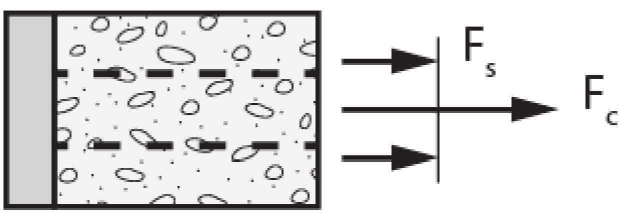
MAAE2202 Midterm Exam Solutions
Exam A - 2013

Question 1:

Givens: $\sigma_p := 500\text{MPa}$ $\qquad \frac{E_s}{E_c} = 10$ $\qquad \frac{A_s}{A_c} = \frac{1}{30}$

(1) FBD (for step c):

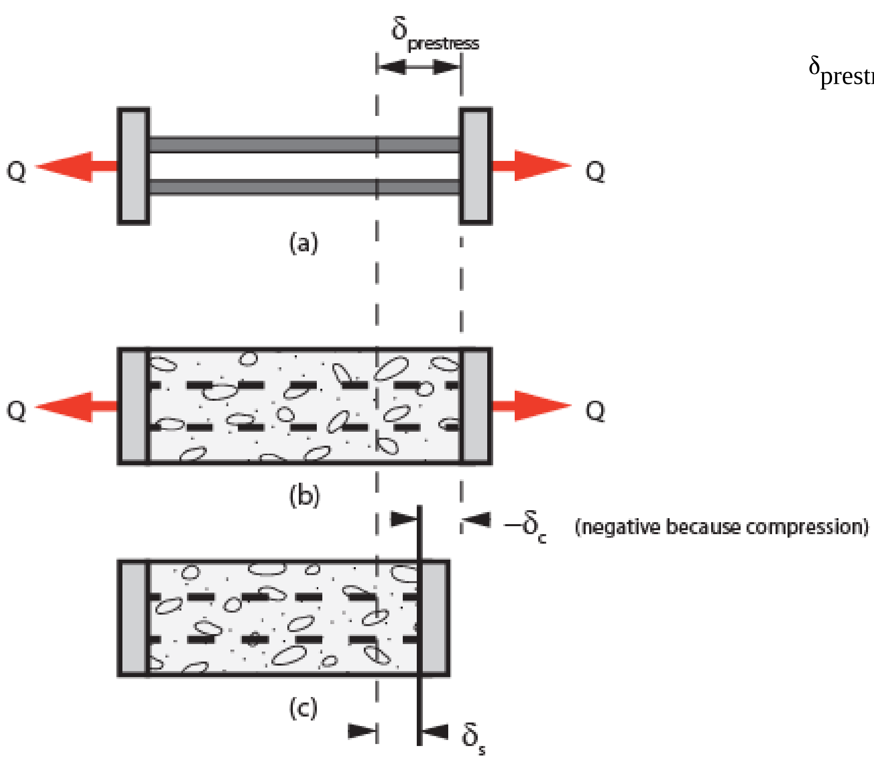


(II) Equilibrium:

ΣF: $F_s = -F_c$ (1)

(III) Compatibility:

Use superposition of all three steps. Deformations shown below:



$\delta_{prestress} = \delta_s - \delta_c$ (2)

(IV) Force-Displacement Relation:

All members are axial loaded: $\delta = \frac{PL}{EA}$ => $\delta_c = \frac{F_c \cdot L}{E_c \cdot A_c} = \sigma_c \cdot \frac{L}{E_c}$ (3)

also: $\sigma = \frac{P}{A}$ (4)

$$\delta_s = \frac{F_s \cdot L}{E_s \cdot A_s} = \sigma_s \cdot \frac{L}{E_s}$$

$$\delta_{prestress} = \frac{Q \cdot L}{E_s \cdot A_s} = \sigma_p \cdot \frac{L}{E_s}$$

Solution:

Sub (3) into (2): $\sigma_p \frac{\not{L}}{E_s} = \sigma_s \frac{\not{L}}{E_s} - \sigma_c \frac{\not{L}}{E_c}$ (5)

(4) into (1): $\sigma_s \cdot A_s = -\sigma_c \cdot A_c$ => $\sigma_s = \frac{-A_c}{A_s} \cdot \sigma_c$ (6)

(6) into (5): $\frac{\sigma_p}{E_s} = \sigma_c \cdot \left[\left(\frac{-A_c}{A_s}\right) \cdot \frac{1}{E_s} - \frac{1}{E_c}\right]$

$$\sigma_p = \sigma_c \cdot \left[\left(\frac{-A_c}{A_s}\right) - \frac{E_s}{E_c}\right] \quad => \quad \sigma_c = \frac{\sigma_p}{\left[\left(\frac{-A_c}{A_s}\right) - \frac{E_s}{E_c}\right]}$$

