ECON 221
October, 2013
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Mid-Term Examination

Answer ALL questions. Non-programmable calculators permitted. No notes allowed.

1. (35 Points) Consider the random variable $X \sim Bin(3, 0.4)$.
   a. What is the mean and variance of X?
   b. In a Table, give the probability distribution function of X. Show work.
   c. Using the Table, compute the mean and variance of X.
   d. Suppose we define a new random variable $Y = (X_1 + X_2 + X_3)/3$, where each $X_i \sim Bin(3, 0.4)$ and independent of one another. What is the mean and variance of Y?

2. (35 points) Consider the following joint probability distribution function for the random variables X and Y.

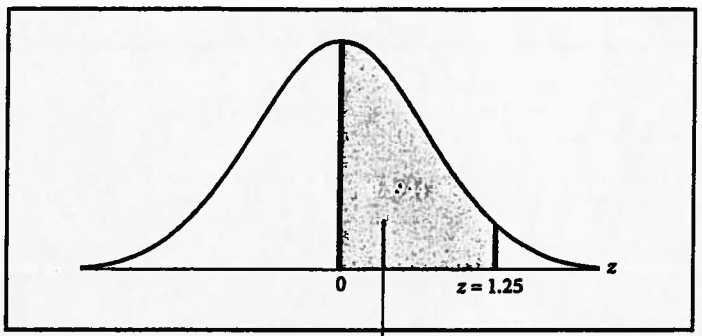
		X	
		1	2
Y	0	.30	.20
	1	.25	.25

   a. What is the mean and variance of the two marginal random variables X, Y?
   b. What is the mean and variance of the conditional random variable $X \mid Y = 1$?
   c. What is Var(2X + 4), E(-X + 5Y – 7)?

3. (20 points)
   a. Suppose $X \sim N(1200, 250^2)$. Compute $Pr(900 < X < 1300)$.
   b. For $Z \sim N(0,1)$, find the cutoff point for the top 10% of the distribution.
   c. For X as in part a., find the cutoff point for the top 10% of the distribution.
   d. $X \sim N(1200, 25^2)$, find the cutoff point for the top 10% of the distribution.

4. (10 points)
   Suppose one has a 10-sided die with faces 1 through ~~24~~ 10. The die is constructed so that each of the 10 sides is equally likely to come up.
   a. How would you go about simulating tosses of this die in EXCEL?
   b. Suppose you simulate 500 tosses in this way. What would you expect the average of these tosses to be? Suppose the average doesn't correspond to this figure?
   c. Suppose you toss the die 1000 times and then you find that the average is even further from the expected value. How would you re-act?

Standard Normal Distribution Table



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997

Dr. William L. Carlson, prepared using Minitab 16.

ECON 221 MT

1. $X \sim Bin(3, .4)$ $[Bin(n,p)]$ (35 pts)

(5) a) $\mu_X = E(X) = np = 3(.4) = 1.2$

$\sigma^2_X = Var(X) = np(1-p) = 3 \times (.4) \times (.6) = .72$

b)

$x=x$	C^3_x	$p^x(1-p)^{3-x}$	$P(X=x)$	$x_i p(x_i)$	$x_i^2 p(x_i)$
0	1	$(.4)^0(.6)^3$	.216	0	0
1	3	$(.4)^1(.6)^2$	.432	.432	.432
2	3	$(.4)^2(.6)^1$	.288	.576	1.152
3	1	$(.4)^3(.6)^0$	.064	.192	.576
			1.000	1.200	2.160

(15)

c) $E(X) = \sum x_i p(x_i) = 1.20$

(5) $Var(X) = E(X^2) - E(X)^2 = \sum x_i^2 p(x_i) - (\sum x_i p(x_i))^2$

$= 2.16 - (1.2)^2 = 2.16 - 1.44 = .72$

d) $Y = \frac{1}{3}(X_1 + X_2 + X_3)$ $X_i \sim Bin(3, .4)$

(10) $E(Y) = \frac{1}{3}(E(X_1) + E(X_2) + E(X_3))$

$= \frac{1}{3}(3 \times (1.2)) = 1.2$

$Var(Y) = \frac{1}{9}(Var(X_1) + Var(X_2) + Var(X_3))$

$= \frac{3}{9} Var(X) = \frac{1}{3}(.72) = .24$

2. (35 pts)

