ENGINEERING ECONOMICS

ECO 1192

TYPICAL PROBLEMS & SOLUTIONS

Problem Set #2

Sections 7 to 13

C.Théoret

These problems and solutions are designed to enhance the understanding of engineering students of key engineering economy subject-matter areas and to assist them in preparing for examinations.

DISCRETE CASH FLOW AND COMPOUNDING							
10,0% DISCRETE RATE OF INTEREST							
N	**(F/P,i%,n)**	**(P/F,i%,n)**	**(A/P,i%,n)**	**(P/A,i%,n)**	**(A/F,i%,n)**	**(F/A,i%,n)**	**(A/G,i%,n)**
1	**1,1000**	**0,9091**	**1,1000**	**0,9091**	**1,0000**	**1,0000**	**0,0000**
2	**1,2100**	**0,8264**	**0,5762**	**1,7355**	**0,4762**	**2,1000**	**0,4762**
3	**1,3310**	**0,7513**	**0,4021**	**2,4869**	**0,3021**	**3,3100**	**0,9366**
4	**1,4641**	**0,6830**	**0,3155**	**3,1699**	**0,2155**	**4,6410**	**1,3812**
5	**1,6105**	**0,6209**	**0,2638**	**3,7908**	**0,1638**	**6,1051**	**1,8101**
6	**1,7716**	**0,5645**	**0,2296**	**4,3553**	**0,1296**	**7,7156**	**2,2236**
7	**1,9487**	**0,5132**	**0,2054**	**4,8684**	**0,1054**	**9,4872**	**2,6216**
8	**2,1436**	**0,4665**	**0,1874**	**5,3349**	**0,0874**	**11,4359**	**3,0045**
9	**2,3579**	**0,4241**	**0,1736**	**5,7590**	**0,0736**	**13,5795**	**3,3724**
10	**2,5937**	**0,3855**	**0,1627**	**6,1446**	**0,0627**	**15,9374**	**3,7255**
11	**2,8531**	**0,3505**	**0,1540**	**6,4951**	**0,0540**	**18,5312**	**4,0641**
12	**3,1384**	**0,3186**	**0,1468**	**6,8137**	**0,0468**	**21,3843**	**4,3884**
13	**3,4523**	**0,2897**	**0,1408**	**7,1034**	**0,0408**	**24,5227**	**4,6988**
14	**3,7975**	**0,2633**	**0,1357**	**7,3667**	**0,0357**	**27,9750**	**4,9955**
15	**4,1772**	**0,2394**	**0,1315**	**7,6061**	**0,0315**	**31,7725**	**5,2789**
16	**4,5950**	**0,2176**	**0,1278**	**7,8237**	**0,0278**	**35,9497**	**5,5493**
17	**5,0545**	**0,1978**	**0,1247**	**8,0216**	**0,0247**	**40,5447**	**5,8071**
18	**5,5599**	**0,1799**	**0,1219**	**8,2014**	**0,0219**	**45,5992**	**6,0526**
19	**6,1159**	**0,1635**	**0,1195**	**8,3649**	**0,0195**	**51,1591**	**6,2861**
20	**6,7275**	**0,1486**	**0,1175**	**8,5136**	**0,0175**	**57,2750**	**6,5081**
21	**7,4002**	**0,1351**	**0,1156**	**8,6487**	**0,0156**	**64,0025**	**6,7189**
22	**8,1403**	**0,1228**	**0,1140**	**8,7715**	**0,0140**	**71,4027**	**6,9189**
23	**8,9543**	**0,1117**	**0,1126**	**8,8832**	**0,0126**	**79,5430**	**7,1085**
24	**9,8497**	**0,1015**	**0,1113**	**8,9847**	**0,0113**	**88,4973**	**7,2881**
25	**10,8347**	**0,0923**	**0,1102**	**9,0770**	**0,0102**	**98,3471**	**7,4580**
26	**11,9182**	**0,0839**	**0,1092**	**9,1609**	**0,0092**	**109,1818**	**7,6186**
27	**13,1100**	**0,0763**	**0,1083**	**9,2372**	**0,0083**	**121,0999**	**7,7704**
28	**14,4210**	**0,0693**	**0,1075**	**9,3066**	**0,0075**	**134,2099**	**7,9137**
29	**15,8631**	**0,0630**	**0,1067**	**9,3696**	**0,0067**	**148,6309**	**8,0489**
30	**17,4494**	**0,0573**	**0,1061**	**9,4269**	**0,0061**	**164,4940**	**8,1762**

INTEREST RATE FORMULAS

Revised: January 2005

SUMMARY MEASURES	**Cash Flow**	**Discrete**	**Discrete**	**Continuous**
	Compounding	**Discrete**	**Continuous**	**Continuous**
Single Sum	***Compound Amount***	$F = P(1+i)^n$ = **P(F/P,i,n)**	$F = Pe^{rn}$ **= P(F/P,r,n)**	$F = Pe^{rn}$ **= P(F/P,r,n)**
	Discount Amount	$P = F(1+i)^{-n}$ = **F(P/F,i,n)**	$P = Fe^{-rn}$ **= F(P/F,r,n)**	$P = Fe^{-rn}$ **= F(P/F,I,n)**
Annuities	***Compound Amount***	$F = A[\frac{(1+i)^n - 1}{i}]$ **F = A(F/A,r,n)**	$F = A[\frac{e^{rn} - 1}{e^r - 1}]$	$F = \bar{A}[\frac{e^{rn} - 1}{r}]$
	Sinking Fund	$A = F[\frac{i}{(1+i)^n - 1}]$ **A = F(A/F,r,n)**	$A = F[\frac{e^r - 1}{e^{rn} - 1}]$	$\bar{A} = F[\frac{r}{e^{rn} - 1}]$
	Discount Amount	$P = A[\frac{(1+i)^n - 1}{i(1+i)^n}]$ **P = A(P/A,r,n)**	$P = A[\frac{1 - e^{-rn}}{e^r - 1}]$	$P = \bar{A}[\frac{e^{rn} - 1}{re^{rn}}]$
	Capital Recovery	$A = P[\frac{i(1+i)^n}{(1+i)^n - 1}]$ **A = P(A/P,r,n)**	$A = P[\frac{e^r - 1}{1 - e^{-rn}}]$	$\bar{A} = P[\frac{re^{rn}}{e^{rn} - 1}]$

SUMMARY MEASURES	Cash Flow	Discrete	Discrete	Continuous
	Compounding	Discrete	Continuous	Continuous
Gradient Series	**Uniform Gradient Series (Conversion to a Uniform Series)**	$A = G[\frac{1}{i} - \frac{n}{(1+i)^n - 1}]$ **A = G(A/G,r,n)**	$A = G[\frac{1}{e^r - 1} - \frac{n}{e^{rn} - 1}]$	**Not defined.**
	Geometric Gradient (Conversion to a Single Sum i.e. Present Worth)	***If i% ≠ k%:*** P = C(P/C,i,k,N) $P = [\frac{C}{(i-k)}](1 - [\frac{(1+k)}{(1+i)}]^n$ ***If i% = k%*** $P = \frac{CN}{(1+i)}$ C ≡ First (≠0) term of series	**Not defined**	**Not defined**

CONTENTS

Topics covered in Problem Set #2:

Topics covered in Problem Set #1:

7.1 The following information is provided on an investment project:

A firm is considering the purchase of a truck for $300,000 fully installed. It is expected to last 3 years with a salvage value of $100,000 at that time. Revenues from operations will be $250,000 each year and operating and maintenance costs will be $75,000 each year

- Depreciate the truck using the DB method (d=25%).
- The before-tax interest rate is 10%.
- The after-tax interest rate is 5%.
- A 50% tax rate on income from operations and depreciation recapturing
- The half-year rule does NOT apply.
- The firm gets a $100,000 loan (at a 10% rate of interest) which is repaid as follows:

Loan Repayment at End of Year	Percentage of Loan Repaid
1	30%
2	30%
3	40%

a) Complete the Table below.

	Years			
Item	**0**	**1**	**2**	**3**
1. Before-Tax Cash Flow (BTCF)	(300,000)	175,000	175,000	175,000 + 100,000
2. Interest on Loan ($)		10,000	7,000	4,000
3. Depreciation ($)		75,000	56,250	42,188
4. Taxable Income ($)		90,000	111,750	128,812
5. Taxes Payable ($)		45,000	55,875	64,406
6. After-Tax Cash Flow (ATCF)	(300,000)	130,000	119,125	110,594 + 113,282

	Years			
Item	**0**	**1**	**2**	**3**
				= 223,876
7. Repayment of Loan	-	30,000	30,000	40,000
8. (=2.) Interest on Loan	-	10,000	7,000	4,000
9. Cash Flow on Equity (CFOE)	(200,000)	90,000	82,125	179,876

b) Determine the project=s book value after three years.

