

**Concordia University**  
**Applied Ordinary Differential Equations, ENGR 213**  
**Final Exam**  
**16 April 2010**  
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Only approved calculators with stickers are allowed.  
Time allotted: Three hours.

1. (14%) Find the general solution of each of the following differential equations. In each case, you may leave the solution in implicit form.

(a)  $\frac{dy}{dx} = \frac{y^2 + 1}{x^2y + y}$

(b)  $(ye^{2xy} + \cos(x + 2y)) dx + (xe^{2xy} + 2 \cos(x + 2y)) dy = 0$

2. (10%) Solve the initial value problem

$$t \frac{dy}{dt} + (t + 1)y = t, \quad y(\ln 2) = 1, \quad t > 0.$$

3. (10%) Solve the following differential equation by an appropriate substitution

$$(x^2 + 3xy + y^2) dx - x^2 dy = 0.$$

You may leave the solution in implicit form.

4. (10%) The Space Shuttle lands in Kennedy Space Center. The spacecraft touches down at  $t = 0$  with a velocity of 100 m/sec. The spacecraft chute is deployed at  $t = 5$  sec. Between touch down and deployment of chute ( $0 \leq t \leq 5$ ), the velocity of the spacecraft  $V(t)$  (in m/sec) is governed by:

$$\frac{dV}{dt} = -0.0001V^2$$

and after the deployment of chute by:

$$\frac{dV}{dt} = -0.002V^2$$

Determine the spacecraft velocity at  $t = 5$  sec and  $t = 15$  sec.

5. (10%) Find the general solution of the following differential equation using the method of *variation of parameters*.

$$x^2y'' + 8xy' + 12y = e^x$$

Hint. For a positive integer  $n$

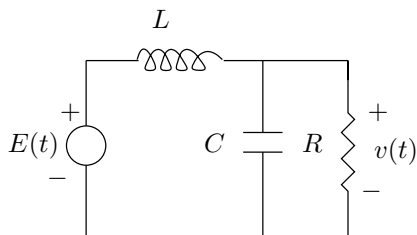
$$\int x^n e^{ax} dx = \frac{e^{ax}}{a} \left( x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \dots - \frac{(-1)^n n!}{a^n} \right)$$

6. (12%) The following circuit consists of an inductor  $L = \frac{1}{2}$  henry, a resistor  $R = 1$  ohm, and a capacitor  $C = \frac{1}{4}$  farad connected to an input voltage  $E(t)$ . The resistor voltage  $v(t)$  satisfies the following ordinary differential equation.

$$LC \frac{d^2v}{dt^2} + \frac{L}{R} \frac{dv}{dt} + v(t) = E(t)$$

Assume that at  $t = 0$  no energy is stored in the inductor and the capacitor, and hence,  $v(0) = 0$  and  $v'(0) = 0$ . Furthermore, for  $t \geq 0$ , the input voltage is  $E(t) = \sin 3t$  volts.

- (a) Find the voltage  $v(t)$  (for  $t \geq 0$ ).  
 (b) Identify in  $v(t)$ , the transient terms and the steady state terms.



7. (10%)

- (a) Find the general solution of the differential equation

$$(x^2 - 1)y'' + y = 0$$

by using the power series method about the ordinary point  $x = 0$ . Write out all terms in the solution up to, and including  $x^5$ .

- (b) Find the singular points of the differential equation. What is the minimum radius of convergence of the power series solution obtained in (a)?

8. (12%) Solve the linear system either by the method of systematic elimination or by a matrix method

$$\begin{aligned} \frac{dx}{dt} &= 4y + \sin(t) \\ \frac{dy}{dt} &= -x + \cos(t). \end{aligned}$$

9. (12%) Solve the initial value problem by using eigenvalues and eigenvectors

$$\begin{aligned} \frac{dx}{dt} &= y - x \\ \frac{dy}{dt} &= 2y \\ \frac{dz}{dt} &= y + z - x, \end{aligned}$$

$$x(0) = 1, \quad y(0) = 1, \quad z(0) = 1.$$