

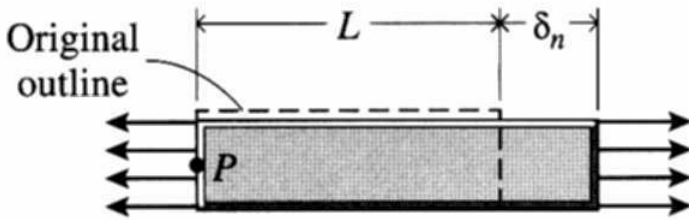
# FORMULA SHEET FOR ENGINEERING 3016

## PART 4 – MECHANICS OF MATERIALS

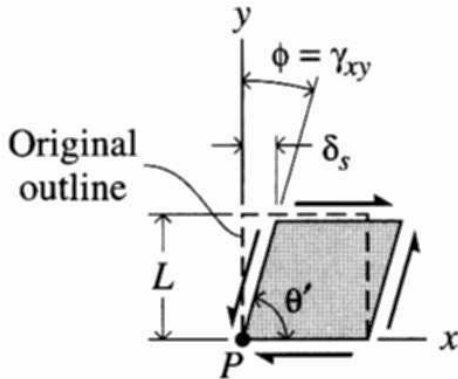
### Chapter 4 Stress, Strain, and Deformation: Axial Loading

Normal stress is normal to the plane  $\sigma = \frac{F}{A}$ ,  $F$  is the normal force,  $A$  is the cross-sectional area.

Shear stress is in the plane  $\tau = \frac{V}{A}$ ,  $V$  is the shear force,  $A$  is the cross-sectional area.



Average normal strain  $\epsilon = \frac{\delta_n}{L}$



Average shear strain  $\gamma = \frac{\delta_s}{L} = \tan \phi$

Hooke's Law:

for normal stress  $\sigma = E\epsilon$

for shear stress  $\tau = G\gamma$

$E$  is the Young's modulus

$G$  is the shear modulus

$G = \frac{E}{2(1+\nu)}$ ,  $\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$  is Poisson's ratio where  $\epsilon_{lat}$

is strain in lateral direction and  $\epsilon_{long}$  is strain in longitudinal direction.

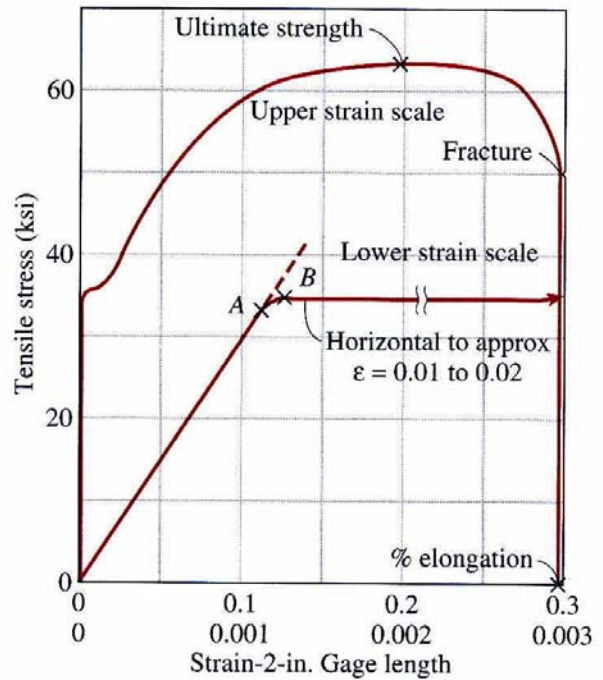
### Deformation of Axially Loaded Members

Member with uniform cross section  $\delta = \frac{PL}{EA}$

Members with multiple loads/sizes  $\delta = \sum_{i=1}^n \frac{P_i L_i}{E_i A_i}$

