MGCR 341: Finance 1
Winter 2012
Vadim di Pietro

Midterm: Version A: Solutions

Student Name:_________________________________

Student Number:_______________________________

Section: circle one: [11:30am to 1:00pm] or [2:30pm to 4:00pm] or [4:00pm to 5:30pm]

Friday March 2nd, 9am to 11am

Time: Do not turn past this page until the exam has begun. The exam will be **120 minutes** in length.

Show your work: In order to receive credit for your answers, you must show your work. Correct answers with no work shown will not receive any credit. Incorrect answers with partial correct work may receive partial credit. The exception to this rule is the True or False section, where no explanations are required.

Answer questions directly on the exam sheet. If you need extra space, you may use the back sides of the pages.

Formula sheet: The exam is closed book, and you may not bring any notes into class. A formula sheet is provided on the last page of this exam. You may detach the formula sheet if you like. You do not need to hand in the formula sheet at the end of the exam.

Calculator: You are allowed a non-programmable non-text-storing calculator. You are allowed a financial calculator, although a simple calculator will suffice.

This exam has a total of **8 pages**, including the cover page and formula sheet.

The exam is worth a total of 100 points.

GOOD LUCK!

1) You have $10,000 to invest at t = 0 for a period of 1 year. Would you rather put the money in a risk free bank account that has a 12% APR with monthly compounding, or purchase two 1-year risk free zero coupon bonds that each have a price of $5,000 and a face value of $5,625? Assume the inflation rate is 0.2% per month and that at t = 0 a small poutine costs $4.95.

/5

$$EAR = \left(1 + \frac{12\%}{12}\right)^{12} - 1 = 12.68\%$$

This means your 10,000 would turn into 10,000(1.1268) = 11,268 after 1 year, which is more than the 2(5,625) = 11,250 you would get back from the 2 bonds. Thus, you prefer the bank account. (Note: The inflation rate and the price of a poutine were irrelevant here.)

2) Today is month t = 0. A bank is offering you the following financial product:

/10

You make a $50,000 lump sum payment to the bank at month t = 25. In addition, you make monthly payments of $4,000 per month from t = 26 to t = 100. In exchange, the bank will give you monthly payments from t = 150 to t = 250, where the first cash flow is X, and each month thereafter the payments will grow by 0.3%.

The annual interest rate (APR) is 12%, with monthly compounding.

The inflation rate is 0.2% per month. At t = 0, a hamburger costs $1.

a) Assuming the contract is fair, write down one equation where the only unknown is X. (You don't need to simplify or solve for X.)

$$\frac{50,000}{1.01^{25}} + \frac{4,000}{0.01}\left(1 - \frac{1}{(1.01)^{75}}\right)\frac{1}{1.01^{25}} = \frac{X}{0.01 - 0.003}\left(1 - \frac{(1.003)^{101}}{(1.01)^{101}}\right)\frac{1}{1.01^{149}}$$

Or, if you did it in terms of values as of t = 25

$$50,000 + \frac{4,000}{0.01}\left(1 - \frac{1}{(1.01)^{75}}\right) = \frac{X}{0.01 - 0.003}\left(1 - \frac{(1.003)^{101}}{(1.01)^{101}}\right)\frac{1}{1.01^{124}}$$

If you did it correctly in terms of other points in time, that was OK as well.

b) Suppose that every month from t = 150 to t = 250 you spend everything the bank gives you on hamburgers. What is the monthly growth rate of the number of hamburgers you are eating each month?

$$1 + g_r = \frac{1 + g}{1 + i}$$

➔ $g_r = \frac{1.003}{1.002} - 1 = 0.0010 \text{ or } 0.10\%$

/15

3) Today is t = 0. You are planning for your retirement. At t = 40 you will want to consume the same amount of goods that $40,000 can purchase today. Each year thereafter you will want to consume 3% more goods than in the previous year, with your final consumption at t = 70. At t = 70 you also want to have $500,000 (nominal) saved in the bank to leave to your children. The inflation rate is 2% per year, and the real risk free rate is 5%.

a) What is the present value of your t = 40 to t = 70 consumption needs plus your final $500,000 savings requirement? Indicate whether you are using the real approach or the nominal approach.

Real approach:

$$PV = \frac{40{,}000}{0.05 - 0.03}\left(1 - \frac{(1+0.03)^{31}}{(1+0.05)^{31}}\right)\frac{1}{1.05^{39}} + \frac{500{,}000/1.02^{70}}{1.05^{70}} = 138{,}069$$

Notes: There are 70-40+1 = 31 consumption needs. This is a forward starting growing annuity with real interest rate of 5% and real growth rate of 3%. The first real cash flow is at t = 40 and so we have to discount the value obtained from the regular formula back 39 periods to get the PV of the consumption needs. The $500,000 is nominal, which corresponds to $500{,}000/1.02^{70}$ in real terms, which is then discounted back 70 years at the 5% real risk free rate.

