STUDENTS OFFERING SUPPORT

Laurier SOS

BU 383 - Financial Management I

Midterm Exam-AID

Review Package

Course Coordinator: Elena Mikhailova

Tutor: Peter Starszyk

Contact: pstarszyk@gmail.com

Table of Contents

Chapter 1: Time Value of Money

The idea behind the Time Value of Money (TVM) is that as long as money can accumulate interest (a return), then any amount of money today is worth more than that same amount of money at any given time in the future due to it's interest earning capacity.

This does not only apply to money; **Capital goods** are a prime example of tangible assets that yield services in the present as well as the future. They can appreciate in value as long as they have the potential to earn a return (ie. cars, homes, factories, etc.). This appreciation in value is referred to as the **Net Productivity of Investment**. For example, buying a home today can potentially double in value over ten years if it is subject to strong economic conditions.

Interest Rate - Rate of Return

Basically, the interest rate (or rate of return) represents the cost of borrowing money from a lender for a specified amount of time. The interest rate can be viewed as compensation for the foregone opportunity cost occurred by the lender, since they could have kept the funds and used it for something other than lending.

Interest rates are always quoted as an annual nominal figure (APR) which can be compounded in different ways; annually, semi annually, quarterly, monthly, semi-monthly, daily, etc.

For example, if you are offered a savings account that pays 5% APR compounded monthly, your monthly rate of return would be 5%/12 = 0.41667%.

Given any APR, you can determine the Effective Annual Rate (EAR) given the number of compounding periods in a year:

$$EAR = \left(1 + \frac{APR}{\#\ of\ compounding\ periods}\right)^{\#\ of\ compounding\ periods} - 1$$

Future Value

The future value of money or a capital good is the total value of it that has accumulated over a period of time **t** at a given rate of return **r**. Where,

PV= Present Value
FV= Future Value
r = Rate of return
t = Time elapsed

It follows that:
$FV = PV * (1+r)^t FV = PV * (1+r)^t = PV * FVIF_{r,t} PV * FVIF_{r,t}$

This introduces the concept of **compounding,** which is defined as the process of leaving your money plus any accumulated interest in an investment for more than one period (hence reinvesting the interest).

Present Value

However, in many cases investors are faced with the problem of figuring out how much they should invest now (present value) in order to accumulate a certain amount over time (future value). This raises the concept of the Present Value.

$$PV = \frac{FV}{(1+r)^t} = \frac{FV}{FVIF_{r,t}}$$

Example:

You would like to save some money in order to buy a $2,000 flat screen TV two years from now for your son's eighteenth birthday. You found a GIC that offers an annual effective interest rate of 7% compounded annually. How much should you invest today in order to accumulate the $2,000 two years from now?

$$PV = \frac{FV}{(1+r)^t}$$

$$PV = \frac{2000}{(1.07)^2} = \frac{2000}{FVIF_{7\%,2}}$$

PV = **$1746.88**

The Value of Capital Goods

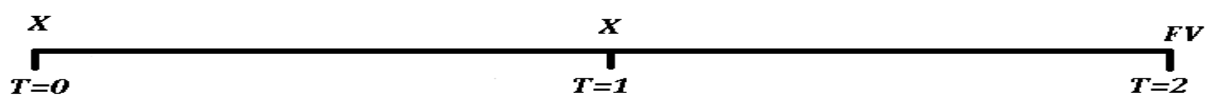
Since many capital goods appreciate in value, the price of a capital good is determined by discounting the asset's cash flows at its respective rate of return. This discount rate reflects the opportunity cost of investing the funds. Discounting the future cash flows (referring to the DCF model) applies to many assets such as stocks, bonds, companies and projects.

Annuities

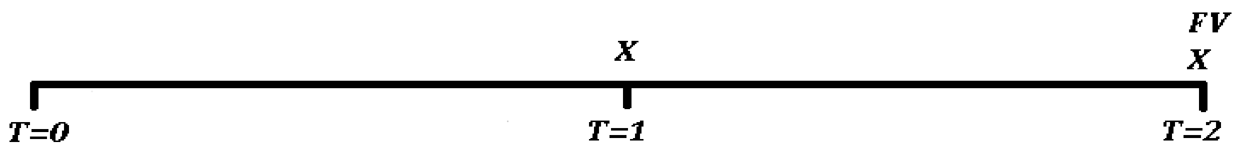
Annuities are financial products that provide a stream of equal cash flows at regular time intervals. These cash flows can be either paid out or received. There are two basic types of annuities that will be focused on in this course; the Annuity **Due** and the Annuity **Immediate**.