Book Value (BV) = $P(1 - d)^{3 =} 300,000(1 - 0.25)^3 = \$126,562.5$

c) Find the net salvage value (NSV) at the end of three years.

NSV = SV - Taxes Payable on capital gains and depreciation recapturing
NSV = SV - t(SV - BV) where At@ is the tax rate
NSV = 100,000 - 0.5(100,000 - 126,563)
NSV = $ 113,282

d) Determine the after-tax present worth of this project.

From the ATCF row (#6):

-300,000 + 130,000(P/F,5%,1) + 119,125(P/F,5%,2) + 223,876(P/F,5%,3)

= $ 125,252

e) Determine the present worth of the owner=s equity.

From the CFOE row (#9):

-200,000 + 90,000(P/F,5%,1) + 82,125(P/F,5%,2) + 179,876(P/F,5%,3)

= $115,588

f) Determine the before-tax present worth of this project.

From the BTCF row (#1):

-300,000 + 175,000(P/A,10%,3) + 100,000(P/F,10%,3)

= $210,339

7.2 From the information in bold characters, complete the table below.

Year	Adjustments to UCC from Purchases & Dispositions	Base UCC Amount for CCA	CCA	Remaining UCC	Tax Savings Due to CCA
where CCA stands for Acapital cost allowance@ UCC stands for Aundepreciated capital cost ***Assume that d=20% (Declining Balance), t=50% and the half-year rule applies.***					
1998	**$500,000**	250,000	50,000	450,000	25,000
1999	**0**	450,000	90,000	360,000	45,000
2000	**300,000**	510,000	102,000	558,000	51,000
2001	**(100,000)**	458,000	91,600	366,400	45,800

7.3 From the information in bold characters, complete the table below .

Year	Adjustments to UCC from Purchases & Dispositions	Base UCC Amount for CCA	Capital Cost Allowance (CCA)	Remaining UCC	Tax Savings Due to CCA
where CCA stands for “capital cost allowance}” UCC stands for “undepreciated capital cost” ***Assume that d=20% (Declining Balance), t=50% and the half-year rule applies.***					

Year	Adjustments to UCC from Purchases & Dispositions	Base UCC Amount for CCA	Capital Cost Allowance (CCA)	Remaining UCC	Tax Savings Due to CCA
1998	**$500,000**	250,000	50,000	450,000	25,000
1999	**(200,000)**	250,000	50,000	200,000	25,000
2000	**300,000**	350,000	70,000	430,000	35,000
2001	**(100,000)**	330,000	66,000	264,000	33,000

7.4 A new tractor has a $60,000 price tag in 1999 and is expected to reduce maintenance and operating costs by $20,000 per year for six years. Assuming an after-tax MARR of 10%, a salvage value of $5,000 in six years, a 20% CCA rate and a corporate income tax of 50%, find the after-tax annual equivalent worth of this tractor.

NOTE: We will convert the present worth OF EACH COMPONENT to an annual equivalent at the end of the solution.

PW(first cost) = -60,000($CCTF_{new}$) where
$CCTF_{new}$ = 1 - {[(0.5)(0.2)(1+0.05)]/[(0.10+0.2)(1+0.1)]} = 0.6818
The present worth of the first cost is $-40,908

The present worth of the salvage value is
6,000(P/F,10%,6)($CCTF_{old}$) where
($CCTF_{old}$) = 1 - {[(0.5)(0.2)]/[(0.1+0.2)]} = 0.6667

Thus the present worth of the salvage value is $2,258

The preset worth of the savings is 20,000(P/A,10%,6)(1-0.5) = $43,550

Therefore, the after-tax present worth of this tractor is $4,900
The after-tax annual equivalent worth is $4,900(A/P,10%,6) = $1,125

7.5 A new tractor has a $60,000 price tag in 1999 and is expected to reduce maintenance and operating costs by $20,000 per year for six years. Assuming that the after-tax MARR is 10%, the tractor=s salvage value will be $5,000 in six years, a 20% CCA rate (with a half-year rule) and a corporate income tax of 50%, find the after-tax annual equivalent

worth of this tractor.

a) Complete the Table below (round to nearest dollar).

	Years						
Item	**0**	**1**	**2**	**3**	**4**	**5**	**6**
1. BTCF (Before-Tax Cash Flows)	(60K)	20,000	20,000	20,000	20,000	20,000	20,000 + 5,000
2. Interest on Loan							
3. Depreciation		6,000	10,800	8,640	6,912	5,530	4,424
4. Taxable Income		14,000	9,200	11,360	13,088	14,470	15,576
5. Taxes Payable		7,000	4,600	5,580	6,544	7,235	7,788
6. ATCF (After-Tax Cash Flows)	(60K)	13,000	15,400	14,420	13,456	12,765	12,212 + NSV = 23,559

NSV (Net Salvage Value) = SV - (SV - BV)tax rate

Book value (BV) = (60,000 - 6,000 -10,800 - 8,640 - 6,912 - 5,530 - 4,424) = 17,694

Therefore, NSV = 5,000 - (5,000-17,694)0.5 = $11,347

The after-tax present worth (t=0) (from Row #6) of this investment is equal to

-60,000 + 13,000(P/F,10%,1) + 15,400(P/F,10%,2) + 14,420(P/F,10%,3)
+13,456(P/F,10%,4) + 12,765(P/F,10%,5) + 23,559(P/F,10%,6)
= $5,794

The after-tax annual equivalent is 5,794(A/P,10%,6) = $1,330 (compared to $1,125 in problem 7.4)

7.6 Would the owner of the tractor in problems 7.4 and 7.5 have been better off using straight-line depreciation ? Use the cash flow approach to find the annual equivalent of the after-tax cash flow.

	Years						
Item	**0**	**1**	**2**	**3**	**4**	**5**	**6**
1. BTCF	(60K)	20,000	20,000	20,000	20,000	20,000	20,000 + 5,000
2. Interest on Loan (no loan)	-	0	0	0	0	0	0
3. Depreciation	-	9,167	9,167	9,167	9,167	9,167	9,167
4. Taxable Income	-	10,833	10,833	10,833	10,833	10,833	10,833
5. Taxes Payable	-	5,417	5,417	5,417	5,417	5,417	5,417
6. ATCF	(60K)	14,583	14,583	14,583	14,583	14,583	14,583 + 5,000 = 19,583

NSV (Net Salvage Value) = SV - (SV - BV)tax rate

BV (book value) = (60,000-55,000) = 5,000

Therefore, NSV = 5,000 - (5,000-5,000)0.5 = $5,000

The after-tax present worth (t=0) of this investment is equal to

-60,000 + 14,583(P/A,10%,5) + 19,583(P/F,10%,6)
= $6,339

The after-tax annual equivalent is 6,339(A/P,10%,6) = $1,455 (compared to $1,125 in problem 7.4 and $1,330 in problem 7.5)

7.7 An asset costs $20,000 and has a scrap value of $5,000 after 5 years. Calculate its annual depreciation, cumulative depreciation and book value during the 5 years using the straight line depreciation method.

Year	Annual depreciation	Cumulative depreciation	Book value
0	Na	na	20,000
1	3,000	3,000	17,000
2	3,000	6,000	14,000
3	3,000	9,000	11,000
4	3,000	12,000	8,000
5	3,000	15,000	5,000

7.8 An asset costs $20,000 and has a scrap value of $5,000 after 5 years. Calculate its annual depreciation, cumulative depreciation and book value during the 5 years using the declining balance approach and a depreciation rate of 20%.

Year	Annual depreciation	Cumulative depreciation	Book value
0	Na	na	20,000
1	4,000	4,000	16,000
2	3,200	7,200	12,800
3	2,560	9,760	10,240
4	2,048	11,808	8,192
5	1,638.4	13,446.4	6,553.6

7.9 Determine the after-tax rate of return on a project which has a first cost of $20,000, a salvage value of $12,000 after two years, and savings of $10,000 and $15,000 in the first year and second year respectively. Assume that the tax rate is 50% and the asset is depreciated over two years using the declining balance method with a 25% rate.

Find i* such that the Net Present Worth of Cash flows is zero.

Therefore: -20,000 + 15,000(1 - 0.5)(P/F,i*,2) + 10,000(1 - 0.5)(P/F,i*,1) + NSV (P/F,i*,2) + PED*t = 0

where NSV = 12,000 -[12,000 - 12,000(1-d)^2]0.5
PED(t) = present worth of depreciation tax credits
= [5000(P/F,i*,1)+3750(P/F,i*,2)]0.5

OR

Find i* such that the Net Annual Equivalent Worth of Cash flows is zero.