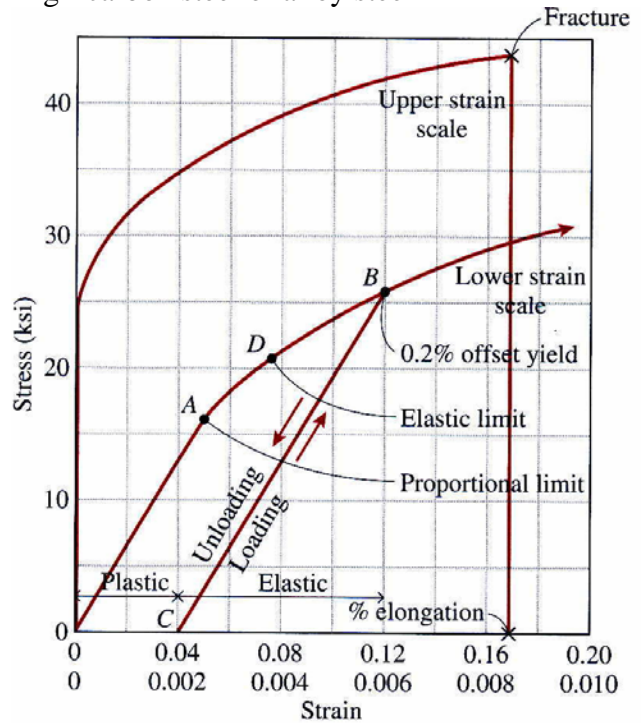
### Stress-Strain Relationships

Low-carbon steel or ductile materials



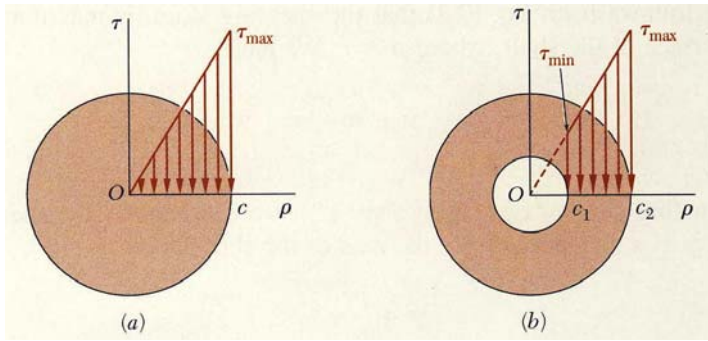
(a) Structural steel

High-carbon steel or alloy steel



(b) Magnesium alloy

## Chapter 7 Torsional Loading: Shafts



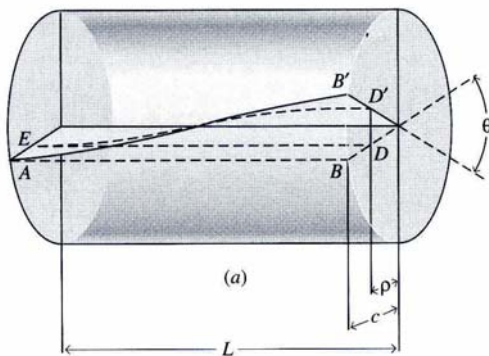
Shear stress at  $c$ ,  $\tau = \frac{Tc}{J}$ ,