The Annuity Due is a stream of cash flows whereby each payment is made at the BEGINNING of each period whereas the Annuity Immediate is a stream of cash flows whereby each payment is made at the END of each period. The following are timeline illustrations for each annuity type where payments of $X are made for two years:

Annuity Due



Annuity Immediate



→ ***Future Value of Annuities***

The future value of an annuity refers to the value of a stream of cash flows at a given time in the future. Each cash flow payment earns a rate of return from the time of payment until the final date. Where,

$FV_t FV_t$ = Future Value of the annuity at the t^{th} payment period

P= Value of each payment (all payments being equal)

r = rate of return

t = number of payments periods

Future Value of Annuity Immediate:

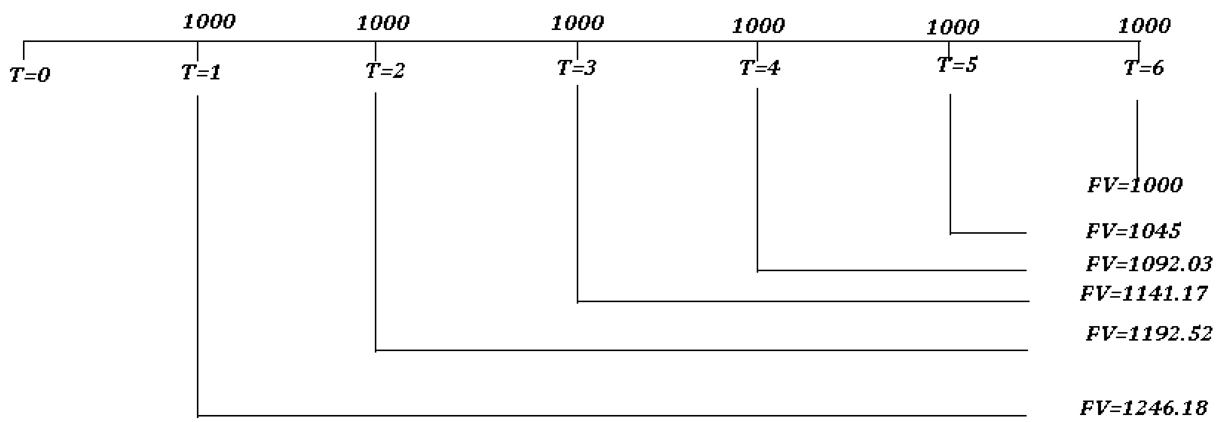
$$FV_t = P * \left[\frac{(1+r)^t - 1}{r}\right]$$

Future Value of Annuity Due:

$$FV_t = P * \left[\frac{(1+r)^t - 1}{r}\right] * (1+r)$$

Example

You just turned 16 and got hired for your first part-time job. You would like to save some money for a trip to Europe with your friends immediately after you graduate from university. As such, you decide to save $1000 at the end of every year (starting this year) until the age of 22. You have found a bank that offers you annual compounding at 4.5%. How much will you have saved up right after graduation?



The sum of all of these future values is the total amount of funds accumulated at the end of the six years. The sum is $**6,716.90**.

Alternatively, the formula for the Future Value of the Annuity Immediate can also be used to arrive at the same figure.

$$FV_t = P * \left[\frac{(1+r)^t - 1}{r}\right]$$

$$FV_6 = 1000 * \left[\frac{(1.045)^6 - 1}{0.045}\right]$$

$$FV_6 = \$6,716.90$$

→ ***Present Value of Annuities***

The present value of an annuity refers to the value of a stream of payments discounted to time zero with respect to a given rate of return. To make this clearer, consider the following scenario: Your father has just retired today and you would like to provide him with an annual income of $50,000 each year for five years starting at the end of this current year. To do this, you purchase a Payout Annuity from Manulife Financial with just a onetime initial investment this very day at a specified rate of return **r.** This initial investment amount is also known as the present value of all of the annuity payments discounted all the way back to the initial day of purchase at a rate of return **r.** Where,

$PV_t PV_t$ = Present Value of the annuity at the t^{th} payment period

P= Value of each payment (all payments being equal)

r = Rate of return

t = Number of payments periods

Present Value of Annuity Immediate:

$$PV_t = P * \left[\frac{1-(1+r)^{-t}}{r}\right]$$

Present Value of Annuity Due:

$$PV_t = P * \left[\frac{1-(1+r)^{-t}}{r}\right] * (1+r)$$

→ ***Growing Annuities and Perpetuities***

We have already discussed annuities, so let's define Perpetuity first. A **Perpetuity** is a series of cash flows that continues indefinitely. As such, we are only able to find the Present value of perpetuity because the future value of any infinite stream of positive cash flows would produce a future value of "infinity" (even if the interest rate were ZERO).

The present value of a perpetuity that generates payments of amount **P** at interest rate **r** is:

End-of-term payments

$$PV = \frac{P}{r}$$

Beginning-of-term payments

$$PV = \frac{P}{r}(1+r)$$

→ ***Present value of Growing Annuities***

A growing annuity is a finite stream of payments (**n** payments) that grow by a rate of **g** each period and earn a rate of return **r**.

