MAT 1320B Fall 2004, Professor: W. Burgess,

Midterm Test 2: October 27, 2004, the questions.

Version A

1. (2 points) If $x^2y^2 + 2xy = 3x$, what is $\frac{dy}{dx}$ at $(1, 1)$?

A. $-1/2$ **B.** $-1/4$ **C.** 0 **D.** $1/4$ **E.** $1/2$ **F.** undefined

2. (2 points) If $y = x^{x+1}$, what is $\frac{dy}{dx}$ at $x = 2$?

A. 2.7726 **B.** 5.5452 **C.** 17.5452 **D.** 0 **E.** 12 **F.** undefined

3. (2 points) If the area of a square is increasing at a rate of 3 cm^2/s when the sides are 2 cm long, at what rate must the length of the sides be increasing ?

A. $\sqrt{2}$ cm/s **B.** 3/2 cm/s **C.** $\sqrt{3}/2$ cm/s **D.** $\sqrt{3}$ cm/s **E.** 3/4 cm/s **F.** 1 cm/s

4. (2 points) Circle the letter of each true statement, and cross out the letter of each false statement, from among the following concerning the graph of a curve $y = f(x)$, where we are given that $f'(x)$ and $f''(x)$ are defined for all x.

A. If $f'(a) = 0$, then the graph has a local extremum at $x = a$
B. If the graph has a local maximum at $x = a$, then $f''(a) > 0$
C. If the graph has an inflection point at $x = a$, then $f''(a) = 0$
D. If $f''(a) = 0$, then the graph has an inflection point at $x = a$

5. (1 point) If Newton's Method is used to find a root of $f(x) = x^4 + 3x^2 - 2$, with $x_1 = 1$, what is x_2 ? Begin by writing down the formula for x_2 in terms of x_1.

6. (2 points) Find the derivative of the function

$$f(x) = 7 \arcsin(x^3)$$

Write clearly and put a $\boxed{\text{box}}$ around your final answer.

7. (2 points) Consider the function

$$f(x) = \sin(e^x)$$

There is exactly one local extremum of $f(x)$ on the interval $1 \leq x \leq 2$. Find its (x, y) coordinates, to 4 decimal places. Is it a local maximum or a local minimum?

8. (3 points) Consider the function

$$f(x) = \frac{\ln(x)}{x^2} \quad \text{for } x > 0$$

Please note that $\lim_{x\to\infty} f(x) = 0$. In this question, you are asked to find the first and second derivative of the function f and to find all important features of its graph. Please work out the details (carefully!) on the bottom of this page and on the next page, and transcribe your final answers neatly to the spaces provided below. Note that a correct answer to some questions may be "none." 1. $f'(x) =$

2. $f''(x) =$

3. x-intercepts:

4. vertical asymptotes:

5. critical point(s) (with (x, y) coordinates):

6. inflection point(s) (with (x, y) coordinates):

7. $\lim_{x\to 0^+} f(x) =$

8. interval(s) of increase of the function:

9. interval(s) where the graph is concave up: Using these data, sketch the graph of f (on this page or the next).

Version B

1. (2 points) If $x^2y^2 + 2xy = 3x$, what is $\frac{dy}{dx}$ at $(1, 1)$?

A. undefined **B.** 0 **C.** $-1/4$ **D.** $1/4$ **E.** $1/2$ **F.** $-1/2$

2. (2 points) If $y = x^{x+1}$, what is $\frac{dy}{dx}$ at $x = 2$?

A. undefined **B.** 0 **C.** 2.7726 **D.** 5.5452 **E.** 12 **F.** 17.5452

3. (2 points) If the area of a square is increasing at a rate of 3 cm^2/s when the sides are 2 cm long, at what rate must the length of the sides be increasing ?

A. 3/4 cm/s **B.** 1 cm/s **C.** $\sqrt{3}/2$ cm/s **D.** $\sqrt{3}$ cm/s **E.** 3/2 cm/s **F.** $\sqrt{2}$ cm/s

4. (2 points) Circle the letter of each true statement, and cross out the letter of each false statement, from among the following concerning the graph of a curve $y = f(x)$, where we are given that $f'(x)$ and $f''(x)$ are defined for all x.

