Multiple Choice Section Questions (1-4)

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Question 1. *Find $f'(e^2)$ when $f(x) = x^2 \ln(x)$*

A) $2 + 5e^2$ **B)** $2e^2 + 2e$ **C)** $5e^2$ **D)** $2e^2 + 2\ln(2)$ **E)** $e^2 + 2\ln(2)$

$$f'(x) = 2x \ln(x) + \frac{x^2}{x} = 2x \ln(x) + x$$

$$f'(e^2) = 2e^2 \underbrace{\ln(e^2)}_{2} + e^2$$

$$= 5e^2$$

Question 2. *Suppose that the demand function for a product is given by $p = -x^2 + 2x + 10$. What is the elasticity of demand when $x = 2$? Is demand elastic or inelastic?*

A) $\eta = -\frac{2}{5}$, elastic **B)** $\eta = -\frac{2}{5}$, inelastic **C)** $\eta = -\frac{5}{2}$, elastic
D) $\eta = -\frac{5}{2}$, inelastic **E)** $\eta = 1$, unit elastic

$p'(x) = -2x + 2$

$p'(2) = -4 + 2 = -2.$

$P(2) = -4 + 4 + 10$
$= 10$

$$\eta = \frac{\frac{10}{2}}{-2} = -\frac{5}{2}$$

$|\eta| > 1 \rightarrow$ elastic

Question 3. *Given the function* $f(x) = \frac{1}{x^2+3}$, *which of the following statements is correct? (Only one is correct.)*

A) $f(x)$ has a local minimum at $x = -3$
B) $f(x)$ has a local maximum at $x = -3$
C) $f(x)$ has an local maximum at $x = 2$
D) $f(x)$ has a local maximum at $x = 0$
E) $f(x)$ has a local minimum at $x = 0$

$$f'(x) = \frac{-2x}{(x^2+3)^2} = 0 \rightarrow x = 0$$

		$\frown$ $x=0$	
f	$\nearrow$	$x=0$	$\searrow$
f'	$+$		$-$

Question 4. *Consider the function* $g(x) = x^4 + 2x^3 - 12x^2 - 4$. *On what interval or intervals is the function concave down?*

A) $(-\infty, -2) \cup (1, \infty)$ B) $(1, \infty)$ C) $(-2, 1)$ D) $(-\infty, \infty)$
E) $(-2, \infty)$.

$$g'(x) = 4x^3 + 6x^2 - 24x$$

$$g''(x) = 12x^2 + 12x - 24$$

$$= 12[x^2 + x - 2] = 12(x+2)(x-1)$$

$$= 0 \rightarrow x = -2$$
$$x = 1$$

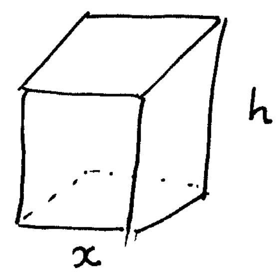
f	$\cup$	-2	$\cap$	1	$\cup$
f''	$+$		$-$		$+$

Long Answer Section Questions (5-8)

Question 5. (14 points) *A storage box with a square base must have a volume of 40 cubic centimeters. The top and bottom cost \$5 per square centimeter and the sides cost \$1 per square centimeter. Find the dimensions which minimize cost.*

Volume $= x^2h = 40$

2 points $\Rightarrow h = \dfrac{40}{x^2}$



2 points

$C(x,y) = (2x^2)(5) + (4xh)(1)$

4 points $= 10x^2 + 4xh$

$= 10x^2 + \dfrac{160}{x}$

4 points $C'(x) = 20x - \dfrac{160}{x^2}$

$= 0 \Rightarrow x^3 = 8$

$x = 2$

$h = \dfrac{40}{x^2} = \dfrac{40}{4} = 10$

$C''(x) = 20 + \dfrac{320}{x^3} > 0$ when $x = 2$

2 points $\hookrightarrow$ so $x = 2$ minimizes the cost

Long Answer Section Questions (5-8)

