UNIVERSITY OF GUELPH
FALL SEMESTER 2010
TERM TEST I
MATH*1080
October 15, 2010

PLEASE FILL IN THE INFORMATION BELOW.

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EXAM

J. Cun

S. Gismondi T Th 8:30 03

PART I

Each question in this part has exactly one correct answer. Remember CIRCLE your answer on this questionnaire and clearly indicate your answer on the computer score card.

1. Assume the yearly cost $C(x)$ (in dollars) per household for water usage is a linear function of the amount of water consumed per year x (in cubic metres). In one year a household used 1000 cubic metres and was charged \$90 and in another year 1600 cubic metres was used and the charge was \$105. Which of the following is the equation for $C(x)$?

(A) $y = \frac{1}{20}x + 40$

(B) $y = \frac{1}{40}x + 65$

(C) $y = -\frac{1}{40}x + 115$

(D) $y = -\frac{1}{20}x + 140$

(E) $y = \frac{1}{25}x + 50$

$(1000, 90)$
$(1600, 105)$
$m = \frac{105-90}{1600-1000}$
$m = \frac{15}{600} = \frac{3}{120} = \frac{1}{40}$
$y = \frac{1}{40}x + b$
$90 = \frac{1}{40}(1000) + b$
$90 = \frac{1000}{40} + b$
$90 = 25 + b$
$b = 65$
$y = \frac{1}{40}x + 65$

2. For x minutes of cell phone usage a month, cell phone provider A charges $A(x)$ dollars a month computed as \$8 plus 5 cents a minute of usage that month while cell phone provider B charges $B(x)$ dollars a month computed as \$10 plus 6 cents a minute of usage for each minute above 500 minutes used that month. For what number of minutes, x, of monthly cell phone usage, where $x > 500$, will both cell phone providers charge the same amount?

(A) 2300 minutes

(B) 2500 minutes

(C) 2600 minutes

(D) 2800 minutes

(E) 2900 minutes

$A(x) = 8 + 0.05x$
$10 + 0.06(x-500)$
$10 + 0.06x - 30$
$0.06x - 20$
$\frac{6(1500)}{100} = \frac{3000}{100}$
$8 + 0.05x = 0.06x - 20$
$\frac{28}{0.01} = \frac{0.01x}{0.01}$
$2800 = x$