A. If $f'(a) = 0$, then the graph has a local extremum at $x = a$
B. If the graph has an inflection point at $x = a$, then $f''(a) = 0$
C. If $f''(a) = 0$, then the graph has an inflection point at $x = a$
D. If the graph has a local maximum at $x = a$, then $f''(a) > 0$ **5.** (1 point) If Newton's Method is used to find a root of $f(x) = x^4 - 3x^2 + 1$, with $x_1 = 1$, what is x_2 ? Begin by writing down the formula for x_2 in terms of x_1.

6. (2 points) Find the derivative of the function

$$f(x) = 5\arcsin(x^4)$$

Write clearly and put a $\boxed{\text{box}}$ around your final answer.

7. (2 points) Consider the function

$$f(x) = \sin(e^x)$$

There is exactly one local extremum of $f(x)$ on the interval $0 \le x \le 1$. Find its (x, y) coordinates, to 4 decimal places. Is it a local maximum or a local minimum?

8. (3 points) Consider the function

$$f(x) = \frac{-\ln(x)}{x^2} \quad \text{for } x > 0$$

Please note that $\lim_{x\to\infty} f(x) = 0$. In this question, you are asked to find the first and second derivative of the function f and to find all important features of its graph. Please work out the details (carefully!) on the bottom of this page and on the next page, and transcribe your final answers neatly to the spaces provided below. Note that a correct answer to some questions may be "none."

1. $f'(x) =$

2. $f''(x) =$

3. x-intercepts:

4. vertical asymptotes:

5. critical point(s) (with (x, y) coordinates):

6. inflection point(s) (with (x, y) coordinates):

7. $\lim_{x\to 0^+} f(x) =$

8. interval(s) of increase of the function:

9. interval(s) where the graph is concave up: Using these data, sketch the graph of f (on this page or the next).

Version C

1. (2 points) If $x^2y^2 + 3xy = 4x$, what is $\frac{dy}{dx}$ at $(1, 1)$?
A. undefined **B.** 0 **C.** 1/2 **D.** $-1/2$ **E.** 1/5 **F.** $-1/5$

2. (2 points) If $y = x^{x+2}$, what is $\frac{dy}{dx}$ at $x = 2$?
A. undefined **B.** 0 **C.** 32 **D.** 43.0904 **E.** 5.5452 **F.** 11.0904

3. (2 points) If the area of a square is increasing at a rate of 3 cm^2/s when the sides are 3 cm long, at what rate must the length of the sides be increasing ?

A. $\sqrt{3}$ cm/s **B.** 1 cm/s **C.** 3/4 cm/s **D.** 1/2 cm/s **E.** 3/2 cm/s **F.** $\sqrt{3}/2$ cm/s

4. (2 points) Circle the letter of each true statement, and cross out the letter of each false statement, from among the following concerning the graph of a curve $y = f(x)$, where we are given that $f'(x)$ and $f''(x)$ are defined for all x.
A. If $f'(a) = 0$, then the graph has a local extremum at $x = a$
B. If the graph has a local minimum at $x = a$, then $f''(a) < 0$
C. If $f''(a) = 0$, then the graph has an inflection point at $x = a$
D. If the graph has an inflection point at $x = a$, then $f''(a) = 0$

5. (1 point) If Newton's Method is used to find a root of $f(x) = x^4 - 5x^2 + 2$, with $x_1 = 1$, what is x_2 ? Begin by writing down the formula for x_2 in terms of x_1.

6. (2 points) Find the derivative of the function

$$f(x) = 7 \arctan(x^4)$$

Write clearly and put a $\boxed{\text{box}}$ around your final answer.

7. (2 points) Consider the function

$$f(x) = \cos(e^x)$$

There is exactly one local extremum of $f(x)$ on the interval $0 \le x \le \frac{3}{2}$. Find its (x, y) coordinates, to 4 decimal places. Is it a local maximum or a local minimum?

8. (3 points) Consider the function

$$f(x) = \frac{\ln(x)}{x^3} \quad \text{for } x > 0$$

Please note that $\lim_{x\to\infty} f(x) = 0$. In this question, you are asked to find the first and second derivative of the function f and to find all important features of its graph. Please work out the details (carefully!) on the bottom of this page and on the next page, and transcribe your final answers neatly to the spaces provided below. Note that a correct answer to some questions may be "none."

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5. critical point(s) (with (x, y) coordinates):

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7. $\lim_{x \to 0^+} f(x) =$

8. interval(s) of increase of the function:

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