The present value at time **t** of such a growing annuity can be computed as follows:

$$PV_t = \frac{P_0}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^t\right]$$

Where P_0 is your first grown payment.

→ ***Present Value of Growing Perpetuities***

A growing perpetuity is an infinite amount of payments that grown at a rate of **g** each period and earn a rate of return **r**.

If P_0 is your first grown payment, then the present value of such a perpetuity can be computed as follows:

$$PV_t = \frac{P_0}{r-g}$$

The Amortized Loan

An amortized loan is a type of loan whereby a borrower borrows a sum of money from a lender, and repays the lender in equal installments at regular intervals. There are **n** installments in total. Though all payments are equal, each payment consists of a principal portion as well as a portion of interest owing for that period. With an amortized loan, the Outstanding Balance Owing (OLB) at any given time **t** is equal the present value of all remaining payments all discounted back to time **t.**

The following formula generates the outstanding principal on a loan at any given time **t**, given that there are **n** payments of **P** in total and a rate of return **r**.

$$OLB_t = P * \left[\frac{1-(1+r)^{-(n-t)}}{r}\right]$$

Example

Suppose you borrow a standard loan of $10,000 from your local bank which you must pay back within the next five years with equal end-of-year installments. Suppose the bank charges you an APR of 4.5 % on the $10,000 loan. What is the outstanding balance on your loan immediately after you make your 3rd payment?

Remember, the Outstanding Loan Balance (OLB), also know as outstanding principal, is the *present value of the remaining payments that have not been made yet,* all discounted back to time **t** at the interest rate **r**.

So, the first step is to calculate the amount of each payment. To do this, equate the original loan to the present value of all five payments at time zero.

$$PV_t = P * \left[\frac{1-(1+r)^{-t}}{r}\right]$$

$$10000 = P * \left[\frac{1-(1.045)^{-5}}{0.045}\right]$$

P=$2,277.92 is the amount of each payment

Now, to find the outstanding principal amount on your loan immediately after your third installment (ie after three year/ two years to maturity) simply discount the last two payments (four and five) back to time 3.

$$OLB_t = P * \left[\frac{1-(1+r)^{-(n-t)}}{r}\right]$$

$$OLB_3 = 2277.92 * \left[\frac{1-(1.045)^{-(5-3)}}{0.045}\right]$$

$$OLB_3 = \$4,265.79$$

A visual representation of the Loan above:

Time	PMT	Interest in PMT	Principal in PMT	OLB
0	**0**	**0**	**0**	**10000**
1	**2277.92**	10000*r = **450**	2277.92-450=**1827.92**	10000-1827.92=**8172.08**
2	**2277.92**	8172.08*r = **367.74**	2277.92-367.74=**1910.18**	8172.08-1910.18=**6261.90**
3	**2277.92**	6261.90*r = **281.79**	2277.92-281.79=**1996.13**	6261.90-1996.13=**4265.77**
4	**2277.92**	4265.77*r = **191.96**	2277.92-191.96=**2085.96**	4265.77-2085.96=**2179.81**
5	**2277.92**	2179.81*r = **98.11**	2277.92-98.11=**2179.81**	2179.81-2179.81= **0**

Home Mortgages

A home mortgage is simply a loan that one takes from a financial institution (lender) to purchase a home at time zero. This mortgage can then be paid back in equal installments at equal time intervals over a specified period of time, given a certain rate of interest (APR). A home mortgage is basically an amortized loan with one KEY DIFFERENCE. In Canada, home mortgages offer APRs that are always compounded semi-annually REGARDLESS of your payment frequency (annual, semi-annual, monthly, etc). So, the first step in solving a mortgage problem is always to find the effective interest rate for each of your payment intervals. Given a standard APR on your mortgage, and your payment frequency per year **m,** your effective interest rate per payment period; **i,** can be computed as follows:

$$i = \left[1 + \frac{APR}{2}\right]^{\frac{2}{m}} - 1$$

Example

You take out a ten year loan for $100,000 at an APR of 8%. You negotiate with your mortgage specialist to make monthly payments. What is your effective rate per payment period? What are your monthly payment amounts?

A ten year loan with monthly payments generates a total of 120 payments over the loan term.

