Midterm Exam 1

Intermedieate Microeconomics 1

Econ 2260A Sections 002 (Class Meeting at 11:30 ~~P.M.~~ A.M.), Fall 2012

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1. In Charlie's world there are only two commodities. Charlie's utility function is

   Quasilinear preference

   $$U(x_1, x_2) = \ln x_1 + x_2, \qquad 4x_1 + 12x_2 = 10$$

   with x_i denoting the quantity of commodity i to be consumed ($i = 1, 2$). Suppose that the price of commodity 1 is \$4, the price of commodity 2 is \$12, and Charlie's income is \$10.

   a. Calculate the tangent point between Charlie's budget constraint and the highest indifference curve that the constraint can touch.

   b. Find out the optimal consumption bundle for Charlie. Explain your answer with a clearly labeled diagram that contains—

      i. the graphs of the budget constraint,

      ii. the indifference curve which the budget costraint is tangent to,

      iii. the optimum,

      iv. the indifference curve that passes thru the optimum,

      v. the slope of the budget constraint.

$$\frac{\delta}{\delta x_1} = \frac{1}{x_1} \qquad \therefore \frac{-MU_1}{MU_2} = \frac{-P_1}{P_2} \qquad \therefore 4(3) + 12x_2 = 10$$

$$\frac{\delta}{\delta x_2} = 1 \qquad \frac{-1}{x_1} = \frac{-4}{12} \qquad 12x_2 = 10 - 12$$

$$4x_1 = 12 \qquad x_2 = \frac{-2}{12}$$

$$x_1 = 3 \qquad x_2 = \frac{-1}{6}$$

2. Alice has income \$40 and lives in a world with only two goods. The price of good 1 is \$5 when her consumption of good 1 does not exceed 3 units and, if her consumption of good 1 is above 3 units, the price for any excess quantity is \$10 per unit. The price for good 2 is constantly \$5.

   a. Draw a clearly labeled diagram that contains Alice's budget constraint, and calculate the following items and write the results in the diagram:

      i. the horizontal intercept of the budget constraint

      ii. the vertical intercept of the budget constraint

      iii. the slope, or slopes if the slope of the budget constraint is not constant, of the budget constraint

      iv. the coordinates of the point at which the budget constraint changes its slope if such a point exists

   b. Alice has opted for consuming exactly 1 pound (unit of good 1) of chocolate and 1 pound (unit of good 2) of playdough. Based on all the above information, circle all the correct statements listed below: lack utility function ∴ don't know what indifference curve looks like

      i. Alice's preference relation is monotone

      ii. Alice's preference relation is not monotone

      iii. There is insufficient information to determine the monotonicity of Alice's preference relation

      iv. Alice's preference relation is convex

      v. Alice's preference relation is not convex

      vi. There is insufficient information to determine the convexity of Alice's preference relation

3. Consider a preference relation defined by

$$(x_1, x_2) \succeq (y_1, y_2) \Leftrightarrow [x_1 \geq y_1 \text{ or } x_2 \geq y_2].$$

I.e., any bundle (x_1, x_2) is at least as preferred as any bundle (y_1, y_2) if and only if at least one of the following inequalities holds: $x_1 \geq y_1$ or $x_2 \geq y_2$. E.g., $(4, 1) \succeq (2, 3)$ because, on the first coordinate, $4 \geq 2$; $(2, 3) \succeq (6, 2)$ since, on the second coordinate, $3 \geq 2$.

a. Is $(4, 1) \succeq (6, 2)$ true or false?

b. Is $(6, 2) \succeq (4, 1)$ true or false?

c. Prove that $\succeq$ is not transitive.

d. Complete the following proof for the claim that $\succeq$ is complete.

Pick any bundles (x_1, x_2) and (y_1, y_2). We want to show that $(x_1, x_2) \succeq (y_1, y_2)$ or $(y_1, y_2) \succeq (x_1, x_2)$ is true. If $(x_1, x_2) \succeq (y_1, y_2)$ then we are done. Hence suppose that $(x_1, x_2) \succeq (y_1, y_2)$ is not true, and we are done if $(X_1, X_2) \succ (Y_1, Y_2)$. Since $(x_1, x_2) \succeq (y_1, y_2)$ is not true, the definition of $\succeq$ implies that "$x_1 \geq y_1$ or $x_2 \geq y_2$" is false, which means "$x_1 < y_1$ and $X_2 < Y_2$." Since "$x_1 < y_1$" implies weak preference, the proof is complete.

4. Burt, living in a two-commodity world, currently has the bundle of 20 units of good A and 30 units of good B. In the diagram where the horizontal axis denotes the quantity of good A and vertical axis denotes that of good B, the slope of Burt's indifference curve passing thru (20, 30) is equal to -4 at (20, 30).

   Burt's preference relation is monotone and strictly convex.

   a. Draw a diagram with axes specified above; on the diagram plot the indifference curve passing thru the bundle (20, 30), the tangent line of the curve at (20, 30), and draw an arrow to indicate the direction of greater preferences. The diagram should be consistent to all the info provided above.

   b. Ann offered to sell Burt 2 units of good A in exchange for some good B, and Burt accepted the offer. From this fact we can deduce that the quantity of good B that Ann asked for in her offer is—

      good 2 = good B

      slope = -4 mean
      willing to give up 4 unit good B
      for 1 unit good A

      i. equal to 4 units of good 2
      ii. less than or equal to 4 units of good 2
      iii. greater than or equal to 4 units of good 2
      iv. equal to 8 units of good 2
      v. less than or equal to 8 units of good 2
      vi. greater than or equal to 8 units of good 2
      vii. equal to 1/4 units of good 2
      viii. less than or equal to 1/4 units of good 2
      ix. greater than or equal to 1/4 units of good 2
      x. unknown, as there is not enough information

5. Bob, living in a two-commodity world, has the utility function $u(x_1, x_2) = x_1^2 + x_2^2$, with x_i denoting the quantity of good i ($i = 1, 2$).

   a. Prove that Bob's marginal rate of substitution at any bundle (x_1, x_2) is equal to $-x_1/x_2$.

   b. On a diagram where x_1 is the horizontal axis and x_2 the vertical axis, graph the indifference curve for the bundles that generate 1 util; write down the equation representing this curve.

   c. Suppose that the prices are $p_1 = 1$, $p_2 = 2$ and the income is 10. Consider the following argument:

      "Solve the equations

$$
\begin{aligned}
-\frac{x_1}{x_2} &= -\frac{1}{2} \quad \text{(the tangency condition)} \\
x_1 + 2x_2 &= 10 \quad \text{(the budget constraint)}
\end{aligned}
$$

      and we obtain $x_1 = 2$ and $x_2 = 4$. Since the tangent point $(2, 4)$ is feasible (i.e., it is nonnegative and belongs to the budget set), the optimum is $(5, 5/2)$."

      i. Is the conclusion of the argument correct? NO

      ii. Draw a clearly labeled diagram to explain your answer; label the optimum and calculate its coordinates.