

**PART A (17 marks)**

**NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.**

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mark

1. Which of the following equations is/are linear?

$$\begin{aligned} (i) \quad & 2x + y^2 + z = 1 \\ (ii) \quad & -x + 3y = 2z + 10 \\ (iii) \quad & xy - y + z + w = 0 \end{aligned}$$

A: none of them	B: (i) only	C: (ii) only	D: (iii) only	E: (i) and (iii) only
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2. Find the system of linear equations corresponding to the following augmented matrix.

$$\left[ \begin{array}{cccc|c} 1 & -1 & 2 & 4 & 0 \\ 0 & 2 & 1 & 0 & -\frac{2}{3} \\ 3 & -2 & 0 & \frac{1}{2} & -1 \end{array} \right]$$

A: $\begin{aligned} x - y + 2z + 4w &= 0 \\ 2y + z &= -\frac{2}{3} \\ 3x - 2y + \frac{1}{2}w &= -1 \end{aligned}$	B: $\begin{aligned} x - y + 2z + 4w &= 0 \\ 2y + z &= \frac{2}{3} \\ 3x - 2y + \frac{1}{2}w &= 1 \end{aligned}$
C: $\begin{aligned} x - y + 2z &= 0 \\ 2y + z &= -\frac{2}{3} \\ 3x - 2y + \frac{1}{2}w &= -1 \end{aligned}$	D: $\begin{aligned} x + y + 4w &= 0 \\ 2y + z &= -\frac{2}{3} \\ 3x - 2y &= -1 \end{aligned}$
E: $\begin{aligned} x - y + 2z + w &= 0 \\ 2y + z &= -\frac{2}{3} \\ 3x - 2y + \frac{1}{2}w &= -1 \end{aligned}$	

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3. Which one of the given vectors is a solution to the following system of linear equations?

$$\begin{aligned} x_1 - x_3 &= 2 \\ x_1 + x_2 + x_3 + x_4 &= 0 \\ 2x_2 + x_3 - x_4 &= -1 \end{aligned}$$

A: (0, -2, 4, 1)	B: (0, 2, -2, 0)	C: (0, 0, -2, 1)	D: (0, 0, 1, 0)	E: (1, 0, -1, 0)
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4. Which of the following matrices is/are in row-reduced echelon form?

$$\begin{aligned} P &= \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 0 \end{array} \right] & Q &= \left[ \begin{array}{cccc|c} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 3 \end{array} \right] \\ R &= \left[ \begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 2 & 0 \end{array} \right] & S &= \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right] \end{aligned}$$

A: Q and R only	B: P only	C: P, Q and R only	D: P and S only	E: S only
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- 1 mark 5. If  $A = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 6 & 5 & 4 & 3 & 2 \\ 2 & 4 & 6 & 8 & 10 \end{bmatrix}$  and  $R$  is the row-reduced echelon form of matrix  $A$ , which one of the following is the first column of  $R$ ?

A: $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	B: $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	C: $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	D: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	E: None of A, B, C or D.
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- 1 mark 6. Find the row-reduced echelon form of the matrix  $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & -2 \\ 1 & 1 & 3 \end{bmatrix}$ .

A: $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$
D: $\begin{bmatrix} -1 & 0 & -4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$	E: $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 4 \\ 0 & 1 & -1 \end{bmatrix}$	

- 1 mark 7. For which values of  $k$  does the matrix shown below row-reduce to the  $2 \times 2$  identity matrix?

$$\begin{bmatrix} 3 & 6 \\ k & 4 \end{bmatrix}$$

A: all real $k$	B: $k = 2$ only	C: $k = 0$ and $k = 3$ only	D: all $k \neq 2$	E: all $k \neq 4$
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- 1 mark 8. Find all the solutions of the system whose augmented matrix is  $\left[ \begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$ .

A: $(0, 5, 7, 0)$	B: $(-3t, 5, t, 7)$	C: $(3t, 5, t, 7)$	D: $(3, 5, 7, 0)$	E: $(3, 5, t, 7)$
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- 1 mark 9. How many solutions does the following system of linear equations have?

$$\begin{aligned} x - y + z &= 2 \\ x + y - z &= 0 \\ y + 2z &= 2 \end{aligned}$$

A: no solutions	B: exactly one solution
C: a one-parameter family of solutions	D: a two-parameter family of solutions
E: a three-parameter family of solutions	

- 1 mark 10. How many solutions does the following system of linear equations have?

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 5 \\ 2x_1 + 4x_2 + 6x_3 + 8x_4 &= 11 \end{aligned}$$

A: a three-parameter family of solutions	B: a two-parameter family of solutions
C: a one-parameter family of solutions	D: exactly one solution
E: no solutions	

- 1 mark 11. If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ , find  $3A - 2B$ .

A: $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$	B: $\begin{bmatrix} 1 & 6 \\ 7 & 8 \end{bmatrix}$	C: $\begin{bmatrix} 1 & 6 \\ 7 & 0 \end{bmatrix}$	D: $\begin{bmatrix} 5 & 6 & 7 \\ 8 & 7 & 6 \end{bmatrix}$	E: $\begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix}$
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- 1 mark 12. If  $A$  is a  $2 \times 3$  matrix and  $B$  is a  $4 \times 3$  matrix and  $C$  is a  $2 \times 4$  matrix then  $AB^T + C$  is

A: a $4 \times 3$ matrix	B: a $2 \times 2$ matrix	C: a $3 \times 2$ matrix
D: a $2 \times 4$ matrix	E: undefined	

- 1 mark 13. If  $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 4 & 1 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 9 & 0 & 2 & 3 \\ 5 & 0 & -1 & 5 \\ 7 & 0 & 3 & 8 \end{bmatrix}$  and  $C = AB$ , what is the value of  $c_{23}$ ?