The effective rate per monthly period is as follows:

$$i = \left[1 + \frac{APR}{2}\right]^{\frac{2}{m}} - 1$$

$$i = \left[1 + \frac{0.08}{2}\right]^{\frac{2}{12}} - 1$$

$$i = 0.6558197\%$$

Using this rate, your monthly payments can be computed by equating the original loan at time zero to the present value of all payments.

$$PV_t = P * \left[\frac{1 - (1 + r)^{-t}}{r}\right]$$

$$100000 = P * \left[\frac{1 - (1.006558197)^{-120}}{0.006558197}\right]$$

P**=$1,206.41**

So, your monthly payments on this loan are **$1,206.41.**

Helpful tips for solving TVM problems!

i. Draw a detailed timeline indicating all cash inflows, cash outflows, unknown figures (i.e. what you are solving for on the timeline), any PV or FV figures that are given, and lastly indicate each cash flow (payment) period

ii. Determine your equation of value: equate all cash inflows to cash outflows

iii. Determine how to calculate the common point(s) in time that you are trying to solve for

Chapter 2: Short-Term Financing

Calculating the Effective Annual Rate of a Short-Term Loan

In general when calculating EAR for a loan with a term of one year we use the following formula:

$$r_{ear} = \left[1 + \frac{\$Interest}{\$Principal}\right] - 1$$

However, if the term of the loan is less than one year, we use the following formula:

$$r_{ear} = \left[1 + \frac{\$Interest}{\$Principal}\right]^{\frac{365}{term}} - 1$$

Many loans have fees that borrowers have to pay which decreases the amount of principal received. In such a situation use the following formula:

$$r_{ear} = \left[1 + \frac{\$Interest + \$Fees}{\$Net\ Amount\ Borrowed}\right]^{\frac{365}{term}} - 1$$

Fees are generally expressed as a percentage of the principal in annual terms. If a free is pro-rated (usually occurs if the term of the loan is less than one year) that means it is reduced based on the term of the loan.

Fee (pro-rated) = (%fee)(term/365)($loan)

If a fee is payable at the beginning you must subtract the amount of the fee from the principal. If the fee is payable at maturity the principal remains constant.

Example

The National Swiss Bank offers you a loan with a standard term of 90 days, nominal interest of 10% and a pro-rated free of 1.2% payable at maturity. What is the effective annual rate on the loan?

Solution

Because no loan amount was given, we assume the loan was $1.

Since our interest is nominal, we need to find our interest for the term of the loan, 90 days.

Interest = $0.1 \times \frac{90\ 90}{365\ 365}$

= 0.024657534

The fee is pro-rated thus we need to reduce the fee to apply only to the term of our loan.

Fee = $0.012 \times \frac{90\ 90}{365\ 365}$

= 0.002958904

$$r_{ear} = \left[1 + \frac{\$Interest + \$Fees}{\$Net\ Amount\ Borrowed}\right]^{\frac{365}{term}} - 1\left[1 + \frac{\$Interest + \$Fees}{\$Net\ Amount\ Borrowed}\right]^{\frac{365}{term}} - 1$$

$$= \left[1 + \frac{0.024657534 + 0.002958904}{\$1}\right]^{\frac{365}{90}} - 1\left[1 + \frac{0.024657534 + 0.002958904}{\$1}\right]^{\frac{365}{90}} - 1$$

= 23.89203%

→ Discount Interest

If a term loan has discount interest you simply deduct the interest from the principal at the beginning of the term.

$$r_{ear} = \left[1 + \frac{\$Interest + \$Fees}{\$Principal - \$Interest}\right]^{\frac{365}{term}} - 1\left[1 + \frac{\$Interest + \$Fees}{\$Principal - \$Interest}\right]^{\frac{365}{term}} - 1$$

→ Installment Loans

Similar to an amortized loans, but payments are calculated as the sum of interest plus principal dived by the number of payments. This increases the effective annual rate.

Payments = $\left[\frac{\$Interest + \$Principal}{Number\ of Payments}\right]\left[\frac{\$Interest + \$Principal}{Number\ of Payments}\right]$

→ Compensating Balance

It is common for a bank to require a compensating balance. A compensating balance is a certain portion of the principal that the borrower is required to keep on deposit in a financial institution, thus not receiving the entire amount of principal. This increases the cost of capital since the borrower is paying interest on principal that they cannot use.

NAB = $[1 - \%Compensating\ Balance] x\ \$Principal[1 - \%Compensating\ Balance] x\ \$Principal$

→ Commercial Paper and Bankers' Acceptance

Commercial paper is an unsecured short-term debt instrument that is issued by corporations. Maturities on commercial papers are usually no longer than 270 days and are issued at a discount.

Bankers' Acceptance is a short-term debt instrument issued by non-financial institutions and guaranteed by the bank.

Quoted Rate = $\left[\frac{Face - Price}{Price}\right] x \frac{365}{term}\left[\frac{Face - Price}{Price}\right] x \frac{365}{term}$

r_{ear} = $\left[1 + \frac{Quoted\ Rate}{(\frac{365}{term})}\right]^{\frac{365}{term}} - 1\left[1 + \frac{Quoted\ Rate}{(\frac{365}{term})}\right]^{\frac{365}{term}} - 1$

<u>OR</u>

Cost Over Term = $\left[\frac{Face + Fee - Price}{Price}\right]\left[\frac{Face + Fee - Price}{Price}\right]$

$$r_{ear} = [1 + \text{Cost Over Term}]^{\frac{365}{\text{term}}} - 1[1 + \text{Cost Over Term}]^{\frac{365}{\text{term}}} - 1$$

→ Line of Credit

It is the maximum amount a person is allowed to borrow from the lender. It is similar to the loan but the main difference is that most institutions will not charge you interest on the portion of the loan that you do not take out. The borrower is allowed to take out any amount of money up to a maximum of the principal. Maturity on a line of credit is usually one year, at which time the entire amount must be repaid.