$J$  is polar second moment of area

For solid cross section  $J = \frac{\pi}{2} c^4$

For hollow cross section  $J = \frac{\pi}{2} (c_2^4 - c_1^4)$

Torsional displacement or angle of twist



For uniform shaft  $\theta = \frac{TL}{GJ}$

For shaft with multi-step  $\theta = \sum_{i=1}^n \frac{T_i L_i}{G_i J_i}$

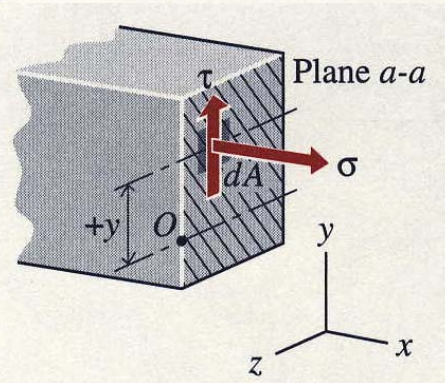
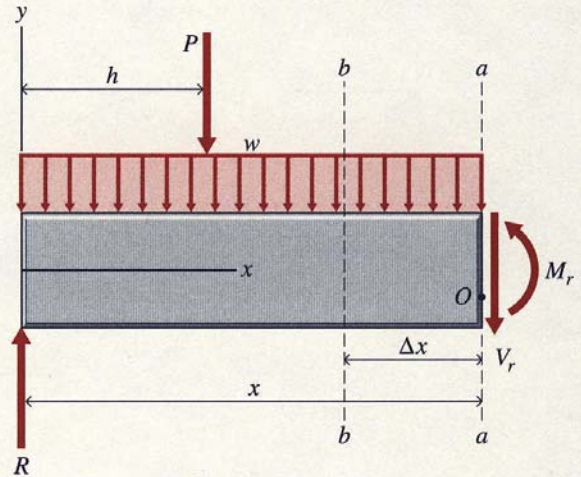
Work of a couple

$u = C\theta$ ,  $C$  is couple,  $\theta$  is angle of twist

Power Transmission by Torsional Shafts

Power =  $T\omega$ ,  $\omega$  is angular velocity

## Chapter 8 Flexural Loading: Stress in Beams



$M_r$  is the resultant of normal stress

$V_r$  is the resultant of shear stress

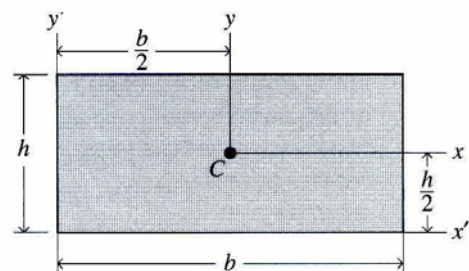
The Elastic Flexural Formula

Normal stress at  $y$ :  $\sigma = -\frac{My}{I}$

Max. normal stress at upper surface  $y = c$ :  $\sigma = -\frac{M_r c}{I}$

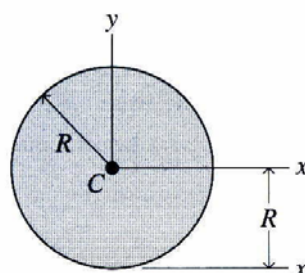
$I$  is the second moment of area

For a rectangular cross section



$$I_x = \frac{bh^3}{12}$$

For a circular cross section



$$I_x = \frac{\pi R^4}{4}$$

Shear Forces and Bending Moments in Beams

the max. bending stress  $\sigma_{max} = \frac{M_{r,max}}{S}$  where  $S = \frac{I}{c}$  is

the section modulus of the beam. If the beam is uniform cross section,  $S$  is constant.

$M_{r,max}$  is the max. bending moment in the beam as  $M_r$  varies along the beam, to find  $M_{r,max}$ , need to draw the bending moment diagram.

Shear force diagram shows the variation of the shear force  $V_r$  along beam

Bending moment diagram shows the variation of the bending moment  $M_r$  along beam