Nominal approach:

$r_{nomimal}$ = (1.05)(1.02) = 0.071 $g_{nominal}$ = (1.03)(1.02)-1 = 0.0506

$$PV = \frac{40{,}000(1.02^{40})}{0.071 - 0.0506}\left(1 - \frac{(1+0.0506)^{31}}{(1+0.071)^{31}}\right)\frac{1}{1.071^{39}} + \frac{500{,}000}{1.071^{70}} = 138{,}069$$

Notes: There are 70-40+1 = 31 consumption needs. This is a forward starting growing annuity with nominal interest rate of 7.1% and nominal growth rate of 5.06%. The first cash flow is at t = 40 and so we have to discount the value obtained from the regular formula back 39 periods to get the PV of the consumption needs.

At t = 40 you will receive your first pension payment of $10,000 (nominal dollars). Each year thereafter your pension will increase by the inflation rate, until you receive your last pension payment at t = 70. You realize that your pension alone will not be enough to cover your consumption and savings needs.

b) How much money (nominal dollars) do you have to have saved in the bank as of t = 39 such that these savings plus your future pension will allow you to meet your total consumption and savings needs? Indicate whether you are using the real approach or the nominal approach.

Real approach:

PV of your pension is given by

$$PV = \frac{10{,}000/1.02^{40}}{0.05}\left(1 - \frac{1}{(1+0.05)^{31}}\right)\frac{1}{1.05^{39}} = 10{,}533$$

Thus you are missing $138{,}069 - 10{,}533 = \$127{,}536$ in PV terms. This corresponds to

$$127{,}536(1+r_{nominal})^{39} = 127{,}536(1+0.071)^{39} = \$1{,}851{,}066$$

nominal dollars at t = 39.

Or, you could say that you are missing

$$127{,}536*(1+r_{real})^{39} = 127{,}536*(1+0.05)^{39} = \$855{,}097$$

real dollars as of t = 39, which corresponds to $855{,}097(1.02^{39})$ = $1,851,066 nominal dollars at t = 39.

Notes: The real growth rate is 0% since your pension increases at the rate of inflation. There are 70-40+1 = 31 pension payments. This is a forward starting constant annuity with real interest rate of 5%. The first cash flow is at t = 40 and so we have to discount the value obtained from the regular formula back 39 periods to get the PV. The $10,000 at t = 40 is nominal, which corresponds to $10{,}000/1.02^{40}$ in real terms.

Nominal approach:

PV of your pension is given by

$$PV = \frac{10{,}000}{0.071 - 0.02}\left(1 - \frac{(1+0.02)^{31}}{(1+0.071)^{31}}\right)\frac{1}{1.071^{39}} = 10{,}533$$

Thus you are missing $138{,}069 - 10{,}533 = \$127{,}536$ in PV terms. This corresponds to

$$127{,}536*(1+r_{nominal})^{39} = 127{,}536*(1+0.071)^{39} = \$1{,}851{,}066$$

nominal dollars at t = 39.

Alternatively:

This question can also be solved in terms of first figuring out the value of your consumption and savings needs as of t = 39, which is $138{,}069(1.071)^{39}$ = 2,003,937. The value of your pension as of t = 39 is

$$PV = \frac{10{,}000}{0.071 - 0.02}\left(1 - \frac{(1+0.02)^{31}}{(1+0.071)^{31}}\right) = 152{,}871$$

Thus as of t = 39 you are missing 2,003,937 – 152,871 = $1,851,066

Finally, there is also a corresponding t = 39 real approach which could be used to get the same answer.

Note: The nominal growth rate of the pension is 2%, since we are told that the pension grows at the rate of inflation, which is 2%.

4) Risky project A requires an initial cash outflow of $300,000 at t = 0 and a second cash outflow of $110,000 at t = 1. The project will generate an expected cash inflow of $22,000 at t = 2, and thereafter the cash inflows are expected to grow in perpetuity at a rate of g.

/10

a) Your boss only wants to accept the project if the IRR is at least 10%. What is the minimal value of g that would make your boss accept the project?