		X = 1	X = 2	Marginals
Y	0	.3	.2	.5
	1	.25	.25	.5
Marginals		.55	.45	1

a) $E(Y) = \sum_y y\,p(y) = 0\cdot(.5) + 1\,(.5) = .5$

$E(Y^2) = \sum_y y^2 p(y) = 0^2(.5) + 1^2(.5) = .5$

$Var(Y) = E(Y^2) - E(Y)^2 = .5 - .5^2 = .5 - .25 = .25$

b) $E(X) = \sum_x x\,p(x) = 1\cdot(.55) + 2\cdot(.45) = .55 + .9 = 1.45$

$E(X^2) = \sum_x x^2 p(x) = 1^2(.55) + 4\cdot(.45) = .55 + 1.80 = 2.35$

$Var(X) = E(X^2) - E(X)^2 = 2.35 - 1.45^2 = .2475$

c) Distribution of $(X \mid Y=1)$

$P(X=1 \mid Y=1) = \dfrac{P(X=1 \text{ and } Y=1)}{P(Y=1)} = \dfrac{.25}{.5} = .5$

$P(X=2 \mid Y=1) = \dfrac{P(X=2 \text{ and } Y=1)}{P(Y=1)} = \dfrac{.25}{.5} = .5$

d)

$E(X \mid Y=1) = \sum_x x\,P_{X|Y=1}(x) = 1\cdot(.5) + 2(.5) = 1.5$

$E(X^2 \mid Y=1) = \sum_x x^2\,P_{X|Y=1}(x) = 1\cdot(.5) + 4\cdot(.5) = 2.5$

$Var(X) = 2.5 - 1.5^2 = 2.5 - 2.25 = .25$

c) $Var(2X+4) = 4\,Var(X) + 0 = 4 \cdot .2475 = .99$

$E(-X+5Y-7) = -E(X) + 5E(Y) - 7$

$= -1.45 + 5*(.5) - 7$

$= -1.45 + 2.5 - 7 = -1.45 - 4.5 = -5.95$ ✓

10/

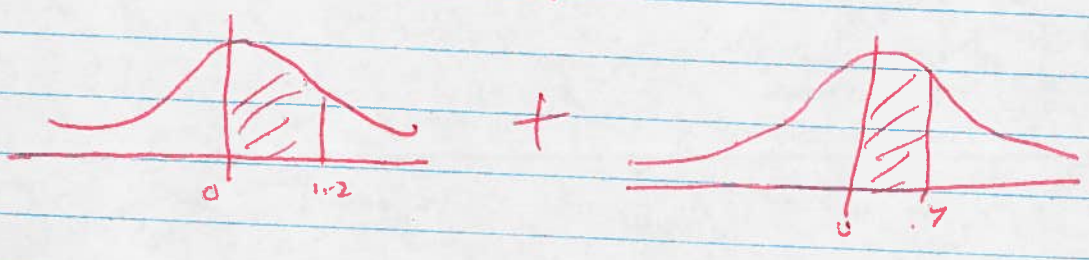
3. (20 pts)

a/ $X \sim N(1200, 250^2)$

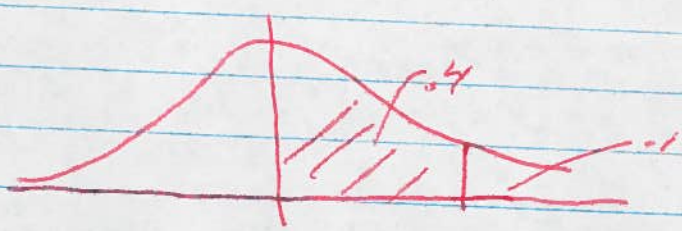
$Pr[900 < X < 1300]$

$= Pr\left[\frac{900-1200}{250} < Z < \frac{1300-1200}{250}\right]$

$= Pr[-1.2 < Z < 2.4]$

$=$ 

$= .3849 + .1554 = .5404$

b/ 

$3 = 1.28$ so $Pr[Z > 1.28] = .10$

c/ $Pr[X > \mu_X + (1.28)\sigma_X] = .10$ (from (b))
$Pr[X > 1200 + (1.28)(250)] = .10$
$Pr[X > 1520] = .10$

d/ $Pr[X > 1200 + (1.28)(25)] = .10$
$Pr[X > 1232] = .10$

4. (10 pts)

a) The discrete random variable is given by:

$X = x$	$P[X = x]$
1	1/10
2	1/10
⋮	⋮
9	1/10
10	1/10

Put these numbers into 2 columns of a spreadsheet. Then

Data → Data Analysis → Random Number Generation
Choose 1 variable, 500 rows
+ Discrete: Here input above 2 columns.

EXCEL gives you 500 rows in a column.

b) $E(X) = \sum x\, p(x) = \frac{1}{10}(1+2+\cdots+9+10)$

$= \frac{1}{10}\frac{10\times 11}{2} = \frac{55}{10} = 5.5$

Take average of output. How close to 5.5 — there will be some roughly error.

c) If you toss the die 1000 times, you'd expect the average to be at least as close as in (b).

If not, why that die is loaded!!