Therefore: -20,000(A/P,i*,2) + 15,000(1 - 0.5)(P/F,i*,2)(A/P,i*,2) + 10,000(1 - 0.5)(P/F,i*,1) (A/P,i*,2) + NSV (A/F,i*,2) + AED*t = 0

where NSV = 12,000 -[12,000 - 12,000(1-d)^2]0.5
AED(t) = present worth of depreciation tax credits
= [5000(P/F,i*,1) + 3750(P/F,i*,2)](A/P,i*,2)0.5

Solve for i*.

7.10 Determine the after-tax annual cost of a $50,000 machine which has operating and maintenance costs of $0.25 per unit. The machine will be sold for $5,000 at the end of five years. Production volumes are 100 units per day, 250 days per year. The declining balance depreciation rate is 25%, the after-tax MARR is 20%, and the tax rate is 50%.

After-tax annual cost = 50,000(A/P,20%,5)+0.25(100)250-NSV(A/F,20%,5) - AED(t)
where NSV = 5,000 -[5,000 - 50,000(1-d)^5]0.5
AED(t) = annual equivalent worth of depreciation tax credits
= [12,500(P/F,20%,1) + 9,375(P/F,20%,2) + 7,031.25 (P/F,20%,3)
+ 5,273.44(P/F,20%,4) + 3,955.08(P/F,20%,5)](A/P,20%,5)0.5

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8.1 A trencher (i.e., a machine to dig trenches) is available for $75,000 with an estimated resale value of $5,000 in 10 years. Its annual operating costs are estimated to be $40 per hour in the first year increasing each year thereafter by $3 per hour. The trencher is capable of digging a trench at an average rate of 50 feet per hour.

The alternate method of digging trenches is to buy a backhoe which has a first cost of $60,000 and a resale value of $5,000 in 10 years. Its annual operating costs are $26 per hour increasing each year thereafter by $1.50 per hour. The backhoe can dig trenches at an average rate of 20 feet per hour.

a) How many feet of trench per year must be dug to break even if MARR=10%?

Let x represent the number of feet of trench per year.

AEW(Trencher) = 75,000(A/P,10%,10)-5,000(A/F,10%,10)
+{[40+3(A/G,10%,10)]/50}x
AEW(Trencher) = 11,889.5 + 1.0235x

AEW(Back hoe) = 60,000(A/P,10%,10) - 5,000(A/F,10%,10)
+{[26+1.50(A/G,10%,10)]/20}x
AEW(Back hoe) = 9385.8 + 1.6726x

Solve for x

Set 11,889.5 + 1.0235x = 9385.8 + 1.6726x; x
= 3,857 feet per year.

b) Which equipment is better (the trencher or the back hoe) if the actual feet of trenches per year fall short of the break-even level determined in question a) above?

Decision: **1. Use back hoe if fewer than 3857 feet per year are required.**
2. Use trencher if more than 3857 feet of trench per year are required.

8.2 Purchase skate making plant:

- price is $3,710,400

- Salvage value in 20 years is $371,040
- N = 20 years
- MARR (before-tax) is 20 per cent
- variable costs = $30.52
- selling price = $55

a) Determine the annual fixed costs.

Let "N" represent the number skates produced.

The annual fixed cost = -3,710,040(A/P,20%,20) + 371,040(A/F,20%,20)
= -$759,830

b) Determine the annual variable costs.

The total variable cost per year is $30.52N

c) Determine the annual revenues.

Total revenues per year are $55N

d) What is the breakeven point (i.e., number of skates)?

At breakeven, Costs = Revenues
Solve for "N"; N = 31,040 pairs of skates

8.3 A manufacturing firm has the following annual revenue and cost functions:

Annual revenue = AR(x) = $1500x - 0.05x^2$
Annual cost = AC(x) = $1,200,000 + $1000x - 0.01x^2$

where Ax@ is the annual production of widgets

a) Determine the break-even production of widgets..

Profits = total revenues - total costs

= 1,500x - 0.05x^2 -(1,200,000 + 1,000x-0.01x^2) = 0
= -0.04x^2 +500x-1,200,000=0

Using the quadratic formula, x1=9,260 and x2=3,240

b) At the break-even production volume, determine the i) fixed costs; and ii) the variable costs.

Total fixed costs are $1,200,000 per year.
Total variable costs = $1{,}000x-0.01x^2 = 1{,}000(6250)-0.01(6{,}250)^2 = \$5{,}859{,}375$

c) Determine the profit maximizing production volume.

From $0.04x^2-500x+1{,}200{,}000$,
d(profits)/dx = 0.08x - 500 = 0; therefore, x = 6,250

d) What is the level of profit in c).

Substitute $x=6{,}250$ in $0.04x^2-500x+1{,}200{,}000$
Total profits = $362,500

8.4 John, a skate lace maker, can produce laces with his current equipment at an estimated annual fixed cost of $3000 and variable costs of $1.25 per pair of laces.

He can also buy the laces at a price of $2 per pair for the first 5,000 pairs, and $1.05 for all pairs purchased beyond 5,000 units.

a) Give the cost equation for making the laces.

Let 'x' represent the number of pairs of laces :
3,000 + 1.25x

b) Give the cost equation for purchasing the laces. Let 'x' represent the number of pairs of laces.

For "x" between 0 and 5,000 units: 2x
For "x" greater than 5,000: 10,000 +(x - 5,000)1.05

c) Find the breakeven level between making and purchasing the laces.

<u>For x less than 5,000 pairs of laces</u>
3,000 + 1.25x = 2x; solve for x; x = 4,000 pairs

<u>For x greater than 5,000 pairs of laces</u>
3,000 + 1.25x = 10,000 + (x - 5,000); solve for x; x = 8,750

d) For what level of pairs of laces is it cheaper to make the laces?

Between 4,000 and 8,750 pars of laces

e) For what level of pairs of laces is it cheaper to buy the laces?

For x less than 4,001 pairs and x greater than 8,750 pairs

8.5 A car manufacturer must decide on purchasing or making specific car parts. The cost of making the parts is C = 8500 + 65x ($)

Purchasing the same car parts from a manufacturer would cost C = 150x ($) where "x" represents the quantity of car parts (in millions).

a) For which quantity of parts is it cheaper (less costly) for the car manufacturer to make the parts?

At breakeven, the cost of making the parts equals the cost of purchasing the parts. Therefore, 8500 + 65x = 150x

Solving for x: x = 100 or 100 million car parts.

The purchase option has variable costs only (no fixed costs). To minimize costs, the car manufacturer would be better off to make the car parts if he expects to need more than 100 million parts per year. (see graph)

b) For which quantity of parts is it cheaper (less costly) for the car manufacturer to buy the parts?

The purchase option has a higher variable cost per car part than the make option. As Ax@ increases, the lower variable unit cost in the make option compensates for the existence of a fixed cost in the make option.

To minimize costs, the car manufacturer would be better off to buy the car parts if he expects to need less than 100 million parts per year.

c) For which quantity of parts is the car manufacturer indifferent between buying and making the parts?

At breakeven (i.e., 100 million units), the car manufacturer is indifferent between purchasing or making the car parts.

8.6 Air Chance is considering the purchase of a new aircraft which has a $30,000,000 price tag and is to be used exclusively on a new route.

The number of passengers per flight is projected to be -1.75x + 400 where "x" is the dollar ($) cost of each ticket. One thousand (1,000) flights are planned each year. The variable costs (i.e., costs per flight) which include fuel, food, drinks, salaries, etc. are estimated at $21,000. The life of the asset is estimated at 20 years with no salvage value. Assume a zero (0)% interest rate and a before-tax calculation.

a) What is the fixed cost per year for this project?
$30,000,000/20 = $1,500,000 per year.

b) Determine the ticket price(s) to break even?
Revenues = [(-1.75x+400)x]1000 = [-1.75x^2 + 400x]1000
Costs = 1,500,000+21,000(1000)
Solve for x using the quadratic formula: x = $128.6 or $100

c) At what ticket price are profits maximized?
Profits = 1750x^2 - 400,000x +22,500,000
d(profits)/dx = 3500x - 400,000 = 0
x = $114.29 per ticket

d) What are the total variable costs per year (i.e., for 1000 flights) at the profit maximizing price level?
1,000(21,000) = $21 million

8.7 The XYZ Company must decide between two projects, A and B. Three scenarios have been developed for each project. Given the information below, which project would you recommend?