Question 5. (14 points) *When the price of a brand of golf ball is 8 dollars per golf ball, 30,000 golf balls are sold. When the price is raised to 10 dollars, 24,000 golf balls are sold. A golf ball costs 8 dollars to make, and the owners of the golf ball company had an initial cost of 22,000 dollars. How many golf balls should the manufacturer sell to maximize profit?* **Be sure to explain why your answer is an absolute maximum.**

x = # of golf balls, p = price/golf ball

p	x
8	30,000
10	24,000

$$m = \frac{\Delta p}{\Delta x} = \frac{-2}{6{,}000} = \frac{-1}{3{,}000}$$

$y = \frac{-1}{3{,}000}x + b$. Plug in $(8, 30{,}000)$ $\quad 8 = 10 + b$, $b = 18$

$$p = \frac{-1}{3{,}000}x + 18$$

$$R(x) = xp = \frac{-1}{3{,}000}x^2 + 18x$$

$$C(x) = 22{,}000 + 8x$$

$$P(x) = \frac{-1}{3{,}000}x^2 + 18x - (22{,}000 + 8x)$$

$$= \frac{-1}{3{,}000}x^2 + 10x - 22{,}000$$

$$P'(x) = \frac{-1}{1500}x + 10 \Rightarrow x = 15{,}000 \text{ is a CP.}$$

It is an absolute max, because $P(x)$ is a C.D. parabola.

Question 6. (16 points) *For the following function find the appropriate information (listed next page) to sketch the graph of the following function.*

$$f(x) = \frac{x}{x-3}$$

$D_f = \mathbb{R} - \{3\}$

$x = 0 \longrightarrow y = 0$

$y = 0 \longrightarrow x = 0$

$(0,0) \leftarrow$ x and y intecept

$\lim_{x \mapsto \infty} f(x) = \frac{1}{1} = 1$

$y = 1$ H.A

$x = 3$ V.A

(NO points if just written 3, or 1)