Chapter 3: Fixed Income Securities

This is an investment that provides a return in the form of fixed periodic payments and the eventual return of principal at maturity. Unlike a variable-income security, where payments change based on some underlying measure such as short-term interest rates, the payments of a fixed-income security are known in advance.

Bonds

A debt investment in which an investor loans money to an entity (corporate or governmental) that borrows the funds for a defined period of time at a fixed interest rate. Bonds are used by companies, municipalities, states and U.S. and foreign governments to finance a variety of projects and activities.

Usually, attached to a bond certificate is a specification of the <u>issuer, principal, holder, maturity and coupons.</u>

The indebted entity (issuer) issues a bond that states the interest rate (coupon) that will be paid and when the loaned funds (bond principal) are to be returned (maturity date). Interest on bonds is usually paid every six months (semi-annually). The main categories of bonds are corporate bonds, municipal bonds and U.S. Treasury bonds, notes and bills, which are collectively referred to as simply "Treasuries".

Two features of a bond - credit quality and duration - are the principal determinants of a bond's interest rate. Bond maturities range from a 90-day Treasury bill to a 30-year government bond. Corporate and municipals are typically in the three to 10-year range.

Companies (and rarely governments) always carry some sort of probability of defaulting on their credit. In other words, since bonds are a form of raising capital by means of debt, any holder of a bond can potentially default on their payments to their bond issuers. Therefore, there are institutions that rate every corporation's probability of default and assign a rating in order to inform investors of the risk associated with these potential bondholders. The two most commonly used rating companies are Standard & Poors (S&P) and Moody's. Under the S&P standard, any corporations with a rating of BBB and above are considered companies that have very little likelihood of credit default and almost considered as "risk free" (be careful with this judgment though!) Anything with a rating of BB/B+ or below is considered risky and is often referred to as a 'Junk Bond'.

Zero Coupon Bond

A Zero coupon bond is an investment in which an issuer lends the holder a sum of money **P** and receives a face value **F** after a time period **n** which has accumulated interest at an APR **i**.

The price (present value) of a zero coupon bond is computed as follows:

$$P = \frac{F}{(1+i)^n}$$

Interest Rates and Inflation - The Fischer Effect

Spot rate **s** - The rate of interest available at an exact point in time for a commodity or a security

Forward rate **f** - The rate of interest that a commodity or security will be available at in a given future time (this is a projected estimate, not a factual figure)

Inflation rate **I** - the rate at which prices of goods and services are rising, and hence decreasing purchasing power to the consumer.

The Fischer Formula allows you to determine your nominal rate **R** of return at a point in time given your real rate of return and the inflation rate:

r = (1+R)(1+I)-1

Now, let's focus on finding spot rates for zero coupon bonds. When you are given a zero coupon bond with a purchase price and a face value that is issued for a certain amount of time **n,** you can easily find the **n**-year spot rate for the bond using the above pricing formula and solving for the interest rate **i.**

Example

You purchase a bond for $93.14 today and are promised a face value of $100 two years from now. What is your 2 year spot rate?

$$P = \frac{F}{(1+s_n)^n}$$

$$93.14 = \frac{100}{(1+s_2)^2}$$

$$s_2 = 3.6172\%$$

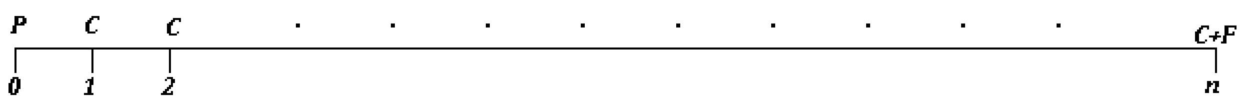
Now suppose you are given spot rates on a bond for this year and would like to determine your expected forward spot rate for next year. Or suppose you are given a spot rate for this year and an expected spot rate for the following year and you would like to determine your expected forward spot rate for the third year. This leads to the following formula, which determines the implied forward rate **f** for the time interval [t,s]:

$$f_{[t,s]} = \left[\frac{(1+s_s)^s}{(1+s_t)^t}\right]^{\frac{1}{s-t}} - 1$$

Basic Bond Pricing

Besides zero coupon bonds, there are bonds that offer actual coupons at regular intervals. These bonds are very similar to annuities in the sense that they generate a steady sequence of cash flows (coupons) for each coupon period for a specified amount of periods.