3. Let $f(x) = sin(x)$ and $g(x) = x^3 + 1$ then which of the following is FALSE?

(A) $(f - g)(x) = sin(x) - x^3 - 1$

(B) Domain of $(\frac{f}{g}) = (-\infty, -1) \cup (-1, +\infty)$

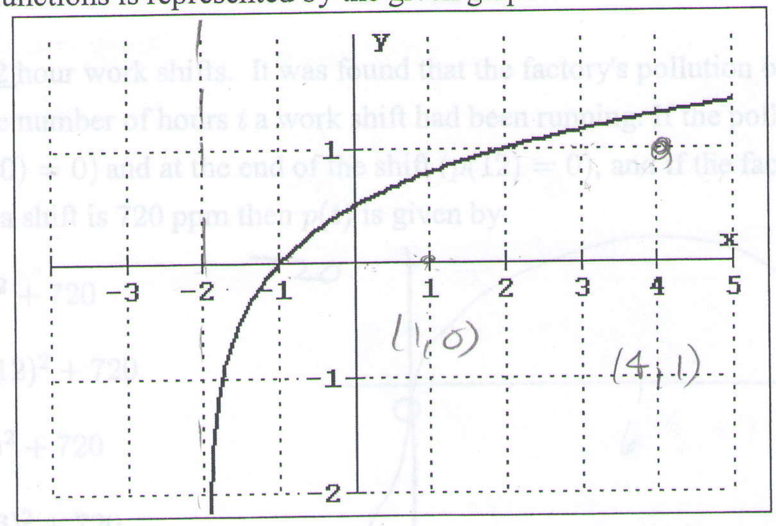
(C) $(f \circ g)(x) = sin(x^3 + 1)$

(D) $(g \circ f)(x) = sin^3(x + 1)$

(E) $(fg)(x) = x^3 sin(x) + sin(x)$

$f(x) = \sin x \quad g(x) = x^3 + 1$
$(\sin x)^3 + 1$

4. Which of the following functions is represented by the given graph?.



(A) $y = log_4(x+2)$

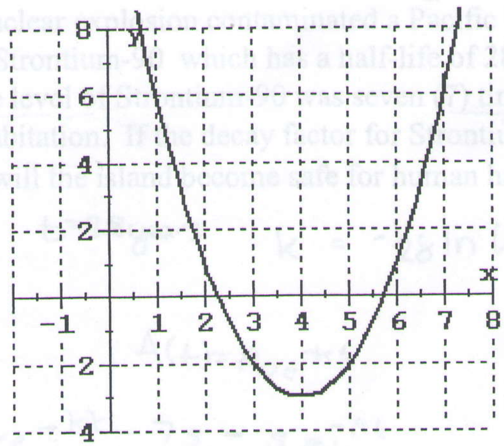
(B) $y = log_3(x+2)$

(C) $y = log_4(x-2)$

(D) $y = log_2(x)$

(E) $y = log_2(x-2)$

5. If the following graph, passing through $(1, 6)$, is the graph of $y = \frac{f(x)}{g(x)}$ where $f(x) = 3x^2 + 7$, then $g(1) =$



(A) $\frac{5}{3}$

(B) 3

(C) 10

(D) $\frac{3}{5}$

(E) 12

$6 = \frac{3x^2+7}{g(1)}$

$6 = \frac{3(1)^2+7}{g(1)}$

$6 = \frac{10}{g1}$

$g_1 6 = 10$

$g_1 = \frac{10}{6} = 5/3$

6. If $f(x) = sin(\frac{x}{5})$ and $g(x) = sin(\frac{x}{5} + \frac{\pi}{5})$ then which of the following correctly completes this statement. The basic period of g is 10π and the graph of g is the graph of f shifted to the left ______.

	Basic Period	Shifted to the left
(A)	10π	$\frac{\pi}{5}$ units
(B)	10π	π units
(C)	$\frac{2\pi}{5}$	π units
(D)	10	π units
(E)	10π	$\frac{1}{5}$ units

$\sin(\frac{1}{5}(x+\pi))$

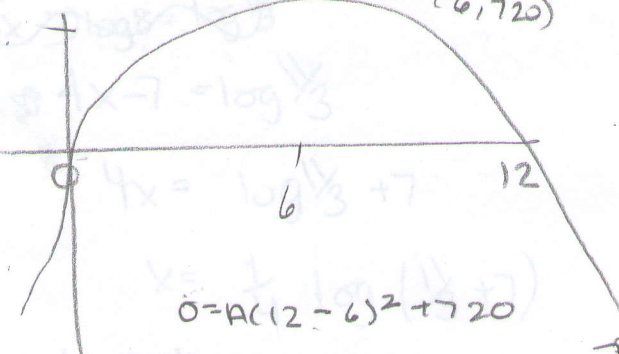
$\frac{2\pi}{1/5} = \frac{2\pi \times 5}{1}$

$5 = 2\pi \div 1/5$

$= 10\pi$

7. A factory operates on 12 hour work shifts. It was found that the factory's pollution output $p(t)$ (ppm) was a quadratic function of the number of hours t a work shift had been running. If the pollution output is 0 at the begining of the shift ($p(0) = 0$) and at the end of the shift ($p(12) = 0$), and if the factory's maximum pollution output during a shift is 720 ppm then $p(t)$ is given by

(6,720)



(A) $p(t) = 20\,(t-6)^2 + 720$

(B) $p(t) = -20\,(t-12)^2 + 720$

(C) $p(t) = \frac{1}{20}\,(t-6)^2 + 720$

(D) $p(t) = -20\,(t-6)^2 + 720$

(E) $p(t) = -\frac{1}{20}\,(t-6)^2 + 720$

$0 = A(12-6)^2 + 720$

$0 = A(6)^2 + 720$

$\frac{-720}{36} = \frac{36A}{36}$

$-\frac{720}{36} = A$

$36 \times 20 = 720$

8. The radioactive fallout from a nuclear explosion contaminated a Pacific Ocean island with the radioactive component Strontium-90 which has a half-life of 28 years. Readings from the island indicated that the level of Strontium-90 was seven (7) times the level considered safe for human habitation. If the decay factor for Strontium-90 is $k = -\frac{1}{28}ln(\frac{1}{2})$, then how many years after contamination will the island become safe for human habitation?