$f'(x) = \frac{-3}{(x-3)^2} < 0$

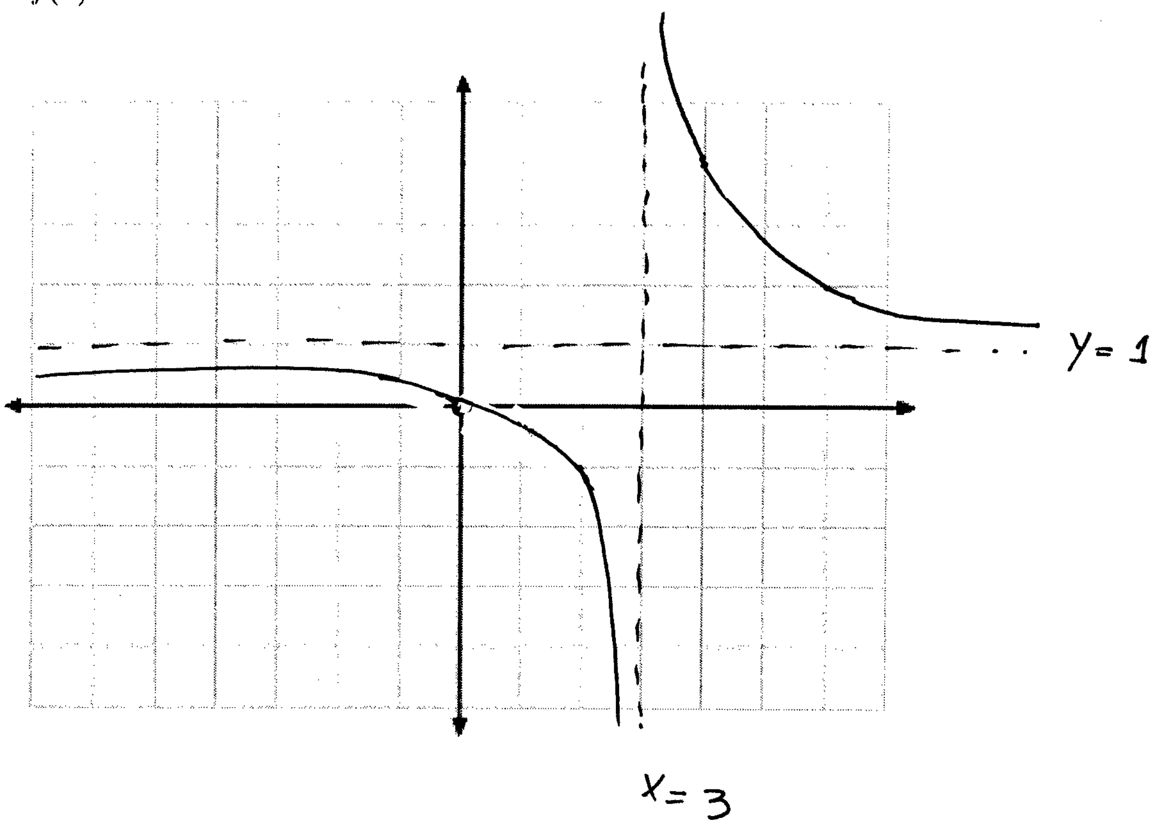
f decreases every where

NO local Extremas.

$f''(x) = \frac{6}{(x-3)^3} \neq 0 \rightarrow$ NO P.O.I.

f	$\cap$	$x=3$	$\cup$
f''	$-$		$+$

Graph of $f(x)$



1. Find the domain of the function
2. Find the y-intercept and plot it
3. Find the x-intercepts and plot them
4. Find the horizontal asymptotes and plot them
5. Find the vertical asymptotes and plot them
6. Find the critical numbers
7. Find the intervals of increase and decrease
8. Identify the relative extrema and plot them
9. Find the intervals of concave up and concave down
10. Identify the points of inflection and plot them
11. Fill in the rest of the graph using (7) and (9)

Question 7. (10 points)

(a) Suppose $f'(x) = 3x^2 + 8x - 4$, *and that* $f(1) = 3$. *Find* $f(x)$.

/4

$$f(x) = \int (3x^2 + 8x - 4)\,dx =$$

2 points → $f(x) = x^3 + 4x^2 - 4x + C$

1 point $f(1) = 1 + 4 - 4 + C = 3$

$\boxed{C = 2}$ → $f(x) = x^3 + 4x^2 - 4x + 2$

1 point

(b) Calculate

/6

$$\int \frac{4x+2}{x^2+x+1}dx$$

2 points

$u = x^2 + x + 1$

$du = (2x+1)\,dx$

2 points

$$\int \frac{4x+2}{x^2+x+1}\,dx = \int \frac{2(2x+1)}{x^2+x+1}dx = \int \frac{2\,du}{u}$$

$$= 2\ln|u| + c = 2\ln|x^2+x+1| + C$$

2 points

Multiple Choice Section Questions (1-4)

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Question 1. *Find $f'(e^2)$ when $f(x) = x\ln(x^2)$*

A) $2 + 2\ln(2)$ B) $6 + \ln(4)$ C) $6 + e^2$ D) $e^2 + 2\ln(2)$ E) 6

$$f'(x) = \ln(x^2) + x \cdot \frac{2x}{x^2}$$

$$= 2\ln x + 2$$

$$f'(e^2) = 2\ln e^2 + 2$$

$$= 4 + 2 = 6$$

Question 2. *Suppose that the demand function for a product is given by $p = -2x^2 + x + 14$ What is the elasticity of demand when $x = 2$? Is demand elastic or inelastic?*

A) $\eta = -\frac{4}{7}$, elastic B) $\eta = -\frac{4}{7}$, inelastic C) $\eta = -\frac{7}{4}$, elastic
D) $\eta = -\frac{7}{4}$, inelastic E) $\eta = 1$, unit elastic

$$p'(x) = -4x + 1 \qquad p(2) = -8 + 2 + 14 = 8.$$

$$p'(2) = -7 \qquad \eta = \frac{\frac{8}{2}}{-7} = -\frac{4}{7}$$

$$|\eta| < 1 \rightarrow \text{inelastic.}$$

Question 3. *Given the function* $f(x) = \frac{-2}{x^2+1}$, *which of the following statements is correct? (Only one is correct.)*