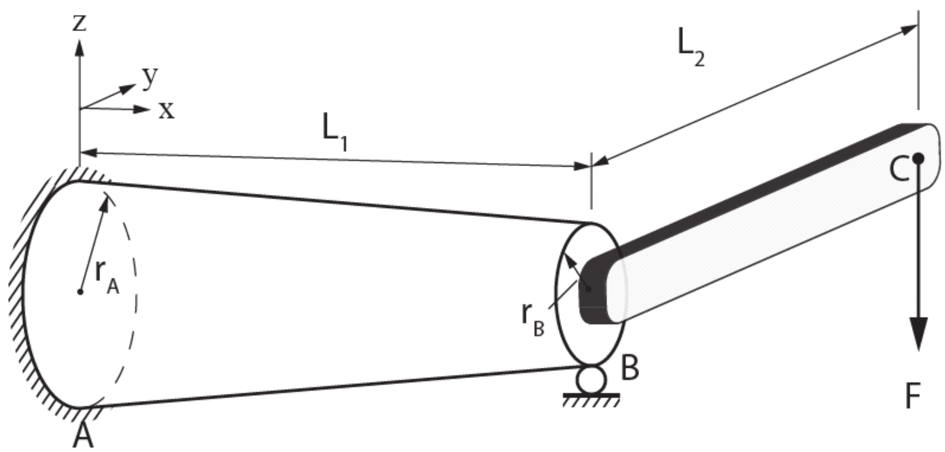
$$\sigma_c := \frac{495MPa}{-25 - 8} = -15 \cdot MPa$$

$$\sigma_s = \frac{-A_c}{A_s} \cdot \sigma_c$$

$$\sigma_s := -25 \cdot (-15MPa) = 375 \cdot MPa$$

The answer makes sense. The steel wires are prestressed in tension, and when the preload is removed, are restrained from fully unloading. Thus the final stress in the wires should be less than the preload and greater than 0. In order to maintain equilibrium, the concrete must be in compression in order to balance the tensile stress in the cables as no other loads are acting on the reinforced concrete block.

Question 2:



Givens:

Geometry

$L_1 := 1.5\mathrm{m}$

$L_2 := 1.2\mathrm{m}$

$r_A := 6\mathrm{cm}$

$r_B := 3\mathrm{cm}$

Material

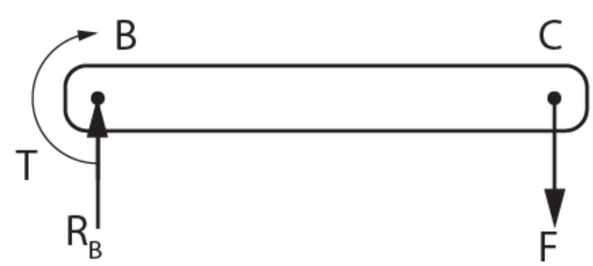
$E := 70\mathrm{GPa}$

$G := 26\mathrm{GPa}$

Loading

$F := 2\mathrm{kN}$

FBD:



Equilibrium:

For link BC: $\Sigma M_B = 0$ $T := -F \cdot L_2 = -2.4 \cdot kN \cdot m$

For link AB: Torque is constant = T

Compatibility:

Link AB and BC are rigidly connected, and link BC is rigid, therefore:

$\delta_C = L_2 \cdot \sin(\theta_{AB})$ or approximately $L_2 \cdot \theta$ for small angles (Positive being up)

Force-Deflection:

we need angle of rotation of shaft AB: $\theta = \frac{T \cdot L}{G \cdot J}$ however, J varies with x! $d\theta = \frac{T \cdot dx}{G \cdot J(x)}$

$J = \frac{\pi}{2} \cdot r^4$ so as a function of x: $J(x) = \frac{\pi}{2} \cdot \left[r_A - \left(\frac{r_A - r_B}{L_1} \right) \cdot x \right]^4$

let: $b := -\frac{r_A - r_B}{L_1} = -0.02$

$J(x) = \frac{\pi}{2} \cdot (a + b \cdot x)^4 = \frac{\pi \cdot b^4}{2} \cdot \left(x + \frac{a}{b} \right)^4$

$a := r_A = 0.06\, m$

$$\theta_{AB} = \int_0^{L_1} \frac{T}{G \cdot J(x)}\, dx = \frac{T}{G} \cdot \frac{2}{\pi \cdot b^4} \cdot \int_0^{L_1} \left(x + \frac{a}{b} \right)^{-4} dx$$

Given: $\int (x + a)^n\, dx = \frac{(x + a)^{n+1}}{n + 1}$

$$\theta_{AB} = \frac{2T}{\pi\, b^4 G} \cdot \left[-\frac{1}{3} \cdot \left(x + \frac{a}{b} \right)^{-3} \right] \Bigg|_0^{L_1}$$

$$\theta_{AB} := \frac{2T}{\pi b^4 G} \cdot \left[-\frac{1}{3} \cdot \left(L_1 + \frac{a}{b} \right)^{-3} \right] = -2.078 \cdot deg$$

(negative indicates direction is opposite f that indicated by FBD)

Fropm compatibility: $\delta_C := L_2 \cdot (\theta_{AB}) = -43.53 \cdot mm$ (negative implies downwards displacement)

The answer makes sense. Given the force acts downwards, it is expected that the vertical deflection of point C will be downwards as well. Also, the angle of rotation is small, thus our small angle approximation is valid.