Project A	Pessimistic Scenario	Expected Scenario	Optimistic Scenario

Parameters	**MARR = 15%; N = 7 years**		
Annual capital cost recovery (K$)	52.9	48.1	45.7
Annual fixed costs (K$)	18	16.5	13
Annual variable costs ($)	2.35	2.25	2.2
Annual equivalent salvage value (K$)	0.181	0.452	0.633
Production volume (thousands of units)	40	50	80
Net annual revenues (K$)	-34.7	-14.1	26.0

Project B	**Pessimistic Scenario**	**Expected Scenario**	**Optimistic Scenario**
Parameters	**MARR = 15%; N = 10 years**		
Annual capital cost recovery (K$)	72.7	69.7	63.8
Annual fixed costs (K$)	45	35.5	25
Annual variable costs ($)	1.1	1.06	1.01
Annual equivalent salvage value (K$)	0.837	0.985	1.13
Production volume (thousands of units)	40	50	80
Net annual revenues (K$)	-30.9	5.2	91.6

Analysis: **Project A will have negative annual equivalent revenues for the pessimistic and expected scenarios. As such, project A offers a higher risk of losses than project B. Furthermore, in an optimistic environment, B=s profits exceed those for A.**

For project B, profits would be experienced under the expected and optimistic scenarios. Overall, the bottom line for project A is worse than that for project B.

Decision: **On balance, project B appears to be the better choice**

although losses would occur under a pessimistic environment.

8.8 A lawn care company needs an additional tractor to complete a "big" job on time. It can buy or lease the tractor. Relevant details are in the following table.

Decision Required: Buy or Lease Tractor		
DETAILS	**BUY**	**LEASE**
First Cost ($)	30,000	----
Life (years)	6	----
Salvage value ($)	4,000	----
Operating Expenses ($)	50/day	----
Maintenance contract ($)	3,000/year	----
Leasing cost ($)	----	130/day
MARR (%)	10	10

a) How many days per year must the tractor be used for the buy and lease options to have the same costs?

Let n= number of days
<u>Buy option</u>
Cost = 30,000(A/P,10%,6) - 4,000(A/F,10%,6) + 3,000 + 50n
= $9,369.30 + 50n
<u>Lease option</u>
Cost = 130n
At breakeven, cost of buying = leasing cost
Therefore, $9,369.30 + 50n = 130n
Solve for n; n = 117.1 days

b) Which option (buy or lease) is recommended if the contractor needs the tractor for fewer days than the breakeven point?

Draw breakeven diagram: leasing the tractor is less costly for fewer than

117.1 days

8.8 The sensitivity of a machine=s annual equivalent worth to its key parameters is shown below:

The base case parameters of the machine are:

- initial investment ($) = 150,000
- annual revenues ($) = 70,000
- annual expenses ($) = 43,000
- salvage value ($) = 0
- useful life (years) = 10
- MARR (%) = 8

ANNUAL EQUIVALENT WORTH VALUES							
	-15%	**-10%**	**-5%**	**Base Case (0%)**	**5%**	**10%**	**15%**
P	**6250**	**5029**	**3809**	**2588**	**1368**	**147**	**-1074**
SV	**2588**	**2588**	**2588**	**2588**	**2588**	**2588**	**2588**
AR	**-7912**	**-4412**	**-912**	**2588**	**6088**	**9588**	**13088**
AC	**9038**	**6888**	**4738**	**2588**	**438**	**-1712**	**-3862**
N	**-16**	**894**	**1761**	**2588**	**3377**	**4128**	**4845**
MARR	**4420**	**3788**	**3178**	**2588**	**2018**	**1466**	**933**

a) Complete table.
b) To what parameter is the annual equivalent of the machine most sensitive in the range +/- 15% from base case value? **Annual revenues**
c) To what parameter is the annual equivalent of the machine least sensitive in the range +/- 15% from base case value? **MARR**

8.9 To which parameter is the following project least and most sensitive in the range +/- 15%

from the base case parameters?

Base Case

- P = $10,000
- SV = $2,000
- AR = $5,000
- AC = $1,000
- N = 5 years
- MARR = 10%

PRESENT WORTH VALUES							
	-15%	**-10%**	**-5%**	**Base Case**	**5%**	**10%**	**15%**
P	**7905**	**7405**	**6905**	**6405**	**5905**	**5405**	**4905**
SV	**6219**	**6281**	**6343**	**6405**	**6467**	**6529**	**6591**
AR	**3562**	**4510**	**5457**	**6405**	**7353**	**8300**	**9248**
AC	**6974**	**6784**	**6595**	**6405**	**6215**	**6026**	**5836**
N	**4657**	**5253**	**5836**	**6405**	**6961**	**7503**	**8033**
MARR	**7093**	**6858**	**6629**	**6405**	**6185**	**5970**	**5760**

From the table, we can conclude that this project is least sensitive to the salvage value and most sensitive to the annual revenues.

9.1 A regional municipality is examining the economic viability of constructing a road in a marshy area. Two (2) different routes have been retained for final study. The benefits and costs (both in $1,000) of each route are given in the following table.

Route	Initial Cost ($)	Annual Savings In Fire Damage ($)	Recrea-tional Benefits ($)	Tolls & Fees ($)	Timber Access Benefits ($)	Annual O&M Costs ($)
A	225	20	17	25	15	2
B	250	15	19	30	15	3

a) Determine the conventional benefit/cost ratio for Routes A and B if MARR=10%.

$(B/C)^A$ = [20+17+25+15]/[225(A/P,10%,50) +2]
= 77/24.7 = 3.1
Therefore, route A is valid

$(B/C)^B$ = [15+19+30+15]/[250(A/P,10%,50) +3]
= 79/28.2 = 2.8
Therefore, route B is valid

b) Using the incremental Benefit/Cost approach, determine which route is better if MARR=10%.

$(B/C)^{(B-A)}$ = [79 - 77]/[28.2 - 24.7] = 0.57; select the smaller project (i.e., A)

9.2 A small Crown Corporation is examining the economic viability of expanding its dining room. Two (2) different proposals have been retained for final study. The benefits and costs (both in $1,000) of each proposal are given in the following table.

Parameter	Proposal A	Proposal B
AC	20	30
P	200	400

Parameter	Proposal A	Proposal B
AR	95	100
SV	50	150
N (years)	6	12
MARR (%)	10	10

CONVENTIONAL B/C RATIO

Proposal A

$$\frac{\text{Benefits}}{\text{CR + (O\&M)}} = \frac{95}{200(A/P,10\%,6) + 20 - 50(A/F, 10\%,6)}$$

$$= 95/59.44 = 1.59 > 1$$

Decision: Proposal A is valid.

Proposal B

$$\frac{\text{Benefits}}{\text{CR + (O\&M)}} = \frac{150}{400(A/P,10\%,12) + 30 - 150(A/F,10\%,12)}$$

$$= 150/81.7 = 1.84 > 1$$

Decision: Proposal B is valid.

DECISION:

If proposals A and B are independent, select both since their respective B/C ratios exceed unity.

If proposals A and B are mutually exclusive, use the incremental approach

(although not necessary in this case) to determine the better proposal.

INCREMENTAL APPROACH: Conventional B/C and Annual Equivalent

$$\spadesuit(B/C) = \frac{(150 - 95)}{(81.7 - 59.44)} = 55/22.26 = 2.47 > 1$$

DECISION: Select the higher cost proposal B.

PLEASE NOTE: The incremental step was NOT NECESSARY in this case. In calculating individual (B/C) ratios, the higher cost proposal (i.e., B) has a larger individual (B/C) ratio than the lower cost proposal A. From this information, we could have selected proposal B over proposal A (where A and B are mutually exclusive).

9.3 The following table indicates the conventional benefit-cost ratios for 6 projects which are ranked, in terms of their respective initial cost, from smallest (A) to largest (F).

	B/C Ratios					
Projects	**A**	**B**	**C**	**D**	**E**	**F**
A	0.9					
B	1.9	1.8				
C	1.7	1.7	1.2			
D	1.5	1.5	0.9	1.15		
E	1.3	1.3	1.1	1.1	1.1	
F	1.1	1.1	0.9	0.8	0.9	1.05

a) Determine which public sector projects are acceptable?

All projects with an individual benefit-cost ratio at least equal to unity are acceptable:

B, C, D, E and F

b) Determine the best public sector project.

Comparing projects B, C, D, E and F:

- **project B is preferred to project A**
- **project C is preferred to project B**
- **project C is preferred to project D**
- **project E is preferred to project C**
- **project E is preferred to project F**

Therefore project E is the best public sector project.

INTRODUCTION

Sooner or later physical assets will become worn out, inadequate or obsolete. At some point, this old asset, known as the defender, will be replaced with a new alternative, called the challenger.

The problem facing management is whether to buy a more efficient asset or continue to use the existing equipment.