The following is an illustration of a life of a bond where **P** is the purchase price of the bond, **C** is the coupon payment for the **n** periods, and **F** is the Face value received by the investor at the end.



So, the Price of a bond with C coupon payments and Face value for n coupon periods is just all of the coupon payments discounted back to time zero at the investor's prospective yield rate, plus the Face value discounted back to time zero also at the investor's prospective yield rate **j**. The following is the basic pricing formula for bonds:

$$P = C * \left[\frac{1-(1+j)^{-n}}{j}\right] + \frac{F}{(1+j)^n}$$

Example

Suppose you are interested in purchasing a ten year bond that pays 5% annual coupons and a face value of $2000 at the end of the ten years. Also, you are hoping to earn an 8% yield on your investment, what is the fair price of the bond today?

C = $2000*5% = $100

$$P = C * \left[\frac{1-(1+j)^{-n}}{j}\right] + \frac{F}{(1+j)^n}$$

$$P = 100 * \left[\frac{1-(1.08)^{-10}}{0.08}\right] + \frac{2000}{(1.08)^{10}}$$

P=1,597.40

Note: When the coupon rate is lower than the investor's yield rate, the bond sells at a discount (P<F) because a lower rate is being discounted by a higher one. When the coupon rate is higher than the investor's yield rate, the bond sells at a premium (P>F) because a larger rate is being discounted by a smaller one. Lastly, when the coupon rate is equal to the investor's yield rate, the bond sells at par value (P=F) because the coupon rate is being discounted by itself.

Summary

Position		
Discount	Coupon rate < yield	P<F
Premium	Coupon rate > yield	P>F
Par	Coupon rate = yield	P=F

Practice Problems

Problem 1
You invest $100 today in a GIC at an annual effective rate of 5% for two years compounded annually. What is your total investment worth at the end of the two years?

Problem 2
Consider the scenario discussed above with Manulife Financial. Suppose you wish to provide the $50,000 annually with start-of-year payments to your father (beginning at the end of this current year) for five years at an annual effective rate of 3.5%. What is the fair price of such a payout annuity? (In other words, what is the present value of this payout annuity?)

Problem 3
Suppose you have just turned twenty years old and would like to start saving for retirement. To do this, you would like to deposit $X dollars in a savings account semi-annually that offers an APR of 7.5%. You expect to retire on your 65^{th} birthday and would like to receive an annual income of $70,000 thereafter for fifteen years. After the fifteen years of income, you would like to have a remaining fund balance of $80,000 which you plan to give to your grandchildren at the time. Suppose you make the $X contributions at the beginning of every half-year until retirement, and then immediately transfer your funds into a payout annuity that offers 4% APR. You then receive equal end-of-year income payments after retirement for the fifteen years. What are your semi-annual contributions? (Solve for X).

Problem 4

Suppose your sister won the lottery and decided to set up a Life Income Fund (LIF) for you and your family today. Suppose that the first five end-of-year payments will grow at a rate of 3%, the first payment being $10,000 in a year from now. After these five payments, you will begin to receive annual payments that will grow at a rate of 2% forever. Suppose the LIF offers 5% APR on your funds, what is the initial amount that your sister invested in the LIF today?

Problem 5

Your friend lends you $25,000 for 60 days at an annual interest rate of 6% and charges you a pro-rated fee of 0.4% payable at the beginning of the term. What is the EAR on your loan?

Problem 6

You take out an installment loan of $5,000 at an annual interest rate of 4.2% The loan matures in 7 years. What are your level payments?

Problem 7

Suppose there are currently two zero coupon bonds on the market. The first one is a 2 year bond with face value of $1000 and price of $920. The second bond is a 3 year bond with a face value of $1000 and price of $850. What is the implied 3 year forward rate? In other words, what is the implied forward rate for time interval [2,3]?

