# PHYS 131 MIDTERM November 1st, 2012

The exam comprises two parts: 8 short-answer questions, and 5 problems. Calculators are allowed, as well as a formula sheet (one-side of an 8½” x 11” sheet) of your own making.

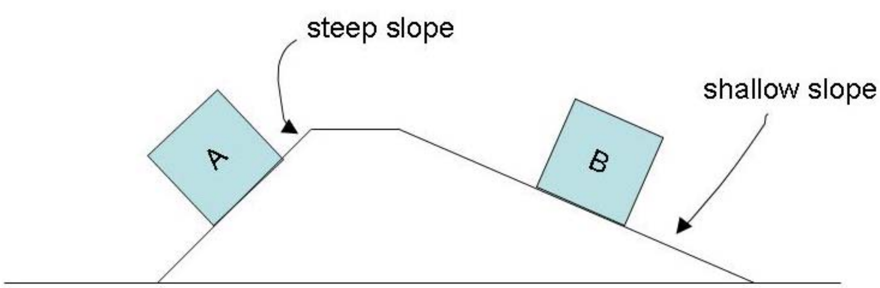
Answer **the short-answer questions** with a few words or a phrase, but be concise, please! For the problems, show **all your work**.

The short answer problems are worth four points each, and the problems are worth 8 points each. Put all answers in the **answer booklets provided**; you may keep this exam.

Good luck !

## Short answer questions (answer all): you should not need to do any calculations for these questions. Answer in **a few words, a short phrase, or a simple sketch.**

1) [4 pts] Two ramps, one of which is steeper than the other, lead from the floor to a loading platform. Two blocks, A and B, start at the floor and are pushed up the ramp. First assume there is no friction. a) Which box, A or B, requires a larger external force to push up the ramp? b) Which requires more work by that external force to move to the platform from the floor? Now assume there is friction (same coefficient of friction on both ramps): c) Which box requires more work (again, done by the external force) to move from the floor to the platform? Explain all your answers, and ignore the size of the blocks compared to the length of the ramps.




SOLUTION: a) To push the blocks up the ramp in the absence of friction, we must balance the component of mg down the ramp (which is mg sin(angle)). So a) requires a larger force. b) From conservation of energy, the work done by the internal force will go into gravitational potential energy which is the same (mgh) in both cases – so they require the same amount of work. c) When friction is present, it does negative work on the system which is d (the slant distance) x $\mu F_N$ where $F_N$ is the normal force, mg cos(angle). For larger angles, both the normal force $F_N$ and the distance up the slope d are smaller – so box A requires less work to get it up the ramp.

2) [4 pts] A ball is dropped from rest from the top of a building of height h, and hits the ground with speed $v_f$. From the ground, a second ball is thrown straight upwards at the same moment that the first ball is dropped. The second ball is thrown with an initial upwards speed of $v_0 = v_f$, the same speed with which the first ball will eventually hit the ground. Ignore air resistance. Will the balls cross at half the height of the building, above the halfway point, or below it? Explain your reasoning.

SOLUTION: The ball dropping starts from rest and accelerates uniformly to $v_f$, whereas the ball rising starts from $v_f$ and decelerates uniformly to zero. Only if the two balls had symmetric motions would the crossing point be at half-height (initially, at every moment the rising ball is moving faster than the falling ball). Formally, we can set up equations like:

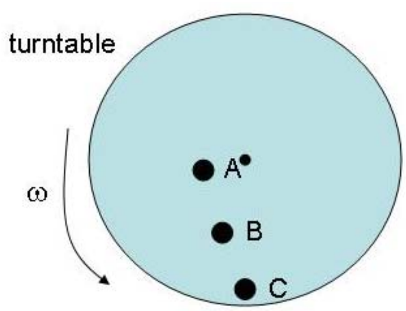
$$y_{dropping} = h - \frac{1}{2} g t^2 \qquad (1)$$

$$y_{rising} = v_f t - \frac{1}{2} g t^2 \text{ , with } v_f = \sqrt{(2gh)} \qquad (2)$$

equating the two gives $t = h/v_f$ and putting that back into (1) gives

$$y_{crossing} = h - \frac{1}{2} g (h/v_f)^2 = h - \frac{1}{2} g (h^2/(2gh)) = \frac{3}{4} h$$

3) [4 pts] Three pennies are on a rotating turntable as shown in the diagram. Initially, due to static friction, the pennies are not moving with respect to the turntable as it rotates at a given angular speed. The speed of rotation is then increased slowly. Assuming each penny has the same coefficients of friction with the turntable surface, which penny starts to slide first? Explain your reasoning.

