Multiple Choice Questions (1-5)

Question 1 *Suppose a function $y = f(x)$ is defined implicitly by the equation*

$$x^2y^3 - 3xy + x = 3$$

The derivative of this function at the point $(1, 2)$ is

A) 1 **B)** 2 **C)** $-\frac{9}{2}$ **D)** $\frac{-11}{9}$ **E)** -3

$$\frac{d[x^2y^3 - 3xy + x = 3]}{dx} \rightarrow 2xy^3 + 3x^2y^2\frac{dy}{dx} - 3y - 3\frac{dy}{dx} + 1 = 0$$

Plug in $(1, 2)$

$$16 + 12\frac{dy}{dx} - 6 - 3\frac{dy}{dx} + 1 = 0$$

$$11 + 9\frac{dy}{dx} = 0 \qquad \frac{dy}{dx} = -\frac{11}{9}$$

Question 2 *If $\log_4(4x - 1) - \log_4(x - 3) = 2$, then $x =$?*

A) $x = \frac{9}{2}$ **B)** $x = \frac{47}{12}$ **C)** $x = \frac{4}{11}$ **D)** $x = \frac{31}{12}$ **E)** $x = \frac{4}{13}$

$$\log_4\left(\frac{4x-1}{x-3}\right) = 2$$

$$\frac{4x-1}{x-3} = 16$$

$$4x - 1 = 16x - 48$$

$$47 = 12x$$

$$x = \frac{47}{12}$$

Question 3 *Let a be a positive number not equal to 0 or 1. If $\log_a(x) = 7$, then $\log_{a^3}(x) =$?*

A) $\frac{7}{3}$ **B)** 21 **C)** $\frac{21}{a}$ **D)** $\sqrt[3]{7}$ **E)** $\frac{7}{a}$

$\log_a(x) = 7 \iff a^7 = x \iff (a^3)^{7/3} = x$

$\iff \log_{a^3}(x) = \frac{7}{3}$

Question 4 *Suppose 1000 dollars is deposited in an account with an annual interest rate of 4%, compounded 2 times per year. How long will it take for the deposit to triple?*

A) $\frac{\ln 3}{2 \ln 1.02}$ **B)** $\frac{\ln 3}{\ln 1.02}$ **C)** $\frac{\ln 1.02}{2 \ln 3}$ **D)** $\frac{\ln 3}{1.02 \ln 2}$ **E)** $\frac{\ln 2}{3 \ln 1.02}$

$P(t) = 1000\left(1 + \frac{.04}{2}\right)^{2t} = 1000(1.02)^{2t}$

Solve $3000 = 1000(1.02)^{2t}$

$3 = (1.02)^{2t}$

$\ln(3) = 2t \ln(1.02)$

$t = \frac{\ln(3)}{2 \ln(1.02}$

Question 5 *Find the equation of the tangent line for the function* $f(x) = \frac{x+3}{x-1}$ *at* $x = 3$.

A) $y = 2x + 1$ **B)** $y = 2x + 4$ **C)** $y = -x + 6$ **D)** $y = 2x - 3$
E) $y = -x + 2$

$$f'(x) = \frac{(x-1)(1) - (x+3)(1)}{(x-1)^2} = \frac{-4}{(x-1)^2}$$

$$f'(3) = \frac{-4}{4} = -1$$

$y = -1x + b$. Plug in $(3, 3)$

$$3 = -3 + b$$

$$b = 6$$

Multiple Choice Questions (1-5)

Question 1 *If* $\log_3(3x-1) - \log_3(x-2) = 3$, *then* $x =$?

A) $x = \frac{43}{12}$ **B)** $x = \frac{9}{23}$ **C)** $x = \frac{53}{24}$ **D)** $x = \frac{31}{11}$ **E)** $x = \frac{4}{13}$

$\log_3\left(\frac{3x-1}{x-2}\right) = 3$. So $\frac{3x-1}{x-2} = 27$ or $3x-1 = 27(x-2)$

$3x-1 = 27x-54$

$53 = 24x$

$x = \frac{53}{24}$

Question 2 *Find the equation of the tangent line for the function* $f(x) = \frac{x+2}{x-3}$ *at* $x = 2$.