MAJOR REASONS FOR REPLACEMENT:

- ! physical impairment - the existing asset is worn out, because of normal use or accident and no longer will render its intended function unless extensive repairs are made
- ! inadequacy - the existing asset does not have sufficient capacity to fill the current and expected demands; to meet the new demands, the existing asset must be either supplemented or replaced.
- ! obsolescence - two types: functional and economic

functional: there has been a decrease in the demand for the output of the asset; a loss of revenue follows

economic: the result of the existence of a new asset that will produce at lower cost than can be obtained with the old asset

rental or lease possibilities - a variation of obsolescence except that the replacement asset does not necessarily have to be different, in any respect, from the existing asset; may have more to do with more advantageous financial or income tax factors.

DIFFERENT TYPES OF ASSET LIVES

- ! economic - the period of time extending from the date of installation to the date of retirement (by demotion or disposal)

- ! ownership - the period of time extending from the date of acquisition to the date of disposal by a specific owner

- ! physical - the period of time extending from the date of acquisition to the date of final disposal by successive owners.

! Useful life - the period of time an asset is kept in productive use

FACTORS TO BE CONSIDERED IN REPLACEMENT ANALYSIS

- ! recognition and acceptance of past error
- ! the possible existence of a sunk cost
- ! remaining life of the old asset
- ! economic life for the proposed replacement asset
- ! method of handling unamortized values
- ! income taxes: possible capital gains or losses

GENERAL MODEL OF REPLACEMENT

! a general form of cost minimization models

$$C = Ax + B/x + k$$

where
"x" is the design variable (e.g., period of use)
"C" are the total costs
"Ax" are the variable costs which vary directly with "x" (operating costs)
"B/x" are the variable costs which vary indirectly with "x" (e.g., depreciation)
"K" are the costs that are independent of "x" (e.g., costs of borrowed funds)

TYPES OF REPLACEMENT ANALYSES

! cyclic - determine the minimum cost life of a physical asset; the asset is replaced by an identical asset

! defender versus the challenger - the existing asset is challenged by a new asset; switch to the challenger when costs are less

SUNK COST

! A sunk cost is any past cost that cannot be changed by any future investment decision. It is a result of a bad decision that was made at some time in the past; for replacement analysis a sunk cost should not be included in the economic comparison.

sunk cost = present book value - present realizable value

10.1 The defender (a truck) was purchased exactly 3 years ago and **is already beyond** its economic life. A new and more efficient truck (the challenger) is now available on the market. The MARR is 10%. Use the outsider approach to perform the following calculations. Additional information on the defender and challenger is provided below.

DEFENDER	**Market Value at Beginning of Year ($)**	**Operating cost ($)**
Fourth year (coming year)	$4,000	$5,000
Fifth year	$3,000	$5,500
Sixth year (last year)	$2,000	$6,000

CHALLENGER

- First Cost = $9,000
- Annual operating cost = $4,000
- Economic Life = 5 years
- Salvage Value (in 5 years) = $2,000

a) Use the annual equivalent method (no taxes) to determine the challenger's annual equivalent cost. Show all calculations.

4,000 + 9,000(A/P,10%,5) - 2,000(A/F,10%,5) = $6,047

b) Use the annual equivalent method (no taxes) to determine the cost of keeping defender during the coming year. Show all calculations**.**

5,000 + 4,000(A/P,10%,1) - 3,000(A/F,10%,1) = $6,400

c) Should the existing truck (i.e., defender) be replaced by the challenger before the defender's fourth year? **DECISION: Switch to challenger NOW.**

10.2 The defender (an existing machine) is 1 year old and can be sold today for $12,500. It cost $26,000 when purchased a few years ago. Its estimated net salvage value 1 year

from now is $8,000, $5,000 in 2 years, and $2,000 in 3 years. If the defender was to continue in service, annual disbursements would be $9,000 in the coming year, $10,500 the following year, and $12,500 the year after that. The defender is beyond its economic life.

The challenger has a first cost of $25,000 and a net salvage value each year for the next 10 years of 75 per cent of the previous year's salvage value (e.g., $18,750 at the end of year 1, $14,062.50 at the end of year 2, etc.). The challenger's annual operating cost will be $8,000 for the first three years and increasing to $9,000 for each of the last 7 years. It has an economic life of 10 years.

a) Should the defender be replaced now? Show all calculations.

AE(Challenger) = 25,000(A/P,10%,10)-1,407.8(A/F,10%,10)
+{8,000(P/A,10%,3)+9,000(P/A,10%,7)(P/F,10%,3)}(A/P,10%,10)
=$12,572

AE (Defender cost next year)= 12,500(A/P,10%,1) - 8,000(A/F,10%,1) +9,000
= $14,750

Yes, replace defender by challenger.

b) Should the defender be replaced one year from now? Show all calculations.

Does not apply as defender has already been replaced.

10.3 A supervisor has recommended that a 2-year old piece of equipment be replaced immediately. He feels that the new equipment is economically more advantageous at a 10% per year rate of return and a 5-year planning horizon. Perform a replacement analysis for a 5-year period. Is the supervisor=s decision valid?

	Existing Equipment	**Proposed Equipment**
Original Purchase Price ($)	30,000	40,000
Current Market Value ($)	15,000	40,000
Estimated useful life (Years)	5	15
Estimated Value in 5 years ($)	7,000	10,000

Salvage Value in 15 Years ($)	-------	5,000
Annual Operating Cost ($)	5,000	3,000

Proposed equipment
Annual equivalent = 40,000(A/P,10%,5) - 10,000(A/F,10%,5) + 3,000
= $11,914

Existing Equipment
Annual equivalent = 15,000(A/P,10%,5) - 7,000(A/F,10%,5) + 5,000
= $11,767.40

Decision: Supervisor is wrong B keep defender.

10.4 Find a new physical asset's minimum-cost life?

Life or duration	Salvage Values	Annual Cost	Capital recovery and return	Annual Equiv of Operating Costs	Annual Equivalent Cost
0	13000	--	--	--	--
1	9000	2500	5300	2500	7800
2	8000	2700	3681	2595	6276
3	6000	3000	3415	2717	6132*
4	2000	3500	3670	2886	6556
5	0	4500	3429	3150	6579

Sample calculation

EAC* = 13000(A/P,10,3) - 6000(A/F,10,3) +
{2500(P/F,10,1) + 2700(P/F,10,2) + 3000(P/F,10,3)}(A/P,10,3)
= $6,132

Therefore, this asset should be retained 3 years to minimize costs.

INTRODUCTION

- To this point, prices have been held constant (no change)
- In the real world, prices are not constant, sometimes increasing (inflation) and sometimes decreasing (deflation)
- Price changes can have significant effects on the value or worth of an investment; therefore they need to be factored into an engineering economic study

WHY IMPORTANT TO CAPTURE PRICE CHANGES?

- Price changes have repercussions on the purchasing power of money
- The purchasing power (or value) of money declines if prices increase and cash flows are not follow responsive (do not fully follow the growth in prices). Inflation makes future dollars less valuable than today or present dollars.
- For example, $100 today will have a purchasing power of $90.91 if prices increase by 10 per cent during the year
- Consequently, investors whether it be in financial (e.g., the purchase of a corporate bond) or physical investments (e.g., a restaurant) must factor the impact of price changes in their decision-making analysis
- It follows that one needs to account for price changes not only on initial or first costs and the salvage value but also on the cash flow generated by a project
- **Investors are not simply satisfied to keep pace with inflation; to be successful, an investment must result in a net gain in buying power over and above the increase required to keep up with inflation**

Example: If an investor wants a real return of 4 per cent on a $1,000 investment when the rate of inflation is 10%, then

- $1000 now must grow to $1,100 in one year just to maintain today's purchasing power
- in addition, there must be a 4% rate of return on the $1,100

- as a result, the formula for one year is

$$F = P(1+f)^N(1+i_r)^N$$
$$= 1000(1.10)(1.04)$$

where f = inflation rate

i_r = real rate of return

i_c = combined rate

THE MEASUREMENT OF PRICE CHANGES

- Consumer Price Index -- consumer basket of goods and services used to track changes in prices
- Wholesale Price Index
  - GDP Implicit Price Deflator

REAL INTEREST RATE

- It is important to state future dollars in terms of a base year
- For the base year, the Present Worth of real dollars (inflation-free dollars) will equal the Present Worth of nominal dollars (dollars containing inflation)
- The PW of real dollars (i.e., inflation-free dollars) is found as follows:

Let \$F be the future worth of \$P now (containing both inflation and a real rate of return)
To remove inflation from these future dollars (\$F), adjust for inflation as follows:

F' = F/(1+f), where f is the inflation rate
To bring these dollars to now or today \$ adjust as follows

$$P = F'/(1+i_r)$$

Therefore, the current (i.e., now) real dollars are given by

$$P = F/\{(1+f)(1+i_r)\}$$

where $(1+i_c) = (1+f)(1+i_r)$

or $i_c = (1+f)(1+i_r)-1$

Therefore, $i_r = \{(1+i_c)/(1+f)\} - 1$

11.1 Assume that P = 1000; F=2000; N=4 years; f = 10% (inflation); t=0 (No taxes)

Find i_c and i_r

We know that $F = P(1+i_c)^N$
Therefore: $2000/1000 = (1+i_c)^4$
Solve for i_c: $i_c = 18.9\%$
From $i_r = (1+i_c)/(1+i_i) - 1 = (1+0.189)/(1+0.1) - 1 = 0.081$ or 8.1%

11.2 Assume that $5,000 is deposited in an account earning interest at 10 percent per year over a five-year period. During this period, inflation is expected to remain at 6 percent per year.

a) Determine the dollar amount in the fund at the end of five years. That is, what is the future worth after five years in actual dollars?