IRR is the discount rate such that if you were to discount all cash flows at that rate you would get a 0 NPV.

$$\mathbf{0 = -300,000 - \frac{110,000}{1+0.10} + \frac{22,000}{0.10-g}\left(\frac{1}{1+0.10}\right)}$$

➔ **g = 0.05 or 5% is the minimal value of g**

b) Suppose the actual growth rate, g, is 5%, and that the opportunity cost of capital for this type of risky project is 12%, and that the risk free rate is 7%. By how much would you be increasing or decreasing shareholder wealth (in PV terms) by accepting this project versus not accepting the project? Make sure to specify if it represents an increase or decrease.

$$\boldsymbol{NPV = -300,000 - \frac{110,000}{1+0.12} + \frac{22,000}{0.12-0.05}\left(\frac{1}{1+0.12}\right) = -117,602}$$

You would be decreasing shareholder value by $117,602.
(Note: NPVs are calculated using the opportunity cost of capital for the project.)

5) Project A is risk free and will generate a cash inflow of $100 at t = 0 and a cash outflow of $111 at t = 1. The risk free rate is 10%.

/10

a) What is the IRR of Project A?

0 = 100 – 111/(1+IRR) → IRR = 11%

b) Assuming, as we usually do, that you have access to capital markets, do you think it makes sense to accept Project A? Justify your answer in 1 sentence.

The project is risk free, so the cost of capital for the project is the risk free rate of 10%. So the NPV is = 100 – 111/1.10 < 0, so the NPV is negative and it doesn't make sense to accept the project, even though the IRR > 10%. Recall that the IRR rule doesn't make sense when the cash inflow is before the outflow. You could also have simply stated that the project represents borrowing money for 1 year at an 11% rate, which doesn't make sense if you can borrow money at the lower 10% risk free rate.

c) Assuming you don't have access to capital markets, is it possible that it makes sense to accept Project A? Justify your answer in 1 sentence.

Yes, because if you can't borrow at a 10% rate, the argument in b is no longer valid.

/15

6) You are given the following information:

- Bond A is a 1-year, 5% coupon bond with face value $3,000 and price = $3,000
- Bond B is a 2-year, zero coupon bond with face value $1,000 and YTM = 6%
- Bond C is a 3-year "step-down" bond, which will make the following payments: $2,260 at t = 1, $1,200 at t = 2, and $11,000 at t = 3. The price of bond C is $12,000.
- Bond D is a 3-year, 10% coupon bond with face value $5,000

a) What is the fair price of bond D?

We must first bootstrap the yield curve.

$r_{0,1}$ is equal to 5% (recall that when price = FV, the YTM = coupon rate)

$r_{0,2}$ is equal to 6% (recall that the YTM of any risk free zero coupon bond is equal to the maturity-matched risk free rate)

The price of bond C must satisfy:

$$12,000 = \frac{2,260}{1 + r_{0,1}} + \frac{1,200}{(1 + r_{0,2})^2} + \frac{11,000}{(1 + r_{0,3})^3}$$

→ $r_{0,3}$ = 7.81%

Now we can compute the fair price of bond D as follows:

$$P = \frac{500}{1 + r_{0,1}} + \frac{500}{(1 + r_{0,2})^2} + \frac{5,500}{(1 + r_{0,3})^3} = 5,311$$

(You may have gotten a slightly different answer if you rounded $r_{0,3}$, but you were not penalized for this.)

b) Suppose bond D is trading at a price of $5,290. Construct an arbitrage using bonds A, B, C, and D. Specifically, one leg of your arbitrage should involve either buying or short-selling exactly one unit of bond D. You must specify how many units of each of the other bonds you will buy or sell. Make sure to clearly state whether you are buying or selling each of the bonds, including bond D.

Bond D is too cheap, so we buy 1 unit of it.

Although we can't be sure that we will need to short the other 3 bonds, we still define A, B, and C as the number of units we short of bonds A, B, and C, respectively.

Next we set up our t = 1, t = 2, and t = 3 net cash flow = 0 conditions. In other words, we want the cash inflows from bond D to be offset by the liabilities on our short positions.

t = 1 condition:
500 = A(3,150) + C(2,260)

t = 2 condition:
500 = B(1,000) + C(1,200)

t = 3 condition:
5,500 = C(11,000)

- ➔ **C = 0.5**
- ➔ **B = -0.1**
- ➔ **A = -0.2**

Thus, your arbitrage consists of:

- **Buying 1 unit of bond D**
- **Shorting 0.5 units of bond C**
- **Buying 0.1 units of bond B**
- **Buying 0.2 units of bond A**

(Note that if you get a negative number, it means you have to do the opposite of what you initially assumed for those bonds.)

7) Given $r_{0,1} = 5\%$ and $r_{0,2} = 10\%$, what is $f_{1,1}$ if there is no arbitrage?