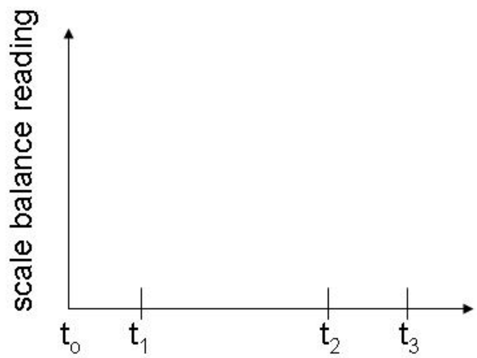



SOLUTION: the centripetal force for each penny is $mv^2/r$ and is provided by static friction. Each penny has $v = 2\pi r/T$ so we get $F_c = m (4\pi^2)r/T^2$. So the outermost penny C starts to slide first - that is, as we increase the rotational speed (decreasing T), its required centripetal force will be the first to get to the limit that static friction can provide, and it will no longer move with the turntable.

4) [4 pts] A popular carnival ride has passengers stand with their backs against the inside wall of a cylinder whose axis is vertical. As the cylinder spins, the floor is lowered and the passengers are “stuck” to the wall. Passengers feel as though they are being pushed against the wall. Explain the physics of the ride. Is there a force pushing the passengers into the wall?

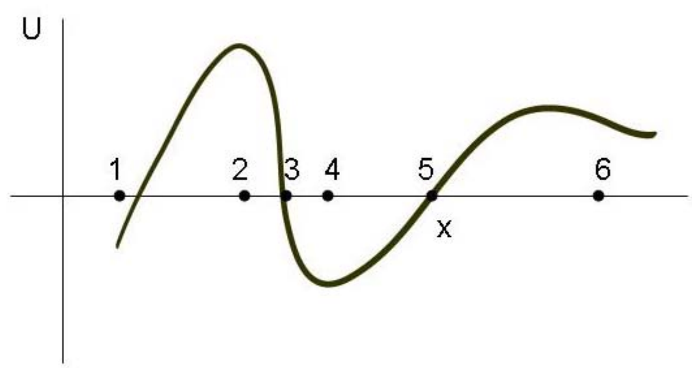
SOLUTION: There is no force pushing outwards. The passengers are in uniform circular motion and the normal force provided by the wall provides the centripetal acceleration. Static friction (proportional to the large normal force) balances mg vertically so they don't drop.

5) [4 pts] You are standing on a spring balance (which indicates the normal force acting on you) in an elevator. Draw a set of axes (shown below) in your exam booklet, and add schematically what you expect to see for the reading of the balance as a function of time, as you start from the ground floor at rest at time $t_o$, accelerate at a constant rate upwards until time $t_1$ (when you reach a constant upwards speed), then decelerate (again, at a constant rate) from time $t_2$ until $t_3$, when you reach the 5th floor. On the vertical scale, indicate where a value 'mg' is, where m is your mass. Explain your reasoning.




SOLUTION: The scale reading is constant and larger than mg from to $t_0$ to $t_1$ (because the net force on you must be upward), constant and equal to mg from $t_1$ to $t_2$ (because the net force on you must be zero), and constant and smaller than mg from $t_2$ to $t_3$ (because the net force must be downward).

6) [4 pts] The plot below shows a potential energy function U(x) as a function of a particle's position x. Identify the numbered points:
   a. where the x-component of the force on the particle is positive;
   b. where the x-component of the force is negative;
   c. where the x-component of the force has the largest magnitude;
   d. that are equilibrium points (where the force is zero), and for each, indicate if the equilibrium is stable or unstable.