Future Worth = 5000(F/A,10%,5) = $30,525 (Actual $)

b) Considering the eroding effect of inflation on purchasing power, what is the future worth after five years of this fund in "now" or real dollars?

$30,525(P/F,inflation,5) = 30,525(P/F,6%,5) = $22,811 [inflation-free dollars]

c) Give one equation which would enable you to calculate the interest rate at which the "now" or real dollars will expend, with their same buying power, into equivalent "then" or actual dollars (that is, the equation required to find the real interest rate on this investment)?

real interest rate = $i_r = (1 + i_c) / (1 + f) - 1$

= (1+0.1)/(1+0.06) - 1 = 0.03774 or 3.774%

11.3 Assume that A = $150 per year; f = 10%, P = 1000; F = 1000 N=3 years
Find i_c and i_r

Using the Present Worth Approach: $1000 = 150(P/A,i^*,3) + 1000(P/F,i^*,3)$
Solve for i*: $i^* = 15\%$ $(=i_c)$

Find i_r: From $i_r = (1 + i_c) / (1 + f) - 1$; $i_r = (1 + 0.15) / (1 + 0.1) - 1 = 0.0455$ or 4.55%

11.4 Introduce a 30% tax rate on interest income in Problem 11.3.

Solve for i_c and i_r. Using the Present Worth Approach:

$1000 = 150 (1-0.3)(P/A,i^*,3) + 1000(P/F,i^*,3)$
Solve for i*: $i^* = 10.5\%$ $(=i_c)$
Find i_r: From $i_r = (1 + i_c) / (1 + f) - 1$; $i_r = (1 + 0.105) / (1 + 0.1) - 1 = 0.0045$ or 0.45%

11.5 Assume that A = \$150 per year and is fully responsive to inflation; f = 10%; P = 1000 and is also fully responsive to inflation; F* = 1000; N = 3 years. Find i_c and i_r.

Using the Present Worth Approach:

$$1000 = 150(1+f)(P/F,i^*,1)+150(1+f)^2(P/F,i^*,2)+ 150(1+f)^3(P/F,i^*,3) + 1000(1+f)^3(P/F,i^*,3)$$

Solve for i*: i* = 26.5% ($=i_c$)

Find i_r: From $i_r = (1 +i_c) /(1 + f) - 1$; $i_r = (1 +0.265) /(1 + 0.1) - 1= 0.15$ or 15.0%

11.6 Introduce a 30% tax rate on interest income in Problem 11.5. Find i_c and i_r..

Using the Present Worth Approach:

$$1000 = 150(1+f)(1-0.3)(P/F,i^*,1) + 150(1+f)^2(1-0.3)(P/F,i^*,2) + 150(1+f)^3(1-0.3)(P/F,i^*,3) + 1000(1+f)^3(P/F,i^*,3)$$

Solve for i*: i* = 21.1% ($=i_c$)

Find i_r: From $i_r = (1 +i_c) /(1 + f) - 1$; $i_r = (1 +0.211) /(1 + 0.1) - 1 = 0.105$ or 10.5%

11.7 You must prepare financially for your son's or daughter=s college attendance in 11 years The relevant information is as follows:

- 4 years of college costs \$9,000/year in today dollars
- inflation is expected to be 8% per year for the next 20 years
- financial institutions (e.g., banks) are expected to pay interest on deposits at a 12% rate compounded annually
- assume that annual college payments are made in a lump sum at the beginning of each year.

How much money must you deposit in a financial institution at the end of each of 10 years (that is, the last deposit by the parents is made one year before the son or daughter enters university?

SAVINGS FOR SON=S or DAUGHTER=S COLLEGE YEARS		
College Year	**Cost of College (Actual College Year Dollars)**	**Value or Worth of College Dollars at the Beginning of the First Year of College**
First	**9,000(F/P,8%,11) = 19431**	**19431**
Second	**9,000(F/P,8%,12) = 20988**	**18740.2**
Third	**9,000(F/P,8%,13) = 22662**	**18066.15**
Fourth	**9,000(F/P,8%,14) = 24480**	**17424.86**
	TOTAL = $87,561	**TOTAL = $73,662.2**
Therefore, F = $73,662.2 **The 10-year annuity becomes:** **A = F(A/F,12%,10) (P/F,12,1) = $3,748**		

11.8 On a very cold January 1, 1995 morning, you decided to plan for a two-year vacation in Italy to start on January 1, 2000 and end on December 31, 2001. From discussions with travel agents and friends, you estimate that it would cost $15,000 (in January 1, 1995 dollars) for each year of your vacation. For the sake of simplicity, assume that all vacation-related expenses are prepaid on January 1 of each vacation year.

To pay for this holiday, you will deposit the same amount of money in a bank account on December 31 of years 1995, 1996, 1997 and 1998. Your vacation savings will be transferred electronically from a Canadian bank to an Italian bank on December 31, 1999 without any loss of interest.

On January 1, 2000 you are to fly to Italy. Assume that money earns 10% interest compounded annually in Canada and inflation is always 5% per year around the world.

a) How much are January 1, 2000 vacation expenses?

Let V1 equal the actual $ in 1st year of trip
= 15000(F/P,5,5) = $19,144.50

b) How much are January 1, 2000 vacation expenses?

Let V2 equal the actual $ in second year of the trip
= 15000(F/P,5,6) = $20,101.50

Let V2* equal the actual $ of the second year discounted by one year
= $20,101.50(P/F,10%,1) = $18,274

c) Determine the cost of the trip in actual dollars.

V1 + V2 = $39,246

d) Calculate the actual dollar amount of the deposits on December 31, 1995, 1996, 1997 and 1998.

Let D represent the December 31 deposits.

These deposits must equal (V1 + V2*) on January 1, 1995

Therefore, D(P/A,10%,4) = V1 + V2* = $37,185

Solve for D: The annual deposit over a 10-year period is $10,664

11.9 A young engineer decides, on her 30th birthday, to set up a retirement fund that pays an interest rate of 10% compounded annually. She believes that a retirement fund equivalent to $500,000 worth of purchasing power in today=s dollars will be adequate to see her through her sunset years starting on her 63rd birthday. Assume a general 5% inflation rate per year through her work and retirement years.

a) What would be the actual dollar amount in her retirement fund on her 63rd birthday(i.e., as she retires)?

$500,000(1+0.05)^{34} = $2.626 million

b) If the engineer plans to make equal annual deposits to her retirement fund (with the first deposit on her 30th birthday and the last on her 62nd birthday), what

should be the annual deposit (A) to her retirement fund?

A(F/P,10%,33)+A(F/A,10%,32) (F/P,10%,1) = $2.626 million
Solve for A; A=$10,782

c) If the engineer dies on the day she receives her 20th annual pension benefit, what is its value expressed in dollars on her 30th birthday (or today dollars)?

Assuming that she receives her first pension cheque on her 64th birthday.

Annual pension = $2,626,674(A/P,10%,20) = $309,809

Value now = 309,809(P/F,5%,35) = $56,168

11.10 Find the real dollars (using today as the base year) corresponding to the actual dollars shown below if inflation is 5% per year.

a) $500 three years from now. **500(P/F,5%,3) = $431.92**

b) $100 in one year. **100(P/F,5%,1) = $95.24**

c) $600 four years ago. **600(F/P,5%,4) = $729.30**

11.11 An investment pays $20,000 in five years.

a) If inflation is 5% per year, what is the real value of the $20,000 in today=s dollars?

$20,000(P/F,5%,5) = $15,670.52

b) If inflation is 5% and the real MARR is 10%, what is the present worth in actual dollars of this investment?

