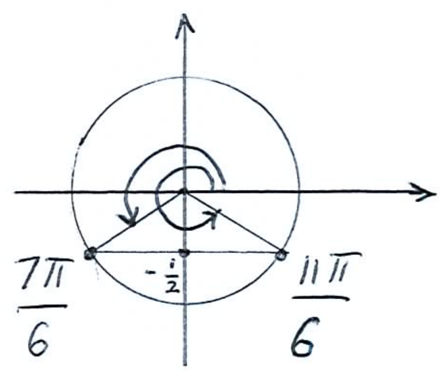
$$\cos(2x) = -\sin x.$$

Using identity: $\cos 2x = 1 - 2\sin^2 x \Rightarrow$
$1 - 2\sin^2 x + \sin x = 0$. Let $y = \sin x$, then
$1 - 2y^2 + y = 0$, or $2y^2 - y - 1 = 0$

$$y = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \Rightarrow \begin{array}{l} y_1 = 1 \\ y_2 = -\frac{1}{2} \end{array}$$

1) $y_1 = 1 \Rightarrow \sin x = 1 \Rightarrow x = \frac{\pi}{2} + 2n\pi$, but in $[0, 2\pi]$ there is only one solution: $x = \frac{\pi}{2}$.

2) $y_2 = -\frac{1}{2} \Rightarrow \sin x = -\frac{1}{2}$.
There are two values of x in $[0, 2\pi]$ that satisfy $\sin x = -\frac{1}{2}$: $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$.



Finally, $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(4 marks) B33. Determine

$$\lim_{x \to -2} \frac{|2x-4| - |x+4|}{|x|}.$$

$$|2x-4| = \begin{cases} 2x-4, & \text{when } 2x-4 \geq 0, \text{ or } x \geq 2 \\ -(2x-4), & \text{when } 2x-4 < 0, \text{ or } x < 2 \end{cases}$$

$$|x+4| = \begin{cases} x+4, & \text{when } x+4 \geq 0, \text{ or } x \geq -4 \\ -(x+4), & \text{when } x+4 < 0, \text{ or } x < -4 \end{cases}$$

$$|x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$

From the above:

$$\lim_{x \to -2} \frac{|2x-4| - |x+4|}{|x|} = \lim_{x \to -2} \frac{-(2x-4) - (x+4)}{-x} =$$

$$= \lim_{x \to -2} \frac{-3x}{-x} = \underline{3.}$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(5 marks) B34. Find $\frac{dy}{dx}$ if $y = \csc\left(3\cot\left(-e^{x^2}\right)\right)$. Do not simplify your answer.

$y = \csc(u)$, where $u = 3\cot(-e^{x^2}) = 3\cot(v)$, where $v = -e^{x^2} = -e^{w}$, where $w = x^2$.

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dw}\cdot\frac{dw}{dx} =$$

$$= \frac{d(\csc(u))}{du}\cdot\frac{d(3\cot(v))}{dv}\cdot\frac{d(-e^{w})}{dw}\cdot\frac{d(x^2)}{dx} =$$

$$= -\csc(u)\cot(u)\cdot(-3\csc^2(v))\cdot(-e^{w})\cdot 2x =$$

$$= -\csc(3\cot(-e^{-x^2}))\cot(3\cot(-e^{-x^2}))\cdot$$

$$\cdot[-3\csc^2(-e^{x^2})]\cdot(-e^{x^2})\cdot 2x$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(5 marks) B35. Find $f'(x)$ if it is known that $f(x) = [g(x)]^2$ and $g(e^x) = \dfrac{1+e^x}{1-e^x}$.

$$g(e^x) = \frac{1+e^x}{1-e^x} \Rightarrow g(x) = \frac{1+x}{1-x}.$$

$$f'(x) = 2g(x)\cdot g'(x) =$$

$$= 2\,\frac{1+x}{1-x}\cdot\left(\frac{1+x}{1-x}\right)' = 2\,\frac{(1+x)}{(1-x)}\cdot\frac{1\cdot(1-x)-(1+x)\cdot(-1)}{(1-x)^2} =$$

$$= 2\,\frac{(1+x)}{(1-x)}\cdot\frac{1-\cancel{x}+1+\cancel{x}}{(1-x)^2} = 2\,\frac{(1+x)}{(1-x)}\cdot\frac{2}{(1-x)^2} =$$

$$= \underline{\frac{4(1+x)}{(1-x)^3}}$$