(A) $28\,\frac{ln(\frac{1}{7})}{ln(\frac{1}{2})}$ years

$t = 28$ years $\quad k = -\frac{1}{28}\ln(\frac{1}{2}) \quad$ S be safe value

(B) $\frac{ln(\frac{1}{7})}{28\,ln(\frac{1}{2})}$ years

$A(t) = A_0 e^{-kt}$

(C) $28\,\frac{ln(\frac{1}{2})}{ln(\frac{1}{7})}$ years

$\frac{1S}{7S} = e^{-kt} \quad 7S = Se^{-kt}$

$7 = e^{-(\frac{1}{28}\ln\frac{1}{2})t}$

$28\ln 7 = \ln\frac{1}{2}t$

$t = \frac{28\ln 7}{\ln\frac{1}{2}}$

(D) $\frac{ln(7)}{28\,ln(\frac{1}{2})}$ years

$\ln 7 = \ln(e^{\frac{1}{28}\ln\frac{1}{2}t})$

(E) $28\,\frac{ln(7)}{ln(\frac{1}{2})}$ years

$\ln 7 = \frac{1}{28}\ln(\frac{1}{2})t$

9. If, in an arithmetic sequence $\{a_n\}_{n=1}^{+\infty}$, $a_5 = 36$ and $a_{21} - a_{10} = 55$ then a_1 equals

(A) 14

$a_{n+1} = a_n + d \quad a_5 = a_1 + 4d \quad a_1 + 20d - (a_1 + 9d) = 55$

(B) 15

$a_n = a_1 + (n-1)d \quad 36 = a_1 + 4(5) \quad \frac{11d}{11} = \frac{55}{11}$

(C) 16

$36 - 20 = a_1 \quad d = 5$

(D) 17

$16 = a_1$

(E) 18

(16, 21, 26, 31, 36, 41, 46, 51, 56) 61

1 2 3 4 5 6 7 8 9

10. If $3(8^{4x-7}) = 11$ then x equals

(A) $\frac{1}{4}log_8(\frac{11}{3}) + \frac{1}{4}log_8(7)$

(B) $4(log_8(\frac{11}{3}) + 7)$

(C) $\frac{1}{4}log_8(\frac{11}{3}) + 7$

(D) $\frac{1}{4}(log_8(\frac{11}{3}) + 7)$

(E) $\frac{1}{4}(log_8(11) - 7)$

$3(8^{4x-7}) = 11$

$\ln 8^{4x-7} = \ln \frac{11}{3}$

$\log_8(8^{4x-7}) = \log_8(\frac{11}{3})$

$4x-7 \ln 8 = \ln \frac{11}{3}$

~~$(4x-7)\log 8 = \log \frac{11}{3}$~~

$4x - 7 = \log \frac{11}{3}$

$4x = \log \frac{11}{3} + 7$

$x = \frac{1}{4}\log(\frac{11}{3} + 7)$

11. If the sequence $\{b_n\}_{n=1}^{+\infty} = \{6, 11, 16, 21, \ldots\}$ then b_{101} equals

5 5 5

(A) 501

(B) 506

(C) 511

(D) 516

(E) 518

$a_{n+1} = a_n + d$

$a_n = a_n + (n-1)d$

$a_n = 6 + (101-1)d$

$a_n = 6 + (100)(5)$

$a_n = 500 + 6$

$= 506$

$a_n = a_1 + (n-1)d$

$g_n = g_1 r^{n-1}$

12. If f is a function where $f(x) \leq 0$ only for $2 \leq x \leq 6$ and the given graph is the graph of the absolute value of $f(x)$, $|f(x)|$, then the function f is which of the following?