The combined or actual MARR
= (1+inflation rate)(1+real MARR) - 1
= (1.05)1.1 - 1 = 0.155

$20,000(P/F,15.5%,5) = $9,730.16

11.12 Johnny Slapshot, this year=s 25-year old hockey superstar, will receive twenty annual

payments of $500,000 from the owners of a hockey team with the first annual payment on his 40th birthday and the last on his 59th birthday.

a) Find the present worth Ain age 25 dollars@ of the twenty payments if the inflation rate and real MARR are 8% and 4% respectively.

The present worth of the actual payments are
500,000(P/A,8%,20)(P/F,8%,15)
= $1,546,410

b) Find the present worth Ain actual dollars@ on his 25th birthday of all the payments if the inflation rate and real MARR are 8% and 4% respectively.

The actual dollar (or with inflation) MARR = 1.04(1.08) - 1 = 0.1232
The present worth of the actual payments are
500,000(P/A,12.32%,20)(P/F,12.32%,14)
= $719,762

c) Find the value of the 10th (i.e., at age 49) $500,000 payment Ain age 40 dollars@ if the inflation rate and real MARR are 8% and 4% respectively.

$500,000(P/F,8%,9) = $250,125

d) Find the value of the 10th payment Ain age 25 dollars@ if the inflation rate and real MARR are 8% and 4% respectively.

$500,000(P/F,8%,24) = $78,850

11.13 The following information is provided on an investment project:

A firm is considering the purchase of a truck for $300,000 fully installed. It is expected to last 3 years with a salvage value of $100,000 at that time. Revenues from operations will be $250,000 each year and operating and maintenance costs will be $75,000 each year

- Depreciate the truck using the DB method (d=25%)
- The half-year rule does not apply.
- The before-tax with inflation interest rate is 20%.
- The before-tax inflation-free interest rate is 15%.
- The after-tax with inflation interest rate is 10%.

- The after-tax inflation-free interest rate is 5%.
- There is 5% annual inflation rate.
- The tax rate = 50%
- The firm gets a $100,000 loan (at a 10% rate of interest) which is repaid as follows:

Loan Repayment at End of Year	Percentage of Loan Repaid
1	30%
2	30%
3	40%

a) Complete the Table below.

	Years			
Item	**0**	**1**	**2**	**3**
1. BTCF (Actual $)	-300000	175000	175000	175000+ 100000
2. BTCF (Constant $)	-300000	166667	158730	237555
3. Interest on Loan	-	10000	7000	4000
4. Depreciation	-	75000	56250	42188
5. Taxable Income	-	90000	111750	128812
6. Taxes Payable	-	45000	55875	64406
7. ATCF (Actual $)	-300000	130000	119125	110,594 +71093.7 =181687.7
8. ATCF (Constant $)	-300000	123810	108050	156948.7
9. Repayment of Loan	-	30000	30000	40000
3. Interest on Loan	-	10000	7000	4000

	Years			
Item	**0**	**1**	**2**	**3**
10. CFOE (Actual $)	-200000	90000	82125	112948.7
11. CFOE (Constant $)	-200000	85714	74490	97569.3

b) Determine the after-tax inflation-free present worth of this project.

From the ATCF real dollar row (row 8):
-300,000 + 123,810(P/F,5%,1) + 108,050(P/F,5%,2) + 156,948.7(P/F,5%,3)
= $51,490

c) Determine the present worth of the owner=s equity after-tax inflation-free cash flow.

From the CFOE real dollar row (row 11):

= -200,000 + 85,714(P/F,5%,1) + 74,490(P/F,5%,2) + 97,569.3(P/F,5%,3)
= $33,477

PURPOSE: Find the annual revenues required to breakeven, i.e., to

- pay the operating costs
- pay income taxes
- recover the original investment "P"
- generate a competitive/acceptable rate of return

CONTEXT: PRE-TAX, EQUITY CAPITAL (NO DEBT), NO INFLATION

Using the Annual Equivalent Method:

1. AE = -P(A/P,i%,N) + SV(A/F,i%,N) + AEOR - AEOC

   where i% is the before-tax, no inflation rate of return

Using the Present Worth Method:

2. PW = -P + SV(P/F,i%,N) + PWOR - PWOC

   where i% is the before-tax, no inflation rate of return

Using a break even approach where Revenues = Costs

Let AE = 0 in Equation 1. and solve for AEOR

AEOR = AEOC - SV(A/F,i%,N) + P(A/P,i%,N)

Explanation: AEOR is revenue required to pay for cost recovery and operating costs less the salvage value

Let PW = 0 in Equation 2. and solve for PWOR

PWOR = PWOC - SV(P/F,i%,N) + P

CONTEXT: TAXES, EQUITY CAPITAL (NO DEBT), NO INFLATION

Using the Annual Equivalent Method:

1. AE = -P(A/P,i%,N) + NSV(A/F,i%,N) + (AEOR - AEOC)(1 - t)

+ AED(t)

where "t" is the income tax rate

"AED" is the equivalent annual depreciation

= annual depreciation with Straight Line
= declining gradient series with SOYD
= Pd/(i+d)(A/P,i%,N) with Declining Balance**

"NSV" is net salvage value =
Gross salavage value - taxes payable on the difference between (SV - BV)

NSV = SV - (SV - BV)t

Please note: Only the DB depreciation method causes SV and BV to be different (not Straight Line nor SOYD)

Using a break even approach:

Let AE = 0 in Equation 1. and solve for AEOR

AEOR = AEOC+{P(A/P,i,N)-NSV(A/F,i%,N)-t(AED)}{1/(1 - t)}

Explanation: "AEOR" is revenue required to pay for

- cost recovery
- operating costs less the salvage value
- income taxes
- generate a rate of return on investment at least equal to the After-Tax MARR

WHAT IF THERE IS DEBT CAPITAL? INFLATION?

Substitute in the above equations:

Debt Capital; No inflation; No taxes

$i_c = i_e(1 - r) + i_d(r)$

$$r = \frac{Debt\ Capital}{Total\ Capital} = \frac{Debt\ Capital}{Debt + Equity\ Capital}$$

Debt Capital; Taxes; No inflation

$i_a = i_e(1 - r) + i_d(r)(1 - t)$

Debt Capital; Taxes; Inflation

$i_{af} = i_{df}(r)(1 - t) + (1 - r)\{(1 + i_e)(1 + f) - 1\}$

Note: if r = 0: $i_c = i_e = i$

12.1 A trucking company is considering the purchase of a new truck which has the following characteristics:

- Cost: $200,000;
- Annual operating cost: $25,000 at the end of the first year increasing at a rate of $2,000 per year thereafter;
- The truck has a five year life;
- The truck's salvage value (=$30,000) and its book value will be identical at the end of the fifth year;
- Depreciation will be based on the Sum-of-Years-Digits Method (SOYD);
- Owners expect a 10% rate of return on their investment
- The tax rate is 50%.
- There is no inflation
- A loan of $100,000 is obtained at a 20% interest rate.

a) Determine the annual equivalent revenues (AER) required to break even.

From $i_a = i_e(1 - r) + i_d(r)(1 - t)$

$= 0.10(1 - 0.5) + 0.2*0.5(1 - 0.5) = 0.10$ or 10%

AEOR = AEOC+{P(A/P,i,N)-NSV(A/F,i%,N)-t(AED)}{1/(1 - t)}

where AEOC = 25,000+5,000(A/G,10%,5) = $34,050.50
P = $200,000
NSV = SV - (SV - BV)t = 30000 - (30000 - 30000)0.5
= $30,000
AED = (5/15)(200,000-30,000)
- (1/15)(200,000-30,000)(A/G,10%,5)
= $19,533.4

= 34050.5+[200,000(A/P,10%,5)-30,000(A/F,10%,5) - 0.5(19533.4)]2
= $32,525.30

b) Identify the cost component which the revenues calculated in a) are to meet.

Find the annual revenues required to breakeven, i.e.,

- **to pay the operating costs**
- **to pay income taxes**
- **to recover the original investment "P"**
- **to generate a competitive/acceptable rate of return (opportunity cost)**

c) If the annual revenues generated by this truck are expected to be $28,000 in year 1 and to grow by $3,000 each year thereafter, should this truck be purchased?

AEOR = $28,000 + 3,000(A/G,10%,5)
= $33,430.10
YES. GO AHEAD.

12.2 You purchased a new computer in early November of this year. You were informed this morning that a better computer is now available. Details pertaining to both computers are found below.