SOLUTION: $F_x = -dU/dx$, so we have:

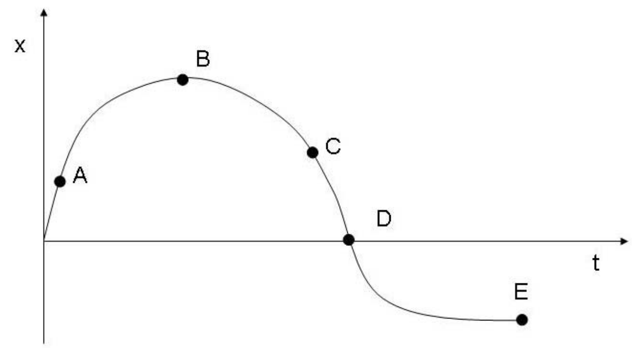
a) points 3 and 6

b) points 1 and 5

c) point 3

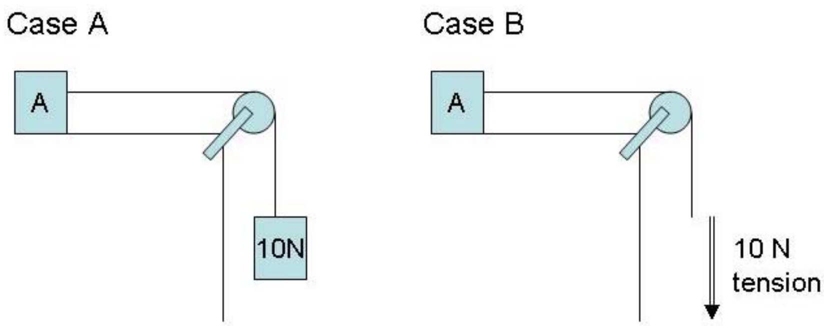
d) point 2 (unstable) and point 4 (stable)

7) [4 pts] The figure below shows the position-vs-time graph for a moving object. At which point (or points):
   a. is the object moving with the largest positive velocity?
   b. is the object moving in the negative x direction?
   c. is the object at rest?
   d. is the object changing its direction of motion?




SOLUTION:
a) point A
b) points C and D
c) points B and E
d) point B

8) In the left-hand drawing (Case A) below, block A is accelerated across a frictionless surface by hanging a 10 N weight over a massless, frictionless pulley with a massless string. In Case B, block A is accelerated across the same frictionless surface by a constant 10 N tension in the string. Are the accelerations of block A in the two cases the same, and if not, which is larger? Explain your reasoning.



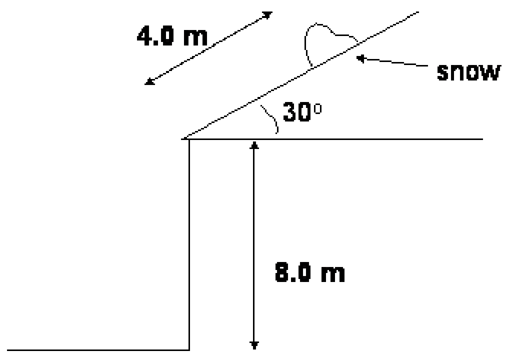

SOLUTION: In case B, the applied force on the block horizontally is 10 N.
In case A, the FBD of the 10 N weight is T (upwards) – mg = – ma. Since a>0 we have T = mg – ma < 10 N. So the acceleration in case A is less than in case B.

## Problems (show your work):

**1)** [8 pts] A patch of snow is on a roof that is inclined at 30.0° to the horizontal (see figure below for dimensions). Initially, static friction keeps the snow in place. As the snow melts, the coefficient of static friction is reduced and eventually the snow starts to slide. At that point, the coefficient of *kinetic* friction is $\mu_k$=0.187.

- **a.** What is the coefficient of static friction at the moment the snow starts to slide?
- **b.** Determine the speed of the snow when it reaches the edge of the roof.
- **c.** What is the time necessary for the snow to fall to the ground from the edge of the roof?
- **d.** What is the horizontal distance from the edge of the roof to where the snow lands?

Note that the snow does not leave the roof horizontally!