(4, -4)

(A) $f(x) = -(x-4)^2 - 4$

(4, -4)

(B) $f(x) = (x-4)^2 - 4$

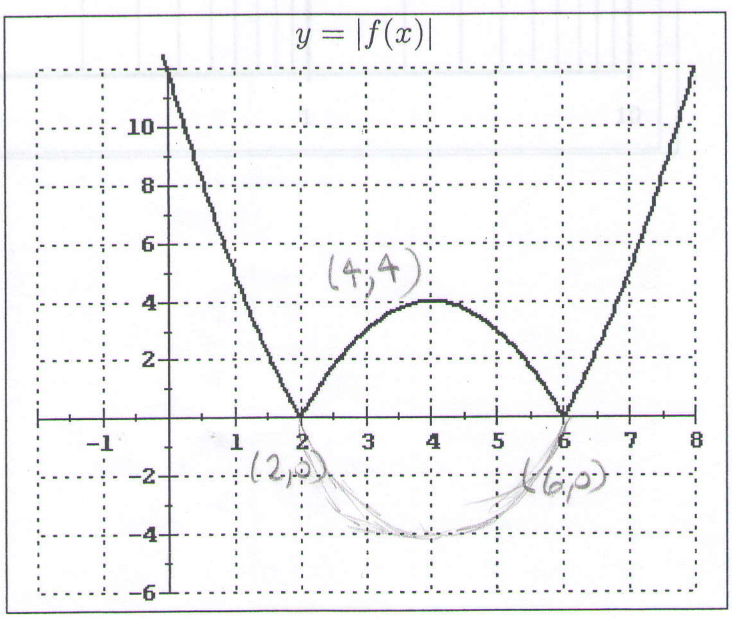
(-4, -4)

(C) $f(x) = (x+4)^2 - 4$

(4, +4)

(D) $f(x) = (x-4)^2 + 4$

(E) $f(x) = -(x-4)^2 + 4$



13. Which of the following equations passes through the point (3, 1200) and when plotted on semi-log graph paper yields a straight line?