A: 1	B: 2	C: 3	D: 0	E: 6
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- 1 mark 14. If  $J = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$  and  $K = \begin{bmatrix} 0 & 1 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}$ , find  $JK^T$ .

A: $\begin{bmatrix} 3 & -1 & 4 \\ 5 & -1 & 6 \end{bmatrix}$	B: $\begin{bmatrix} 3 & 5 \\ -1 & -1 \\ 4 & 6 \end{bmatrix}$	C: $\begin{bmatrix} 5 & 1 & 0 \\ 9 & 1 & 6 \end{bmatrix}$	D: $\begin{bmatrix} 5 & 3 & 1 \\ 2 & -5 & 4 \end{bmatrix}$	E: undefined
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- 1 mark 15. Let  $M = \begin{bmatrix} a & b & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$  and  $N = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$ . If  $MN = I$ , where  $I$  is the  $3 \times 3$  identity matrix, what is  $b$ ?

A: -2	B: 2	C: 0	D: 1	E: -1
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- 1 mark 16. Let  $Q = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ . Evaluate  $(Q - 2I)^2$ , where  $I$  is the  $3 \times 3$  identity matrix.

A: $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	B: $\begin{bmatrix} 1 & -4 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$	C: $\begin{bmatrix} 1 & -4 & -2 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix}$
D: $\begin{bmatrix} 1 & -4 & -2 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$	E: $\begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$	

- 1 mark 17. Let  $A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$ . Which of the following matrices gives the matrix  $A^n$  for  $n > 1$ ?

A: $\begin{bmatrix} 2n & 2^n \\ 0 & 1 \end{bmatrix}$	B: $\begin{bmatrix} n^2 & -2n - 1 \\ 0 & 1 \end{bmatrix}$	C: $\begin{bmatrix} 2n & -2n \\ 0 & 1 \end{bmatrix}$
D: $\begin{bmatrix} n^2 & -n^2 \\ 0 & 1 \end{bmatrix}$	E: $\begin{bmatrix} 2^n & -2^n + 1 \\ 0 & 1 \end{bmatrix}$	

**PART B (8 marks)**

**SHOW YOUR WORK FOR ALL QUESTIONS IN PART B**

*3 marks* 18. Suppose  $\left[ \begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 2 & 2 & 2 \\ 0 & 1 & k & 1 \end{array} \right]$  is the augmented matrix of a system of linear equations.

(a) Find the value of  $k$  for which the system has infinitely many solutions.

(b) Using the value of  $k$  you found in part (a), find all solutions to the system. Express your answer in parametric form.

*2 marks* 19. Let  $B = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & -1 \end{bmatrix}$ . Find  $B^T B$ .

- 3 marks* 20. Find the Row-Reduced Echelon Form of the matrix  $\begin{bmatrix} 1 & 0 & 2 & 0 & 3 \\ 2 & 1 & 3 & 2 & 4 \\ 0 & 2 & -2 & 5 & -3 \end{bmatrix}$ .

At each step, state what elementary row operation is being used.

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Instructor's Name (**Print**)

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Student's Name (**Print**)

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Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO  
LONDON CANADA  
DEPARTMENT OF MATHEMATICS

Mathematics 1229A Test 2

Friday, November 2, 2012

**Code 111**

7:00 p.m. - 8:30 p.m.

INSTRUCTIONS

1. Fill in the tops of this page **and the back of this page** completely. Be sure to print your name **legibly**.
2. Fill in the top of the scantron card completely. **Both print AND code** your Student Number, Section Number (see below) and Exam Code (shown above).
3. CALCULATORS AND NOTES ARE NOT PERMITTED.
4. DO NOT UNSTAPLE THE BOOKLET.
5. There are two parts to this examination: PART A (17 marks) in multiple choice format and PART B (8 marks) in written answer format.
6. In Part A, **circle** the correct answer to each question **on this paper** AND fill in the appropriate box on the **scantron** card with an HB pencil. This question paper will be returned to you.
7. In Part B, write your answer in the space provided.
8. Questions are printed on both sides of the paper. They begin on Page 1 and continue to Page 5. Be sure that your booklet is complete.
9. You must hand in this question paper, your scantron card, and all rough work sheets.
10. Circle your section in the list below.

Instructor	Campus/College	Time	Section
Combariza	Main	9:30 MWF	001
Nguyen	Main	12:30 MWF	002
Olds	Main	1:30 MWTh	003
Kossovskiy	Main	7:00 Tu	004
Florence	Brescia	9:30 MWTh	530
Pourkia	Huron	10:30 TuThF	550
Pourkia	Huron	8:30 MWF	551
Kuzmin	King's	1:30 TTh	570
Valluri	King's	9:30 TTh	571
Pourkia	King's	7:00 MW	572

11. TOTAL MARKS = 25.

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Student Number (**Print**)

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Student's Name (**Print**)

FOR GRADING ONLY

PAGE	MARK
1-3	
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TOTAL	