

## Midterm Exam Summer 2013

**ADM2303X**

**Last Name:** \_\_\_\_\_ **First Name:** \_\_\_\_\_

**Student #:** \_\_\_\_\_

*This exam booklet contains 4 problems. If yours does not, please inform professor now. Please answer all questions in the exam booklet. Only answers in this exam booklet will be marked. Show all work.*

1. One page (8 ½ by 11 inches) review sheet, both sides, is allowed.
2. Calculator permitted for arithmetic use only.
3. NO COMMUNICATION DEVICES (computers, phones, etc.) MAY BE WITHIN SIGHT.

| Question     | Points | Out of    |
|--------------|--------|-----------|
| 1            |        | 2         |
| 2            |        | 5         |
| 3            |        | 3         |
| 4            |        | 4         |
| 5            |        | 3         |
| 6            |        | 3         |
| <b>TOTAL</b> |        | <b>20</b> |

**Statement of Academic Integrity**

The Telfer School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

**Statement to be signed by the student:**

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: \_\_\_\_\_

Note: an examination copy or booklet without that signed statement will not be graded and will receive a midterm exam grade of zero.

**1. (2 points)** A shareholder believes that in one year, there is a 20% chance that his stock will be worth \$75, a 50% chance that it will be worth \$100 and a 30% chance that it will be worth \$140.

a) Find the expected value. (1 point)

$$E(X) = \sum xP(x) = \$75(.20) + \$100(0.50) + \$140(0.30) = \$107$$

b) Find the standard deviation. (1 point)

$$\sigma = SD(X) = \sqrt{Var(X)}, \text{Var}(X) = \sum (x - \mu)^2 P(x)$$

$$\text{Var}(X) = \sum (x - \mu)^2 P(x)$$

$$= (75 - 107)^2(0.20) + (100 - 107)^2(0.50) + (140 - 107)^2(0.30) = 556$$

$$\delta = \sqrt{556} = 23.57$$

**2. (5 points)**

The following contingency table shows the number of adults with credit scores 725 or above and below 725 as well as their marital status.

| Credit Score     | Single (C) | Married (D) |
|------------------|------------|-------------|
| Below 725 (A)    | 39         | 33          |
| 725 or Above (B) | 15         | 63          |

a) Identify the marginal probabilities for this table. (1 point)

| Credit Score     | Single (C) | Married (D) | Total |
|------------------|------------|-------------|-------|
| Below 725 (A)    | 0.26       | 0.22        | 0.48  |
| 725 or Above (B) | 0.10       | 0.42        | 0.52  |
| Total            | 0.36       | 0.64        | 1.00  |

$$P(A) = 0.48$$

$$P(B) = 0.52$$

$$P(C) = 0.36$$

$$P(D) = 0.64$$

b) Identify the joint probabilities for this table. (1 point)

$$P(A \text{ and } C) = 0.25$$

$$P(A \text{ and } D) = 0.22$$

$$P(B \text{ and } C) = 0.10$$

$$P(B \text{ and } D) = 0.42$$

c) Identify the conditional probabilities for this table. (1 point)

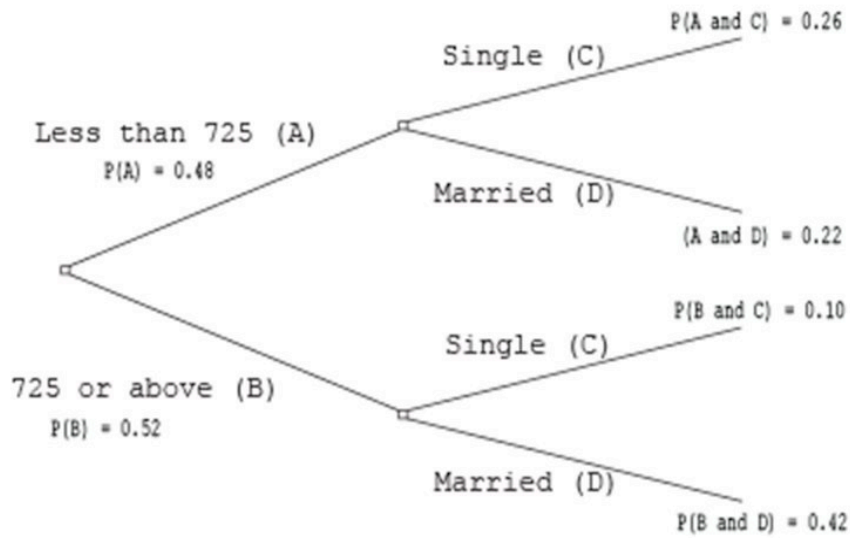
$$P(A | C) = \frac{P(A \text{ and } C)}{P(C)} = \frac{0.26}{0.36} = 0.722$$