A) $y = 2x+1$ **B)** $y = 2x+4$ **C)** $y = -x-3$
D) $y = -5x+2$ **E)** $y = -5x+6$

$f'(x) = \frac{(x-3)(1) - 1(x+2)}{(x-3)^2}$

$f'(2) = \frac{-1-4}{1} = -5$

$y = -5x+b$. Plug in $(2,-4)$

$-4 = -10+b$

$b = b$

Question 3 *Suppose 1000 dollars is deposited in an account with an annual interest rate of 3%, compounded 3 times per year. How long will it take for the deposit to double?*

A) $\frac{\ln 2}{2 \ln 1.01}$ B) $\frac{\ln 2}{3 \ln 1.01}$ C) $\frac{\ln 1.01}{2 \ln 3}$ D) $\frac{\ln 2}{1.01 \ln 2}$ E) $\frac{\ln 2}{\ln 1.01}$

$$P(t) = 1000\left(1 + \frac{.03}{3}\right)^{3t}$$

Solve $2000 = 1000(1.01)^{3t}$

$$2 = (1.01)^{3t}$$

$$\ln(2) = 3t \ln(1.01)$$

$$t = \frac{\ln(2)}{3\ln(1.01)}$$

Question 4 *Let a be a positive number not equal to 0 or 1. If $\log_a(x) = 9$, then $\log_{a^4}(x) =$?*

A) 36 B) $\frac{36}{a}$ C) $\frac{9}{4}$ D) $\sqrt[4]{9}$ E) $\frac{9}{a}$

$$\log_a(x) = 9 \iff a^9 = x \iff (a^4)^{9/4} = x$$

$$\iff \log_{a^4}(x) = \frac{9}{4}$$

Question 5 *Suppose a function $y = f(x)$ is defined implicitly by the equation*

$$x^2y^3 - 2xy + y = 1$$

The derivative of this function at the point $(2, 1)$ is

A) 1 **B)** $\frac{-11}{9}$ **C)** $-\frac{7}{2}$ **D)** $-\frac{2}{9}$ **E)** -3

$$\frac{d[x^2y^3 - 2xy + y = 1]}{dx}$$

$$2xy^3 + 3x^2y^2\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} + \frac{dy}{dx} = 0$$

Plug in $(2,1)$

$$4 + 12\frac{dy}{dx} - 2 - 4\frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$9\frac{dy}{dx} = -2$$

$$\frac{dy}{dx} = \frac{-2}{9}$$

Long Answer Questions (6-7)

Question 6 (10 points)

- *(6 points) Using only the definition of derivative as a limit, calculate $f'(x)$ where*

$$f(x) = \frac{1}{3x+4}$$

- *(4 points) Using the chain rule, calculate the derivative of the same function.*

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{3(x+\Delta x)+4} - \frac{1}{3x+4}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{(3x+4) - (3(x+\Delta x)+4)}{[3(x+\Delta x)+4][3x+4]}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{-3\Delta x}{(3(x+\Delta x)+4)(3x+4)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3\Delta x}{(3(x+\Delta x)+4)(3x+4)} \cdot \frac{1}{\Delta x} = \lim_{\Delta x \to 0} \frac{-3}{[3(x+\Delta x)+4][3x+4]}$$

$$= \frac{-3}{(3x+4)^2}$$

Chain Rule $f(x) = (3x+4)^{-1}$

$$f'(x) = (-1)(3x+4)^{-2}(3) = \frac{-3}{(3x+4)^2}$$

Question 7 (10 points)

A manufacturer of toasters knows that when she sets the price of a toaster at 24 dollars, she will sell 1400 toasters. For each 3 dollars increase in price, she will sell 120 fewer toasters. Let $p = D(x)$ be the demand function where x is the number of toasters sold and p is the price per toaster. Assume the demand function is linear.

- *(5 points) Find the demand function.*
- *(5 points) Find the revenue function.*

x = # of toasters

p = price / toaster

Demand function is linear, so we find 2 points

p	x
24	1400
27	1280

$$m = \frac{\Delta p}{\Delta x} = \frac{24-27}{1400-1280} = \frac{-1}{40}$$

$p = y = \frac{-1}{40}x + b$. Plug in $(1400, 24)$

$$24 = \frac{-1}{40}(1400) + b$$

$$= 35 + b$$

$$b = 59.$$

So $D(x) = -\frac{1}{40}x + 59$

$$R(x) = xD(x)$$

$$= -\frac{1}{40}x^2 + 59x$$