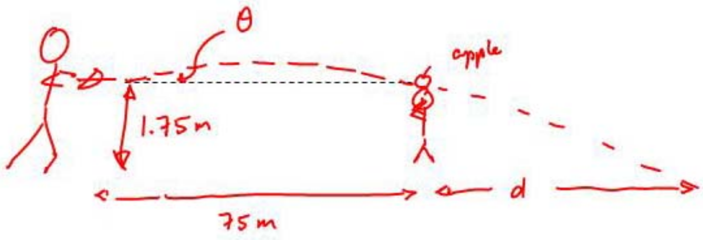



SOLUTION: (a) At the moment the snow starts to slide, we have $\mu_s mg\cos(\theta) = g\sin(\theta)$ and so $\mu_s = \tan(\theta) = 0.577$. For (b), we can use energy, or calculate the forces. Doing the latter gives us (taking +ve x to be down the slope): $F_{x\text{-net}} = mg\sin(\theta) - \mu_k mg\cos(\theta) = ma$, giving $a= g[\sin(\theta) - \mu_k\cos(\theta)] = 3.31\ m/s^2$. So $v^2 = 2ad$ (where d=4.0m), giving v= 5.15 m/s. For (c), we identify that $v_{y0} = -5.15\sin(\theta)$ and we have $y = y_0 + v_{y0}t + \frac{1}{2}at^2$, leading to a quadratic in t that gives us: t = 1.04 s (the large negative solution is incorrect). Finally, for (d) we simply apply $x = v_x t$ since there is no acceleration in x, and we get D=5.15cos(θ)(1.04s) = 4.64m.

**2)** [8 pts] William Tell is preparing to shoot an apple from the head of his youngest child (the others, sadly, have fallen victim to his poor aim). The apple is at the same height as his crossbow and arrow, namely 1.75 m above the level ground. The distance is 43.0 m, and William knows that the launch speed of his crossbow is 51.0 m/s.

- **a.** At what angle θ upwards from the horizontal should he aim the arrow to hit the apple? Calculate **both** solutions.
- **b.** Which of the two solutions that William could choose is preferable for William's son, and why? Assume the arrows are sharp, and the apple is a standard Macintosh.

**c.** For the preferred solution you identified in b), assume that William misses the apple laterally (that is, his arrow gets to the apple at the correct height but is slightly to the side of the apple). What is the distance d that the arrow goes beyond his son before it hits the ground?




SOLUTION: For (a) and (b) we can use the range equation: $R = v^2\sin(2\theta)/g$ . That gives $\sin(2\theta) = 0.162$ , leading to $\theta = 4.66^o$ and $85.3^o$. For (b) we'd choose the low-angle solution, since the high angle solution has the arrow arriving at $85.3^o$ from the horizontal – ie, penetrating through the apple and hitting the boy. For (c), we set up the equation for the vertical position to find the time:
$0 = h - v\sin(\theta)t - \frac{1}{2}gt^2$ where h=1.75 m, then apply $v_x t$ to find the horizontal distance d. We find: t= 0.310 s, and d = 15.8 m.

**3)** [8 pts] One of your CAPA problems involved a volcano, of height H above the surrounding sea level, ejecting rocks at all angles θ (measured from the horizontal) at a speed v.

**a.** Find the horizontal range R (from the volcano) that the rocks achieve at sea level, for H=1450 m, v=175 m/s, and $\theta=32.0^o$.

**b.** Find a closed-form analytic equation for R as a function of H, v, and θ.

**c.** Confirm, for your equation from b):

   i. that the equation gives you the same answer you found in a) for the numeric data given, and

   ii. that for H=0 the equation gives our standard range equation.

SOLUTION: It's easier to go to (b) first. The horizontal range R in terms of the launch angle θ, the speed v, and the height H, is $R = v_x t = v\cos(\theta) t$. We get t from the vertical equation: $0 = H + v\sin(\theta) t - \frac{1}{2} g t^2$, to which the solution is:

$$t = [-v\sin(\theta) \pm \text{sqrt}(v^2\sin^2(\theta) + 2Hg)]/(-g)$$

We need a positive time t, and so (considering the negative sign in the denominator) we need the negative sign in the "±" choice. That gives:

