TEST SOLUTIONS

1

① a) $\lim_{x\to 4} \frac{x-4}{x^2-16} = \lim_{x\to 4} \frac{x-4}{(x-4)(x+4)}$ ① $= \lim_{x\to 4} \frac{1}{x+4}$ ① $= \frac{1}{8}$ ①

b) $\lim_{x\to 2^+} \frac{x+2}{x^2-4} = \lim_{x\to 2^+} \frac{x+2}{(x+2)(x-2)}$ ① $= \lim_{x\to 2^+} \frac{1}{x-2}$ ① $= \infty$ ①

c) $\lim_{x\to 0} \frac{\sin 4x}{2x} = \underbrace{2}_{①} \underbrace{\lim_{x\to 0} \frac{\sin 4x}{4x}}_{①} = 2(1) = 2$ ①

d) $\lim_{x\to \infty} \sqrt{x^2+1} - x = \lim_{x\to\infty} \frac{\sqrt{x^2+1}-x}{1} \cdot \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}$ ②

$= \lim_{x\to\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} = \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}+x} = 0$ ①

e) $\lim_{x\to\infty} \frac{x^2+1}{3x^2+2x+1} = \lim_{x\to\infty} \underbrace{\frac{1+\frac{1}{x^2}}{3+\frac{2}{x}+\frac{1}{x^2}}}_{②} = \frac{1}{3}$ ①

($\frac{1}{x^2} \to 0$, $\frac{2}{x} \to 0$, $\frac{1}{x^2} \to 0$)

2) We suspect $x=0$

I) $f(0)=2$ ①

② - for limit work

II) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^3+1 = \underbrace{1 = L = 1}_{①} = \lim_{x\to 0^-} x^3+1 = \lim_{x\to 0^-} f(x)$

III) verify: $f(a)=L \to \underbrace{f(0)=2 \neq 1 = L}_{①}$

$\therefore$ $f(x)$ __IS__ discontinuous at $x=0$. ① - for some kind of conclusion

(3) $LS = \cos\left(\frac{3\pi}{2} - \theta\right) = \cos\frac{3\pi}{2}\cos\theta + \sin\frac{3\pi}{2}\sin\theta$ (1)

$= \underbrace{(0)}_{1}\cos\theta + \underbrace{(-1)}_{1}\sin\theta = \underbrace{-\sin\theta}_{1} = \underbrace{RS}_{1}$; must show $LS = RS$ (proper form!)

(4)

$LS = \dfrac{1 + \cot x}{1 + \tan x} = \dfrac{1 + \frac{\cos x}{\sin x}\ (1)}{1 + \frac{\sin x}{\cos x}\ (1)} = \left.\dfrac{\frac{\sin x + \cos x}{\sin x}}{\frac{\cos x + \sin x}{\cos x}}\right\}(1)$

$= \underbrace{\dfrac{(\cancel{\sin x + \cos x})}{\sin x} \cdot \dfrac{\cos x}{(\cancel{\sin x + \cos x})}}_{1} = \dfrac{\cos x}{\sin x}\ (1) = \overset{(1)}{\cot x} = \underbrace{RS}_{1}$

proper form!

(5) $LS = \dfrac{1}{\tan x + \cot x} = \dfrac{1}{\underbrace{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}_{2}} = \dfrac{1}{\left.\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right\}(1)}$

$= \dfrac{1}{\frac{(1)}{\cos x \sin x}} = \underbrace{\sin x \cos x}_{1} = RS$ (1) → form!

using $\boxed{\sin^2 x + \cos^2 x = 1}$ (2)