MAT1300B – Test 1 – Fall 2013

Professor: Weixuan Li

Instructions:

1. You have 80 minutes to complete this exam.

2. This exam consists of two parts. Part I has five multiple-choice questions (2 points each) and part II has three long answer questions (10 points). The total is 20 points.

3. Write your answer to multiple-choice questions in given boxes. Write your answer to questions in Part II in the space after each question. If you need more space, use the back of the pages, but indicate clearly where your answer is.

4. No partial marks for multiple-choice questions. For questions in Part II, you must provide enough details to show how the result is obtained. Partial marks will be awarded for questions in Part II, if your answer is partially correct.

5. **Closed-book exam, no books, no calculators, no notes.**

Last name: ______________________________

First name: ______________________________

Student number: ______________________________

Part I. Multiple Choice Questions (1-5)

Question	1	2	3	4	5
Answer					

Question 1 *If* $\log_a x = 8$, *then* $\log_{a^2} x =$

A) $\sqrt{8}$ **B)** 4 **C)** 64 **D)** 16 **E)** $\frac{8}{a}$

Solution: (B). $x = a^8 = (a^2)^4, \log_{a^2} x = 4.$

Question 2 *If* $\ln(4x-2) - \ln(x+3) = 1$, *then* $x =$

A) $e+1$ **B)** $e+2$ **C)** $\frac{3e+2}{4-e}$ **D)** $\frac{e-2}{4-e}$ **E)** $\frac{3e-2}{3-e}$

Solution: (C).

$$\ln\frac{4x-2}{x+3} = 1, \frac{4x-2}{x+3} = e, 4x-2 = ex+3e, (4-e)x = 3e+2, x = \frac{3e+2}{4-e}.$$

Question 3 *For which values x, the graph of the function $y = \frac{x^2+3}{x-1}$ has horizontal tangent lines?*

A) -1 and 2 **B)** -3 and 1 **C)** -3 and 2
D) -1 and 3 **E)** -3 and 3

Solution: (D).
$y' = \frac{2x(x-1)-(x^2+3)}{(x-1)^2} = \frac{x^2-2x-3}{(x-1)^2}$ Let $y' = 0.x^2 - 2x - 3 = 0, x = -1, 3.$

Question 4 *Suppose a function $y = f(x)$ is defined implicitly by the equation*

$$x^2y + 3x - y^2 + 3 = 0.$$

The derivative of this function at the point $(-2, 3)$ is

A) 1 **B)** $\frac{3}{2}$ **C)** $-\frac{9}{2}$ **D)** $\frac{9}{10}$ **E)** -3

Solution: (C).
Take the derivative on both sides with respect to x:

$$2xy + x^2y' + 3 - 2yy' = 0, \quad -12 + 4y' + 3 - 6y' = 0, \quad 2y' = -9, \quad y' = -\frac{9}{2}.$$

Question 5 *Suppose an amount is deposited in an account with an annual interest rate of 6%, compounded monthly. How long will it take for the deposit to double?*

A) $\frac{\ln 2}{12 \ln 1.005}$ **B)** $\frac{\ln 2}{\ln 1.005}$ **C)** $\frac{\ln 1.005}{12 \ln 2}$ **D)** $\frac{\ln 2}{1.005 \ln 12}$ **E)** $\frac{12 \ln 1.005}{\ln 2}$

Solution: (A).
$A(t) = A(0)(1 + 0.006/12)^{12t}. A(t) = 2A(0).2 = 1.005^{12t}, 12t = \frac{\ln 2}{12 \ln 1.005}, t = \frac{ln2}{12 \ln 1.005}.$

Pat II. Long Answer Questions (6-8)

Question 6 (3 points) *Find the derivative of the function* $y = \sqrt{2x+5}$ *by the chain rule.*

Solution: $y = (2x+5)^{\frac{1}{2}}$. Let $u = 2x + 5$. Then $y = u^{\frac{1}{2}}$. By the chain rule,

$$y'_x = y'_u u'_x = \frac{1}{2}u^{-\frac{1}{2}} \times 2 = u^{-\frac{1}{2}} = \frac{1}{\sqrt{2x+5}}.$$

Question 7 (4 points) *Using only* **the definition of derivative as a limit**, *calculate* $f'(2)$ *where*

$$f(x) = \sqrt{2x+5}.$$

Solution:

$$f'(2) = \lim_{h\to 0} \frac{\sqrt{4+2h+5} - \sqrt{4+5}}{h} = \lim_{h\to 0} \frac{\sqrt{9+2h} - 3}{h} = \lim_{h\to 0} \frac{(\sqrt{9+2h} - 3)(\sqrt{9+2h}3)}{h(\sqrt{9+2h}+3)}$$

$$= \lim_{h\to 0} \frac{2h}{h(\sqrt{9+2h}+3)} = \lim_{h\to 0} \frac{2}{\sqrt{9+2h}+3} = \frac{1}{3}.$$

Question 8 (3 points) *A store has been selling Häagen-Dazs bar at the price of \$25.00 per box and, at this price, the store can sell 1600 boxes. If the price is \$28.00 per box, then the store can only sell 1300 boxes. Let* $p = f(x)$ *be the demand function where* x *is the number of boxes sold and* p *is the price per box. Assume the demand function is linear.*

- *(2 points) Find the demand function.*
- *(1 point) Find the revenue function.*

Solution: As a linear function, $p = mx + b$. The slope $m = \frac{28-25}{1300-1600} = -0.01$. Since $25 = (-0.01) \times 1600 + b, b = 16 + 25 = 41$. This function is $p = -0.01x + 41$.

Revenue function is: $R(x) = xp = -0.01x^2 + 41x$.