A) $f(x)$ has a local minimum at $x = -1$
B) $f(x)$ has a local maximum at $x = -1$
C) $f(x)$ has an local maximum at $x = 2$
D) $f(x)$ has a local maximum at $x = 0$
E) $f(x)$ has a local minimum at $x = 0$

$$f'(x) = \frac{(-2)(-1)2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2} = 0 \rightarrow x = 0$$

f	$\searrow$	$x=0$	$\nearrow$
f'	$-$		$+$

Question 4. *Consider the function* $g(x) = x^4 - 2x^3 - 12x^2 - 12$. *On what interval or intervals is the function concave up?*

A) $(-\infty, -1) \cup (2, \infty)$ B) $(-1, \infty)$ C) $(-1, 2)$ D) $(-\infty, \infty)$
E) $(2, \infty)$.

$$g'(x) = 4x^3 - 6x^2 - 24x$$
$$g''(x) = 12x^2 - 12x - 24$$
$$= 12[x^2 - x - 2]$$
$$= 12(x-2)(x+1)$$
$$x = -1$$
$$x = 2$$

g''	$\cup$	-1	$\cap$	2	$\cup$
g	$+$		$-$		$+$

Long Answer Section Questions (5-8)

Question 5. (14 points) *A storage box with a square base must have a volume of 20 cubic centimeters. The top and bottom cost \$5 per square centimeter and the sides cost \$2 per square centimeter. Find the dimensions which minimize cost.*

$$V = x^2 h = 20$$

$$h = \frac{20}{x^2}$$

$$C(x,y) = 5 \cdot 2 \cdot x^2 + (4xh)2$$

$$= 10x^2 + 8x\left(\frac{20}{x^2}\right)$$

$$= 10x^2 + \frac{160}{x}$$

$$C'(x) = 20x - \frac{160}{x^2}$$

$$= 0 \implies x^3 = 8$$

$$\boxed{x = 2}$$

$$h = \frac{20}{x^2} = \frac{20}{4} = \boxed{5}$$

$$C''(x) = 20 + \frac{320}{x^3} > 0 \rightarrow \text{when } x = 2$$

So $x = 2$ maximizes the cost

Long Answer Section Questions (5-8)

Question 5. (14 points) *When the price of a brand of golf ball is 8 dollars per golf ball, 32,000 golf balls are sold. When the price is raised to 12 dollars, 24,000 golf balls are sold. A golf ball costs 6 dollars to make, and the owners of the golf ball company had an initial cost of 22,000 dollars. How many golf balls should the manufacturer sell to maximize profit?* **Be sure to explain why your answer is an absolute maximum.**

p = price
x = # of golf balls

p	x
8	32,000
12	24,000

$m = \frac{\Delta p}{\Delta x} = \frac{-4}{8{,}000} = \frac{-1}{2{,}000}$

$y = \frac{-1}{2{,}000}x + b$. Plug in $(8, 32{,}000) \Rightarrow b = 24$

So $p = \frac{-1}{2{,}000}x + 24$. So $R(x) = xp = -\frac{1}{2{,}000}x^2 + 24x$

$C(x) = 22{,}000 + 6x$

$P(x) = R(x) - C(x) = \frac{-1}{2{,}000}x^2 + 24x - (22{,}000 + 6x)$

$= \frac{-1}{2{,}000}x^2 + 18x - 22{,}000$

$P'(x) = \frac{-1}{1{,}000}x + 18 \Rightarrow 18{,}000$ is a CP

It is an absolute max, because $P(x)$ is a CD parabola.

Question 6. (16 points) *For the following function find the appropriate information (listed next page) to sketch the graph of the following function.*