$y = cb^{kx}$

~~1200 = 1200~~

(A) $y = 60\,(2)^{\frac{x}{3}}$

(B) $y = 960(\frac{1}{2})^{x}$

$1200 = 60(2)^{3/3}$

$= 120$

$1200 = 300(4)^{3/3}$

$= 300(4)^{1}$

$= 1200$

(C) $y = 100x + 900$

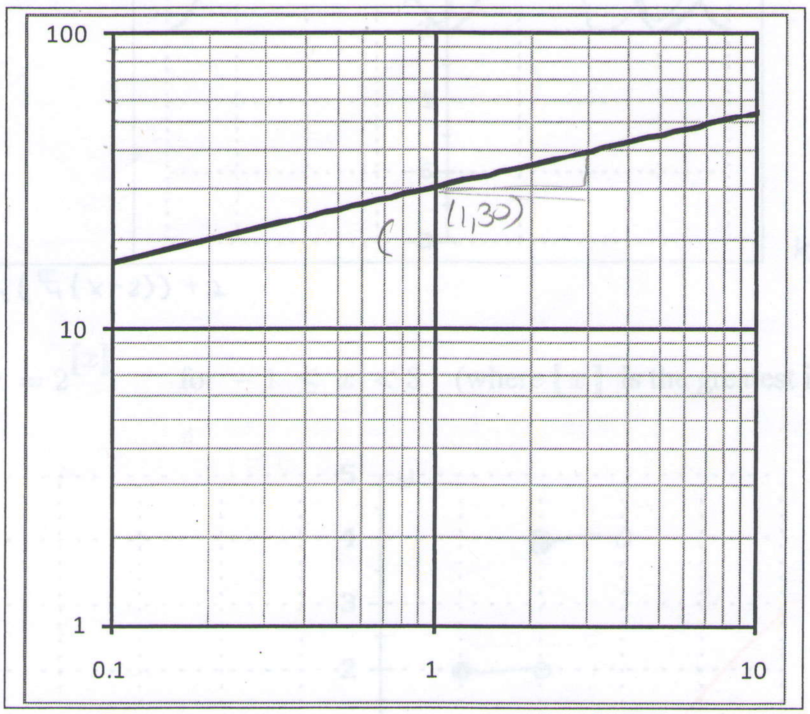
(D) $y = 1200\,x^{\frac{2}{3}}$

$1200 = 300(4)^{3/3}$

$= 1200.$

(E) $y = 300\,(4)^{\frac{x}{3}}$

14. Which one of the following function's graph would be most likely to produce this straight line?



(A) $y = 0.25x + 30$

(B) $y = 30\,x^{0.25}$

(C) $y = 30\,e^{0.25x}$

(D) $y = 20\,x^{0.25}$

(E) $y = 0.25\,x^{30}$

log-log

$y = cx^k$

$y = cx^k$

PART II

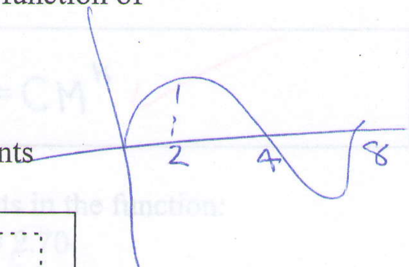
Work each problem. Show all your work in the space provided.

15. Plotted data from an experiment produced a graph modelled by the periodic function of the form

$\boxed{2}$

$$y = K\sin(m(x-a)) + b\,.$$

Use the graph to determine this model by computing the values of the constants K, m, a, and b.