/5

There are 2 ways of transferring $1 from t = 0 to t = 2, and if there is no arbitrage both methods should give you the same amount at t = 10. Thus:

$(1.05)(1+f_{1,1}) = 1.10^2$

➔ $f_{1,1} = 15.24\%$

8) You are given the following information: $r_{0,3} = 4\%$, $f_{3,2} = 6\%$, $f_{5,1} = 7\%$, $f_{6,2} = 9\%$, $f_{9,3} = 11\%$, the price of a 9-year zero-coupon bond with face value $2,000 is $1,000.

/12

a) What is $f_{6,6}$ if there is no arbitrage?

There are 2 ways of transferring $1 from t = 0 to t = 12, and if there is no arbitrage both methods should give you the same amount at t = 10. Thus:

$(1.04)^3(1.06)^2(1.07)(1+f_{6,6})^6 = (2000/1000)(1.11)^3$

➔ $f_{6,6} = 12.46\%$

b) If a bank were to quote you a forward interest rate of $f_{6,6} = 10\%$, explain how would you construct an arbitrage opportunity. Assume that part of the arbitrage involves buying or selling $1 worth of the 9-yr bond. Make sure to specify whether you are buying or selling $1 worth of the 9-year bond, and also specify whether you will be lending or borrowing at the various rates. Finally, make sure to specify the amount of money you plan to borrow or lend at the various interest rates and forward rates.

Step 1: The rate is too low, so we want to lock in $f_{6,6} = 10\%$ for borrowing.
Step 2: Do the same thing to the elements on the same side of the equality as $f_{6,4}$. Thus, we want to lock in $r_{0,3} = 4\%$, $f_{3,2} = 6\%$, $f_{5,1} = 7\%$, and $f_{6,6} = 10\%$ for borrowing.
Step 3: Do the opposite to the elements of other side of the equality. Thus you want to invest money using the 9-year bond. In other words, you have to buy the 9-year bond. And you lend money using $f_{9,3} = 11\%$.

Step 4: Determine the magnitudes of the above positions.

In order to buy $1 worth of the bond, you have to buy 1/1000 units.
This position will give you back $2 at t = 9. So you have to reinvest $2 at the rate $f_{9,3} = 11\%$.
In order to get the above $1 at t = 0:
Borrow $1 at a rate of $r_{0,3} = 4\%$.
To pay off the previous loan you will then borrow 1.04^3 at the rate $f_{3,2} = 6\%$
To pay off the previous loan you will then borrow $(1.04^3)(1.06^2)$ at the rate $f_{5,1} = 7\%$
To pay off the previous loan you will then borrow $(1.04^3)(1.06^2)(1.07)$ at the rate $f_{6,6} = 10\%$

/18

8) (18 points, 20 mins) True or False: write *True* or *False* directly to the left of the statements below. No explanation is required.

***Please note that the questions may have been slightly different in the different versions, and that the answers for those versions may have been different as well. For example, in some versions, the word "down" may have been replaced by "up", etc…, thus changing the answer to the question.**

a) At t = 0, you buy a 3-year zero coupon bond, and a 3-year 8% coupon bond. At t = 0, t = 1, and t = 2, the yield curve is flat at 5%, while at t = 3, $r_{3,1}$ = 12%, $r_{3,2}$ = 14% and $r_{3,3}$ = 16%. The realized compound yield from t = 0 to t = 3 will necessarily be the same for both bonds (you may assume that coupons are always reinvested and rolled over at the 1-year rate). **True. The realized compound yield will equal 5% for both. Note that the rates at t = 3 don't matter because both bonds mature at t = 3 and have no cash flows left.**

b) At t = 0 you bought a 10-year, 8% coupon bond with a 4% YTM. At t = 1 the YTM is still 4%. The realized compound yield from t = 0 to t = 1 is necessarily 8%. **False. It is 4%.**

c) If the yield curve is upward sloping, then short-term interest rates are expected to go down according to the expectations hypothesis. **False. They are expected to go up.**

d) If $r_{0,1}$ is 5% and $r_{0,2}$ is 5%, then according to the expectations hypothesis the expected value of next year's 1-year interest rate, $E[r_{1,1}]$, is 5%. **True.**

e) If bond A has a coupon rate of 5% and a YTM of 9%, then the price of bond A is below its face value. **True. Price < FV when coupon rate < YTM.**

f) Machine A is more expensive than machine B and also has higher annual maintenance costs. Machine A needs to be replaced every 7 years and has an equivalent annual cost of $3,000. Machine B needs to be replaced every 4 years and has an equivalent annual cost of $3,100. Assuming both machines produce the exact same output per year, then one should prefer machine A over machine B. **True, since it has the lower EAC. Although A is more expensive than B (to buy and to maintain), the fact that it needs to be replaced less often can make its EAC lower than that of B, which is what really matters.**