MAT 1308A ASSIGNMENT # 1

Due: 24 September 2013 IN CLASS

NOTE: As per the syllabus, late assignments will not be accepted. If for some reason you won't be able to make it to lecture on the 23^{rd}, please make arrangements to drop off your assignment at some other time beforehand.

1) True or false. If the statement is false, give an example where it fails to be true.

a) Horizontal shift transformations do not alter the domain of a function.

b) Vertical shift transformations do not alter the domain of a function.

c) Reflections (either across the x or y axis) do not alter the domain of a function.

d) Horizontal shift transformations do not alter the range of a function.

e) Vertical shift transformations do not alter the range of a function.

f) Reflections (either across the x or y axis) do not alter the range of a function.

2) For each equation, find the largest possible domain for the function it defines, its range, the formula for that function in terms of x, and sketch its graph.

a) $3x + 2y = 10$.

b) $x^2 = \pi - y$.

c) $y^2 = -2 + x$.

d) $x - \sqrt{y} = 1$.

3) Put the following quadratic functions in vertex form by completing the square. Use your knowledge of transformations to sketch the graph of the function. State the domain and range.

a) $f(x) = x^2 + 10x + 24$.

b) $f(x) = -2x^2 + 4x + 5$.

4) Find the formula for the functions described below. Sketch their graphs and state the the domain and range for each.

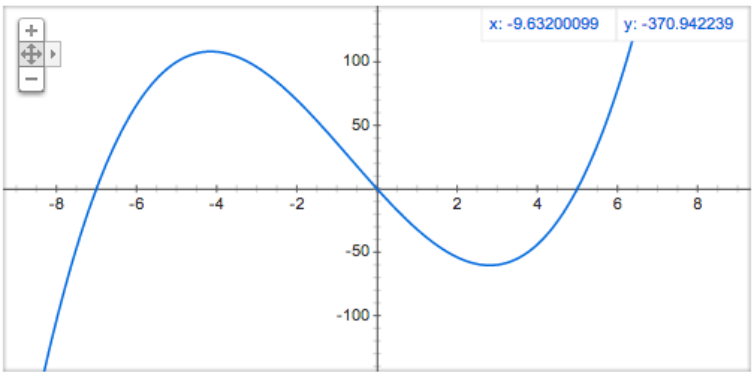
a) The function $f(x) = x^2$, shifted left three units and up five.

b) The function $f(x) = 5x - 1$, shifted three units up, reflected across the x-axis, and then shifted two units right.

c) The function $f(x) = x^3$, reflected across the y-axis, shifted two units down, then reflected across the y-axis.

d) The function $f(x) = \frac{1}{x-4}$, reflected across the y-axis, then shifted right two units, then shifted down six units.

5) Is it possible to write the formula for a cubic function whose graph is



in the form

$$f(x) = (x - h)^3 + k$$

for some $h, k \in \mathbb{R}$? If yes, give the formula. If not, explain why, and give the function with this graph in polynomial form.

6) Give the domain, range, and asymptotes (horizontal and vertical) of the following rational functions.

a) $f(x) = \dfrac{-6}{x+4}$.

b) $f(x) = \dfrac{2x}{3x-5}$.

c) $f(x) = \dfrac{-x^4}{x^3+17}$.

d) $f(x) = \dfrac{26x^2}{51-x^5}$.

e) $f(x) = \dfrac{x+2}{x^2+x-2}$.

7) For each function, draw its graph and use it to find the left- and right-hand limits at the specified point a, if it exists. Does the limit exist at the point?

a) $f(x) = \begin{cases} -x^2 & \text{if } x \neq 2, \\ 1 & \text{if } x = 2, \end{cases}$
at $a = 0, 2$.

b) $f(x) = \dfrac{x+3}{x^2-9}$, at $a = -3, 0, 3$.

c) $f(x) = \sqrt{x+1}$, at $a = 0, -1$.

d) $f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 0, \\ 6x - 1 & \text{if } 0 < x < 2, \\ 0 & \text{if } x \geq 2, \end{cases}$
at $a = -20, 2$.

8) Find the following limits without using the graph of the function (be sure to show some work!).

a) $\lim_{x \to -5}(3x^2 - 8x - 2)$.

b) $\lim_{x \to -1} \left(\dfrac{7x+7}{x^3+7x+9} \right)$.

c) $\lim_{x \to 4} \left(\dfrac{3x+12}{x^2-16} \right)$.

d) $\lim_{x \to 2} \left(\dfrac{3x^2-12}{16-x^4} \right)$.

e) $\lim_{x \to 1} \left(\dfrac{x^5-3x+2}{7x^5+4x^2-10x-1} \right)$.