Solutions

Problem 1

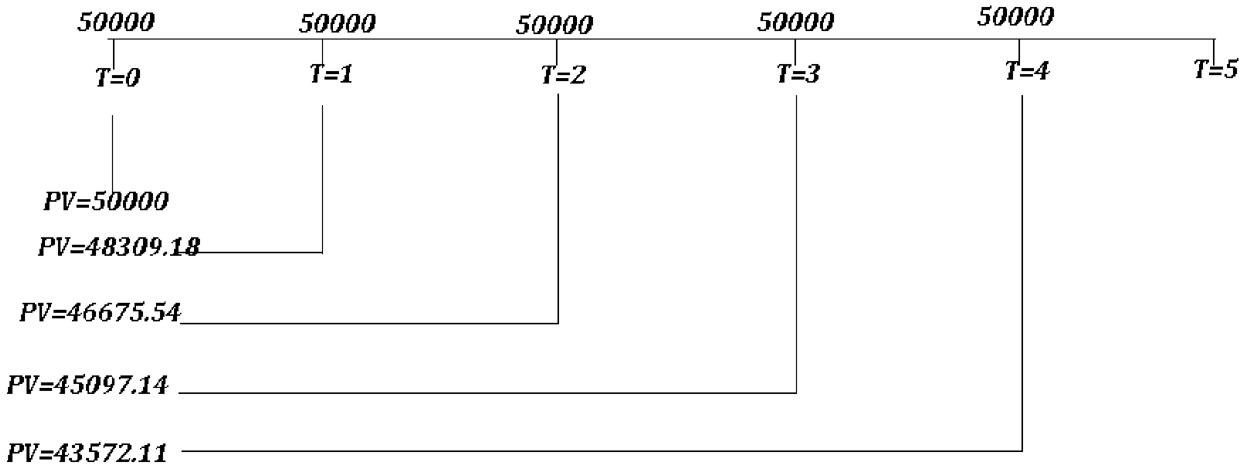
$FV = PV * (1+r)^t$

$FV = 100 * (1.05)^2 FV = 100 * (1.05)^2 = 100 * FVIF_{5\%,2} 100 * FVIF_{5\%,2}$

FV = $110.25

Problem 2

Well, given that the payments will be generated at the beginning of each payment period, this should trigger the words ANNUITY DUE in your mind!



The sum of the present values of all of the cash flows adds up to $233,653.97. Therefore, you can purchase the above annuity at its present value of $233,653.97.

Alternatively, you can use the Present Value Annuity Due formula to arrive at the same answer.

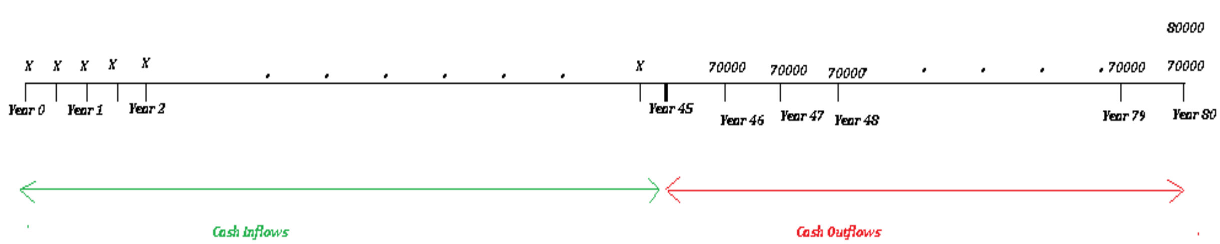
$$PV_t = \left[\frac{1-(1+r)^{-t}}{r}\right] * (1+r)$$

$$PV_5 = \left[\frac{1-(1.035)^{-5}}{0.035}\right] * (1.035)$$

$$PV_t = \$233,653.97$$

Problem 3

The following illustration is a timeline of the entire cash flow problem:



The idea behind this problem is to equate the future value of the accumulated \$X savings at year 45 to the present value of the \$70,000 income payments and \$80,000 balance at year 45. The reason why they must equal is because the total accumulation (Future Value) of your semiannual contributions must be JUST ENOUGH for you to earn \$70,000 for 15 years thereafter and have a remaining balance of \$80,000.

Given that the \$X payments are made semi-annually, and the APR=8%, it is evident that semi-annual compounding would earn a rate of 7.5%/2 = 3.75%. These semiannual contributions also generate an annuity DUE sequence of cash flows which we must find the future vale of at year 45.
Therefore, the FV of all \$X payments are year 45 are:

$$FV_t = P * \left[\frac{(1+r)^t - 1}{r}\right] * (1+r)$$
$$FV_{90} = X * \left[\frac{(1.0375)^{90} - 1}{0.0375}\right] * (1.0375)$$
$$FV_{90} = \mathbf{X * 732.4606188}$$

Now, we would like to find the present value of all cash flows after retirement at year 45 in order to equate the two FV with PV to solve for X. Therefore, the present value of all annual income payments and ending balance at year is: (Note: these cash flows are discounted for the 15 years after retirement).

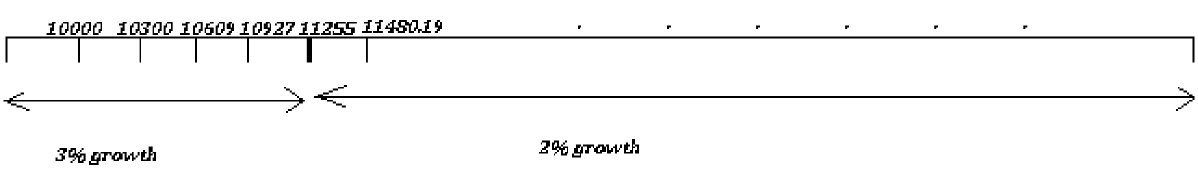
$$PV_t = P * \left[\frac{1-(1+r)^{-t}}{r}\right] PV_t = P * \left[\frac{1-(1+r)^{-t}}{r}\right] + \frac{Ending\ Balance Ending\ Balance}{(1+r)^t \qquad (1+r)^t}$$
$$PV_{15} = 70000 * \left[\frac{1-(1.04)^{-15}}{0.04}\right] + \frac{80000}{(1.04)^{15}}$$
$$PV_{15} = \mathbf{\$822,708.28}$$