$$P(A | D) = \frac{P(A \text{ and } D)}{P(D)} = \frac{0.22}{0.64} = 0.344$$

$$P(B | C) = \frac{P(B \text{ and } C)}{P(C)} = \frac{0.10}{0.36} = 0.278$$

$$P(B | D) = \frac{P(B \text{ and } D)}{P(D)} = \frac{0.42}{0.64} = 0.656$$

d) Construct a decision tree for this table. (1 point)



e) Determine whether Events  $A$  and  $C$  are independent or dependent. What conclusions can be made? (1 point)

$$P(A) = 0.48$$

$$P(A | C) = \frac{P(A \text{ and } C)}{P(C)}$$

$$P(A \text{ and } C) = 0.26$$

$$P(C) = 0.36$$

$$P(A | C) = \frac{0.26}{0.36} = 0.722$$

$$P(A | C) \neq P(A)$$

Events  $A$  and  $C$  are dependent. Married adults appear to have higher credit scores than single adults.

**3. (3 points)**

A survey of Americans ages 18-24 years old found the following political party affiliations.

| Party       | Percentage |
|-------------|------------|
| Independent | 50%        |
| Democratic  | 30%        |
| Republican  | 20%        |

Consider the following information:

- Of those affiliated with the Independent party, 56% were male.
- Of those affiliated with the Democratic party, 40% were male.
- Of those affiliated with the Republican party, 35% were male.

Use Bayes' Theorem to determine the probability that a person who is male is affiliated with the Independent party.

Define  $A_1$  = Independent party,  $A_2$  = Democratic party,  $A_3$  = Republican party,  $B$  = male.

$$P(A_1) = 0.50$$

$$P(A_2) = 0.30$$

$$P(A_3) = 0.20$$

$$P(B | A_1) = 0.56$$

$$P(B | A_2) = 0.40$$

$$P(B | A_3) = 0.35$$

$$P(A_1 | B) = \frac{P(A_1) P(B|A_1)}{P(A_1) P(B|A_1) + P(A_2) P(B|A_2) + P(A_3) P(B|A_3)}$$

$$P(A_1 | B) = \frac{(0.50)(0.56)}{(0.50)(0.56) + (0.30)(0.40) + (0.20)(0.35)}$$

$$P(A_1 | B) = \frac{0.28}{0.28 + 0.12 + 0.07} = \frac{0.28}{0.47} = 0.596$$

**4. (4 points)**

The Columbia/Boone County Office of Emergency Management reported that 911 operators receive an average of 7.2 calls per hour. Assume that the number of 911 calls follows the Poisson distribution.

a) What is the probability that three or more 911 calls will be made during the next hour? (2 point)

$$\lambda = 7.2$$

$$P(0) = \frac{(7.2^0)(2.71828^{-7.2})}{0!} = \frac{(1)(0.000747)}{1} = 0.0007$$

$$P(1) = \frac{(7.2^1)(2.71828^{-7.2})}{1!} = \frac{(7.2)(0.000747)}{1} = 0.0054$$

$$P(2) = \frac{(7.2^2)(2.71828^{-7.2})}{2!} = \frac{(51.84)(0.000747)}{2} = 0.0194$$

$$P(x \geq 3) = 1.0 - P(x < 3) = 1.0 - 0.0007 - 0.0054 - 0.0194 = 0.9745$$

b) What is the probability that exactly four 911 calls will be made during the next 30 minutes? (2 point)

$$\lambda = 3.6$$

$$P(4) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.6^4)(2.71828^{-3.6})}{4!}$$

$$P(4) = \frac{(167.9616)(0.027324)}{24} = 0.1912$$

**5. (3 points)**

According to Harris Interactive, 35% of college students prefer a digital textbook such as a tablet, e-reader, or computer. A random sample of 150 college students was selected. Using the normal approximation to the binomial distribution, what is the probability that 55, 56, or 57 students from this sample prefer a digital textbook?

$$\mu = np = (150)(0.35) = 52.5$$

$$\sigma = \sqrt{npq} = \sqrt{(150)(0.35)(1-0.35)} = 5.842$$

$$z_{57.5} = \frac{x - \mu}{\sigma} = \frac{57.5 - 52.5}{5.842} = 0.86$$

$$z_{54.5} = \frac{x - \mu}{\sigma} = \frac{54.5 - 52.5}{5.842} = 0.34$$

$$P(54.5 \leq x \leq 57.5) = P(0.34 \leq z \leq 0.86) = 0.8051 - 0.6331 = 0.1720$$

**6. (3 points)**

The heights of professional basketball players are normally distributed with mean =190 cm and standard deviation = 5 cm.

a) Calculate the probability that a randomly selected player is over 193 cm in height. (1.5 point)

$$P(X > 193) = 1 - P(X < 193) = 1 - P(Z < .6) = 1 - 0.725747 = 0.274253$$

b) How tall must a player be to be amongst the tallest 10% of all professional basketball players? (1.5 point)

$$P(X > x) = 0.10 \text{ or } P(X < x) = .90 \rightarrow z = 1.28 \rightarrow x = 196.408$$