$8 = \frac{2\pi}{k}$

$k = \frac{\pi}{4}$

2

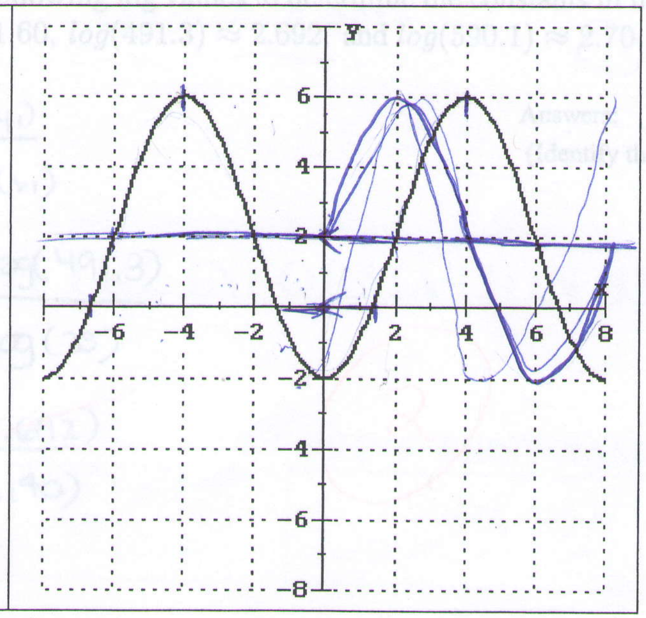
Answers

$K = \boxed{4}$

$m = \boxed{\frac{\pi}{4}}$

$a = \boxed{2}$

$b = \boxed{2}$



$b = \frac{6 + (-2)}{2}$

$b = 2$

$k = \frac{6 - (-2)}{2}$

$= 4$

$y = 4\sin(\frac{\pi}{4}(x-2)) + 2$

16. Sketch the graph of $y = 2^{[\![x]\!]}$ for $-1 \le x < 3$ (where $[\![x]\!]$ is the greatest integer of x)

$\boxed{2}$

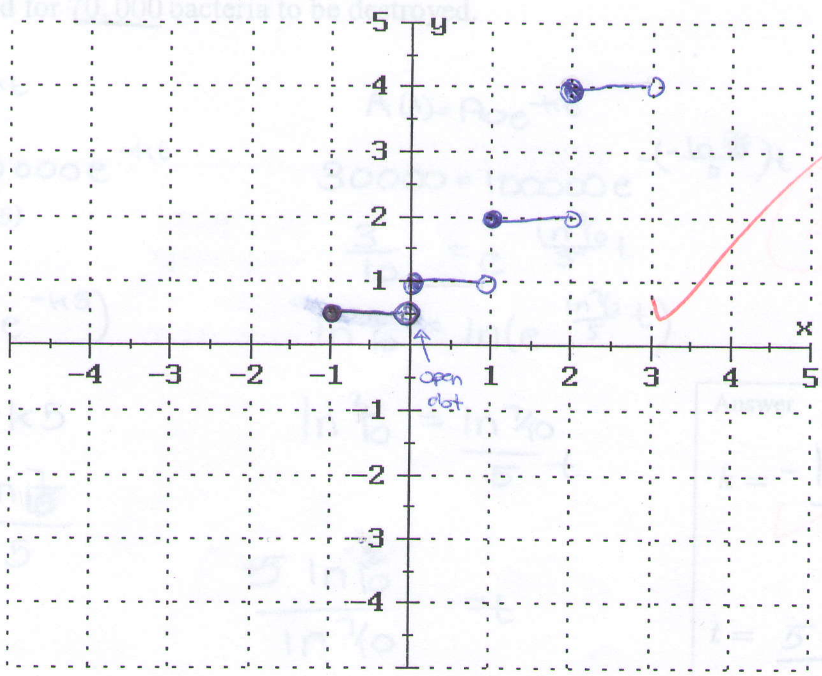
2

x	$[\![x]\!]$	$y = 2^{[\![x]\!]}$
$[-1, 0)$	-1	$\frac{1}{2}$
$[0, 1)$	0	1
$[1, 2)$	1	2
$[2, 3)$	2	4

2^{-1}

2^{0}

$\frac{1}{2}$



17. In determining the mathematical model that gives the pregancy length D (in days) as a function of the body weight M (in gm) of mammals, the data plotted on log-log graph paper produced a straight line passing through the points corresponding to the (M, D) data points $(1,\ 140)$, $(25,\ 491.3)$, and $(40,\ 590.1)$.

2

$y = cx^k$

(a) State the general model of D as a function M.

$y = cx^k$

Answer

$D = CM^k$

(b) Use the data points and the following log values to determine the constants in the function: $log(25) \approx 1.40$, $log(40) \approx 1.60$, $log(491.3) \approx 2.692$, and $log(590.1) \approx 2.70$

$$m = \frac{\log(y_2) - \log(y_1)}{\log(x_2) - \log(x_1)}$$

$$= \frac{\log(590.1) - \log(491.3)}{\log(40) - \log(25)}$$

$$= \frac{(2.70) - (2.692)}{(1.60) - (1.40)}$$

$$\begin{array}{r} 2.700 \\ -2.692 \\ \hline 0.008 \end{array}$$

$$\frac{8}{200} = \frac{4}{100} = \frac{0.008}{0.20}$$

$$= \frac{2}{50} = \frac{1}{25}$$

Answers:

(Identify the constant and give its value)

$C = 140$

$K = \frac{1}{25}$

2

18. When an antibiotic was introduced into a culture of 100,000 bacteria, the number of bacteria decreased exponentially. If, in 5 hours, 30,000 bacteria were destroyed, determine the decay factor k and the time, in hours, that is required for 70, 000 bacteria to be destroyed.

2

$$A(t) = A_0 e^{-kt}$$

$$70000 = 100000e^{-kt}$$

$$\frac{7}{10} = e^{-k(5)}$$

$$\ln\frac{7}{10} = \ln(e^{-k5})$$

$$\ln\frac{7}{10} = -k5$$

$$k = -\frac{\ln\frac{7}{10}}{5}$$

$$A(t) = A_0 e^{-kt}$$

$$30000 = 100000e^{-(-\frac{\ln\frac{7}{10}}{5})t}$$

$$\frac{3}{10} = e^{\frac{\ln 7/10}{5}t}$$

$$\ln\frac{3}{10} = \ln(e^{\frac{\ln 7/10}{5}t})$$

$$\ln\frac{3}{10} = \frac{\ln 7/10}{5}t$$

$$\frac{5\ln\frac{3}{10}}{\ln 7/10} = t$$

2

Answer

$k = -\frac{\ln\frac{7}{10}}{5}$

$t = \frac{5\ln\frac{3}{10}}{\ln 7/10}$