$$t = [v\sin(\theta) + \text{sqrt}(v^2\sin^2(\theta) + 2Hg)]/g$$

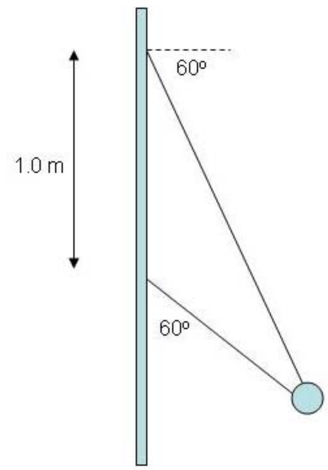
This gives us the closed-form solution for R:

$$R = v\cos(\theta)[v\sin(\theta) + \text{sqrt}(v^2\sin^2(\theta) + 2Hg)]/g$$

(c) i) This gives R = 4.32 km, which is also the answer for (a). (ii) For H = 0, we get $R = 2v^2\cos(\theta)\sin(\theta)/g$ and $2\cos(\theta)\sin(\theta) = \sin(2\theta)$, leading to $R = v^2\sin(2\theta)/g$ which is our standard range equation.

**4)** [8 pts] Two wires are attached to a 2.0 kg ball as shown below (the diagram is not to scale, but the key dimensions are indicated). The ball revolves in a horizontal circle at constant speed v.

**a.** For what speed v is the tension the same in the two wires?
**b.** What is the tension?

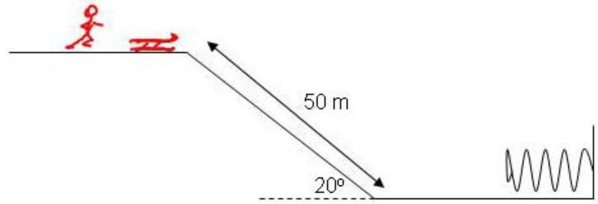



SOLUTION: The triangle created by the bar and the two wires is an isosceles one, so the lower wire is 1.0 m long. That gives the distance of the ball from the axis to be $1.0\cos(30^o) = 0.866$ m. Now we have $T\cos(30) + T\cos(60) = mv^2/r$ and $T\sin(30) + T\sin(60) = mg$. That leads to $v = \sqrt{rg\cdot[\cos(30)+\cos(60)]/[\sin(30)+\sin(60)]} = \sqrt{rg} = 2.91$ m/s. Putting this back into the equation for the horizontal forces gives: T = 14.3 N.

**5)** [8 pts] For the Winter Olympics in 2014 in Sochi, Russia, the Russian delegation has proposed a new event called the 'bob-spring'. An athlete will sprint 50 m from rest then leap onto a bobsled of mass 20 kg. The person+bobsled will then slide down an ice-covered ramp (assume it's frictionless) of 50 m in length, sloped at $20^o$, and slide into a carefully calibrated spring of spring constant k=2000. N/m. The athlete who compresses the spring the furthest wins the gold medal.

An athlete has been in training for this bob-spring. She knows she can achieve a speed v=12.0 m/s at the end of her sprint (assume this is the speed she starts with when she's already on the sled). Her mass is 42.3 kg.

**a.** How far will she compress the spring?
**b.** The Olympic committee is worried about the details (shape, angle, etc) of the ramp. If you were asked to consult with them, what would you tell them – should they be worried about the details, and if so, why?
**c.** Assuming that most athletes, independent of their mass, achieve about the same speed at the end of the sprint, does the athlete's mass matter? Should the committee have different categories for athletes of different masses? Explain your position.




SOLUTION: This is a conservation of energy question. We have ½ $mv^2$ + mgh = ½ $kx^2$, where h = 50sin(20) = 17.1 m.That gives (a) x = 3.86 m. For (b), the only thing that matters is the total drop [50 sin(20)], not the shape or slope distance or angle, because gravity is a conservative force and no non-conservative forces (friction) are acting. For (c), mass clearly matters because the potential energy of the athlete (mgh) depends on her mass, while the total energy of the system is ultimately converted to spring potential ½ $kx^2$, which doesn't depend on mass. Thus, larger mass will lead – for the same athlete speed at the beginning – to a larger compression distance. There should be different 'weight' categories.