Lastly, equate the Future Value of all semiannual contributions to the present value of the annual income payments and ending balance at year 45.

$$FV_{90} = PV_{15}$$
$$\mathbf{X * 732.4606188 = \$822,708.28}$$
X=\$1,123.22

Therefore, in order for this payment schedule to work, you must make semiannual contributions of \$1,123.22 from the age of 20 to the age of 65 (time of retirement).

Problem 4



First, we know that the first payment for the perpetuity is at year 6, so find the PV of the perpetuity at year 5 and discount it back to time zero.

$$PV_t = \frac{P_0}{r-g}$$
$$PV_5 = \frac{11480.19}{0.05 - 0.02}$$
$$PV_5 = 382673$$
$$PV_0 = \frac{382673}{(1.05)^5}$$
$$PV_0 = \$\mathbf{299,834.31}$$

This is the present value of the perpetuity alone. Next, find the present value of the five payment growing annuity. NOTE: do not use $10,000 as $P_0 P_0$ in this case because that is not the first grown payment. The first grown payment is the first payment that grew after the initial payment of $10,000 which is $10,300. So, find the present value of the annuity at time one using the first grown payment, then discount it back by one year to year 0, and discount the initial payment by one year to year zero as well.

Present value of the 5 payment annuity is as follows:

$$PV_t = \frac{\frac{P_0}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^t\right]}{(1.05)^1} + \frac{10000}{1.05^1}$$
$$PV_t = \frac{\frac{10300}{0.05-0.03}\left[1 - \left(\frac{1.03}{1.05}\right)^4\right]}{(1.05)^1} + \frac{10000}{1.05^1}$$
$$PV_0 = \$\mathbf{45,839.21}$$

So, in order to produce this LIF, your sister invested a total amount of 299834.31+45839.21 = $345,673.52 today.

Problem 5

Free (pro-rated) = (0.004)($\frac{60\ 60}{365\ 365}$)($25,000)
= $16.44

Interest =(0.06)($\frac{60\ 60}{365\ 365}$)($25,000)
=$246.58
NAB = $25,000 - $16.14
= $24,983.86

r_{ear} = $\left[1 + \frac{\$\text{Interest} + \$\text{Fees}}{\$\text{Net Amount Borrowed}}\right]^{\frac{365}{\text{term}}} - 1\left[1 + \frac{\$\text{Interest} + \$\text{Fees}}{\$\text{Net Amount Borrowed}}\right]^{\frac{365}{\text{term}}} - 1$

$= \left[1 + \frac{\$246.58+\$246.58}{\$24983.86}\right]^{\frac{365}{60}} - 1\left[1 + \frac{\$246.58+\$246.58}{\$24983.86}\right]^{\frac{365}{60}} - 1$

= 6.57813%

Problem 6

Interest = \$5,000 x 0.042
= \$210

Payments = $\left[\frac{\$\text{Interest} + \$\text{Principal}}{\text{Number of Payments}}\right]\left[\frac{\$\text{Interest} + \$\text{Principal}}{\text{Number of Payments}}\right]$

$= \left[\frac{\$210+\$5,000}{7}\right]\left[\frac{\$210+\$5,000}{7}\right]$

= \$744.29

Problem 7

First, find the spot rates corresponding to the two zero coupon bonds.

For the 2 year bond:

$$P = \frac{F}{(1 + s_n)^n}$$

$$920 = \frac{1000}{(1 + s_2)^2}$$

$$s_2 = \mathbf{4.2572}\%$$

For the 3 year bond:

$$P = \frac{F}{(1 + s_n)^n}$$

$$850 = \frac{1000}{(1 + s_3)^3}$$

$$s_3 = \mathbf{5.5667}\%$$

Now, to find the implied 3 year forward rate:

$$f_{[t,s]} = \left[\frac{(1 + s_s)^s}{(1 + s_t)^t}\right]^{\frac{1}{s-t}} - 1$$

$$f_{[2,3]} = \left[\frac{(1.055667)^3}{(1.042572)^2}\right]^{\frac{1}{3-2}} - 1$$

$$f_{[2,3]} = \mathbf{8.2352\%}$$

References

IFinance. Web. Fall 2010. <http://finance.tempic.com/cgi/index.cgi?r8wk2m=38gsl3oz&classModuleID=3>.

"Investopedia.com: Financial Dictionary." *Investopedia.com - Your Source For Investing Education*. Web. Fall 2010. <http://www.investopedia.com/dictionary/>