$$f(x) = \frac{x}{x+2}$$

$D_f = \mathbb{R} - \{-2\}$

$x = 0 \rightarrow y = 0$

$(0,0)$ x, y intercept

$y = 0 \rightarrow x = 0$

$\lim\limits_{|x| \to \infty} f(x) = \frac{1}{1} = 1$

$y = 1$ H. A

$x = -2$ V. A

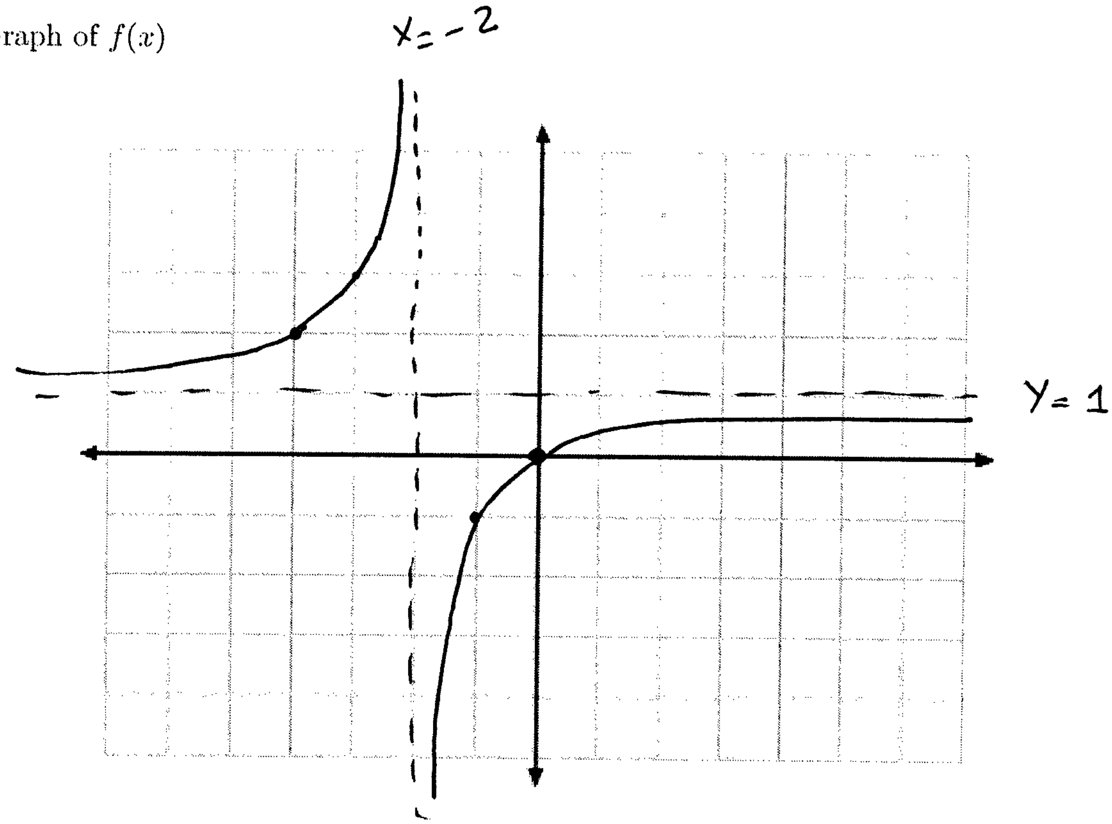
$f'(x) = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2} > 0$ f increases everywhere.

$f' \neq 0 \rightarrow$ No local Extremas

$f''(x) = \frac{-4}{(x+2)^3} \neq 0 \rightarrow$ NO P.O.I.

		$x=-2$	
F	$\cup$		$\cap$
f''	$+$		$-$

Graph of $f(x)$



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10. Identify the points of inflection and plot them
11. Fill in the rest of the graph using (7) and (9)

Question 7. (10 points)

(a) Suppose $f'(x) = -3x^2 + 4x - 2$*, and that* $f(1) = 2$*. Find* $f(x)$*.*

$$f(x) = \int (-3x^2 + 4x - 2)\,dx$$
$$= -\cancel{x}^3 + 2x^2 - 2x + C$$

$$f(1) = -1 + 2 - 2 + C = 2$$
$$C = 3 \qquad f(x) = -x^3 + 2x^2 - 2x + 3$$

(b) Calculate

$$\int \frac{6x+3}{x^2+x+1}dx$$

$u = x^2 + x + 1$

$du = (2x+1)dx$

$$\int \frac{6x+3}{x^2+x+1}\,dx = \int \frac{3(2x+1)\,dx}{x^2+x+1}$$

$$= \int \frac{3\,du}{u} = 3\ln|u| + C$$

$$= 3\ln|x^2+x+1| + C.$$