Sign convention

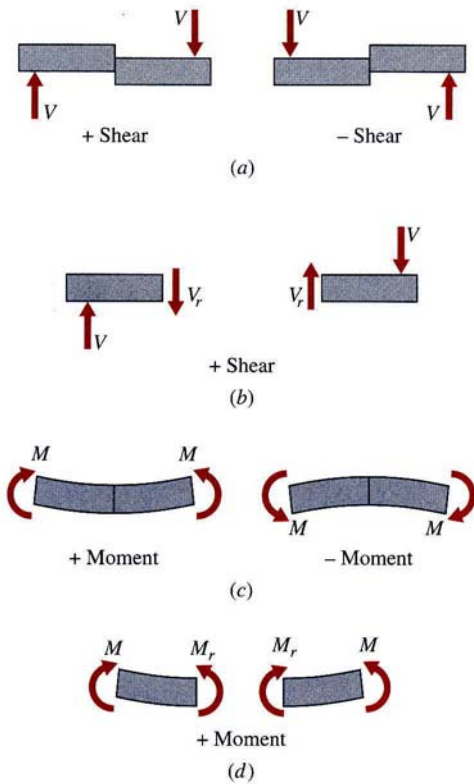


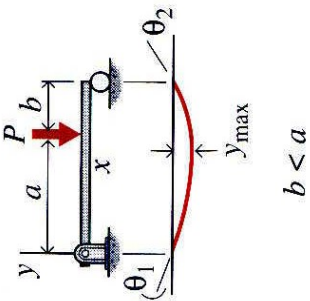
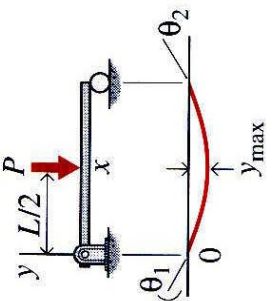
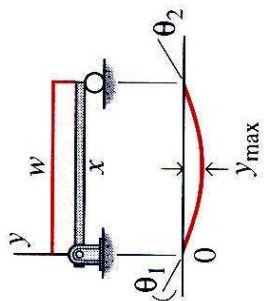
Figure 8-19

Procedure

1. Find the reactions at supports.
2. Determine how to divide the beam into different segments.
3. Starting from the far left end, section the beam at an arbitrary location  $x$  within the chosen segment.
4. Draw FBD for the portion of the beam to the left.
5. Apply equilibrium equations.
6. Repeat the process for each different segment of the beam.

TABLE A-19 Beam Deflections and Slopes

| Case | Load and Support (Length $L$ ) | Slope at End (+ $\Delta$ )              | Maximum Deflection (+ upward)            | Equation of Elastic Curve (+ upward)       |
|------|--------------------------------|---|--|--|
| 1    |                                | $\theta = -\frac{PL^2}{2EI}$ at $x = L$ | $y_{max} = -\frac{PL^3}{3EI}$ at $x = L$ | $y = -\frac{Px^2}{6EI}(3L - x)$            |
| 2    |                                | $\theta = -\frac{wL^3}{6EI}$ at $x = L$ | $y_{max} = -\frac{wL^4}{8EI}$ at $x = L$ | $y = -\frac{wx^2}{24EI}(x^2 - 4Lx + 6L^2)$ |

|   |  |   |  |   |
|---|--|---|--|---|
| 5 |  <p style="text-align: center;"><math>b &lt; a</math></p> | $\theta_1 = -\frac{Pb(L^2 - b^2)}{6LEI}$ <p style="text-align: center;">at <math>x = 0</math></p> $\theta_2 = +\frac{Pa(L^2 - a^2)}{6LEI}$ <p style="text-align: center;">at <math>x = L</math></p> | $y_{\max} = -\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$ <p style="text-align: center;">at <math>x = \sqrt{(L^2 - b^2)}/3</math></p> $y_{\text{center not max}} = -\frac{Pb(3L^2 - 4b^2)}{48EI}$ | $y = -\frac{Pbx}{6EIL}(L^2 - b^2 - x^2)$ <p style="text-align: center;"><math>0 \leq x \leq a</math></p>      |
| 6 |   | $\theta_1 = -\frac{PL^2}{16EI}$ <p style="text-align: center;">at <math>x = 0</math></p> $\theta_2 = +\frac{PL^2}{16EI}$ <p style="text-align: center;">at <math>x = L</math></p>                   | $y_{\max} = -\frac{PL^3}{48EI}$ <p style="text-align: center;">at <math>x = L/2</math></p>   | $y = -\frac{Px}{48EI}(3L^2 - 4x^2)$ <p style="text-align: center;"><math>0 \leq x \leq \frac{L}{2}</math></p> |
| 7 |   | $\theta_1 = -\frac{wL^3}{24EI}$ <p style="text-align: center;">at <math>x = 0</math></p> $\theta_2 = +\frac{wL^3}{24EI}$ <p style="text-align: center;">at <math>x = L</math></p>                   | $y_{\max} = -\frac{5wL^4}{384EI}$ <p style="text-align: center;">at <math>x = L/2</math></p>   | $y = -\frac{wx}{24EI}(x^3 - 2Lx^2 + L^3)$   |