Details

	Defender	Challenger
Initial Cost	40,000	

Current Market Value ($)			30,000			
			50,000			
Salvage value ($)				5,000		
				4,000		
Economic Life (Years)						
			10			
				10		
Annual Operating Costs ($)			25,000			
			20,000			
Tax Rate (%)						
		50				
			50			
Depreciation Method				DB		
				FWO		
Depreciation Rate (%)			20			
			n.a.			
Debt/Capital Ratio (%)			40			
			40			
Cost of Debt Capital (%)			25			
			20			
Cost of ownership capital (%)				10		
				10		
Inflation rate (%)				5		
				5		

a) Determine the annual revenue requirements (AER) for the defender (Note: The defender is not beyond its economic life).

Note that $i_a = i_d(r)(1 - t) + (1 - r)\{(1 + i_e) - 1\}$
Therefore, $i_a = 0.25(0.4)(1 - 0.5) + 0.1(1 - 0.4) = 0.11$

AER = 25,000 + {30,000(A/P,11%,10) - 0.50[30,000(A/P,11%,10)
{0.2/(0.2+0.11)}]
- (30,000[5,000 -30,000(1 - 0.2)^{10}]0.5}
= $28,580.5

b) Determine the annual revenue requirements (AER) for the challenger.

AER = 20,000 + {50,000(A/P,11%,10) -0.50(30,000(A/P,11%,10) -

4,000(A/F,11%,10) - 0.5[(10/55)(50,000-4,000) -(1/55)(50,000 - 4,000)(A/G,10,10)]}2
= $30,530

c) If you had to decide now on the lower cost alternative, would you keep the defender for another year or would you switch to the challenger?

KEEP DEFENDER.

MAJOR COMPONENTS OF FINANCIAL STATEMENTS

Statement	Major Components
Income Statement	summarizes an enterprises=s revenues and expenses over a specified accounting period revenues less operating costs equal income or profit before taxes less taxes equal net profit
Balance Sheet	a snapshot of a business=s financial position at a particular point in time normally the last business day of an accounting period assets (economic resources of the business): current assets: assets that can be converted to cash within a short time (cash, accounts receivable) long-term assets: assets such as equipment, land and buildings that are not expected to be converted to cash in the relatively near future liabilities: claims other than those of the owners on a business=s assets current liabilities: due within a short period of time such as one year; examples are accounts and taxes payable long-term liabilities: those that are not expected to draw on the business=s current assets; examples include bonds and loans owners= equity: the interest of the owners of

	a firm in its assets; the difference between a firm=s assets and liabilities; it is made up of two components 13. par value capital (capital raised from the initially authorized shares of the firm) 14. retained earnings which includes the cumulative sum of earnings from normal operations from past years, and from transactions like the sale of plant assets
Changes in Financial Position or Source/application of funds	it shows how much cash was generated by a company=s operation and by other sources during the period it also shows the amounts of cash used for investments as well as for other non-operating disbursements

FINANCIAL RATIO ANALYSIS

Performance measures are calculated values that allow conclusions to be drawn from data or answers to questions such as:

- is the firm able to meet its short-term commitments?
- are profits generated from assets sufficient?
- how dependent is the firm on creditors?

Financial ratios are ratios of key amounts taken from financial statements; their calculation is usually easy from their interpretation more difficult.

A good interpretation of financial ratios normally requires comparisons with industry standards (thresholds) and with previous year ratios (trend analysis)

FINANCIAL RATIOS
Liquidity Ratios

These ratios measure the ability of a firm to whether unforeseen cash flow fluctuations (i.e., how liquid the firm is).

The more the ratio exceeds one, the better more liquid is the firm

- current ratio (or working capital ratio)
- acid-test ratio (or quick ratio)

Current Ratio	= Current assets/current liabilities
Acid-test Ratio	= quick assets/current liabilities where quick assets= cash+accounts receivable, notes receivable+short term investments in marketable securities (excludes inventory and prepaid expenses)

Leverage or debt-management Ratio

Provides information on the extent to which a firm relies on debt for its operations.

Generally, the higher the ratio, the lower the risks.

Equity Ratio	Total owners= equity/{total liabilities and equity} = total owners= equity/total assets

Efficiency or asset-management ratios

Inventory turnover ratio This ratio examines how efficiently a firm uses it resources to manage its inventories. The concerns with this ratio include: - that sales are generated over a period of time and inventories are measured	= Sales-to-Inventory [a high ratio is preferred to a lower ratio]

at a point in time; better to use an average inventory concept S that sales are generally measured with more current market prices than costs (which are usually based on historical prices). S this ratio overstates the true turnover of inventories during a year.	
Return-on-total-assets or net profit ratio S indicates the productivity of assets in generating profits S a profitability ratio	= profits after taxes/total assets

13.1 The income statement and balance sheet entries for calendar year 1995 for the XYZ Company are shown in Table 1. The cut-off time period for short-term assets and liabilities is one year.

Table 1

Selected Balance Sheet and Income Statement Items XYZ Company Limited (1995)	**(Thousand $)**
Accounts Payable	10 000
Accounts Receivable	15 000
Accumulated depreciation on plant and equipment	11 000
Cash	2 100
Accrued wages (payable in 6 months)	4 000
Common shares	5000
Contributed capital	5 000
Cost of goods sold	60 000
Income taxes payable in 6 months	4 000
Depreciation expense	2 000
General operating expense	8 000
Interest Expense	1 500
Inventories	18 000
Land	3 000
Long term bond investments	4 500
Net plant and equipment	7 500

Selected Balance Sheet and Income Statement Items XYZ Company Limited (1995)	**(Thousand $)**
Mortgage on building and equipment (5-year)	20 000
Total Sales	450 000
Prepaid Expense (due in 3 months)	3 000
Property taxes payable (in 6 months)	3 100
Salaries Expense	80 000
Selling Expense	7 000
Working capital loan (90 days)	2 000

a) Complete XYZ Company=s Balance Sheet for 1995 (from the information above).

Balance Sheet for XYZ Company at December 31, 1995	**$**
ASSETS	
Current Assets	
Accounts receivable	15 000
Cash	2 100
Inventories	18 000
Prepaid expenses (due in 3 months)	3 000
Total Current Assets	38 100
Long Term Assets	
Long term bond investments	4 500

Balance Sheet for XYZ Company at December 31, 1995	**$**
Land	3 000
Gross plant and equipment	18 500
Less: Accumulated depreciation on plant and equipment	11 000
Net plant and equipment	7 500
Total Long Term Assets	15 000
TOTAL ASSETS	53 100
LIABILITIES	
Current Liabilities	
Accounts payable	10 000
Accrued wages	4 000
Income taxes payable in 6 months	4 000
Working capital Loan	2 000
Property taxes payable in 6 months	3 100
Total Current Liabilities	23 100
Long Term Liabilities	
Mortgage on building (5-year)	20 000
Total Long term liabilities	20 000
TOTAL LIABILITIES	43 100
OWNERS= EQUITY	
Common shares	5 000

Balance Sheet for XYZ Company at December 31, 1995	**$**
Contributed capital	5 000
Total Owners= Equity	10 000
TOTAL LIABILITIES & OWNERS= EQUITY	53 100

b) Complete XYZ Company=s Income Statement for 1995 (from the information above).

Income and Expense Statement for XYZ Company from January 1 to December 31, 1995	**$**
REVENUES	
Total sales	450 000
TOTAL REVENUES	450 000
EXPENSES	
Cost of goods sold	60 000
Depreciation expense	2 000
General operating expense	8 000
Salaries	80 000
Selling expenses	7 000
Interest expense	1 500
TOTAL EXPENSES	158 500
PROFIT BEFORE TAXES	291 500
Income Taxes @50% of Profit Before Taxes	145 750

Income and Expense Statement for XYZ Company from January 1 to December 31, 1995	**$**
PROFIT AFTER TAXES	145 750

c) Determine XYZ Company=s Acurrent@ ratio.

Current assets/current liabilities = [38 100/23 100] = 1.65

d) Determine XYZ Company's inventory-turnover@ ratio.

Total sales/Inventory = [450 000/18 000] = 25

e) Determine XYZ Company=s Aacid-test or quick@ ratio.

Current assets less (inventories & prepaid expenses)/current liabilities = [(38 100 - 18 000 - 3 000)/23 100] = 0.74

f) Determine XYZ Company=s AEquity@ ratio.

Total owners= equity/Total Assets = [10 000/53 100] = 0.188

g) Determine XYZ Company=s Areturn-on-assets@ ratio.

Profits after taxes/Total assets = [145 750/53 100] = 2.74 or 274%

h) From the industry norms in the Table 2 below, does the XYZ Company meet the liquidity standards?

Table 2

Industry Norms for Financial Ratios Applicable to XYZ Company in 1995

Ratios	**Industry Norms**	**XYZ Actual Ratios**	**XYZ meets the norm?**

Current ratio	2.5	1.65	No
Acid-test ratio	1.75	0.74	No
Equity Ratio	0.6	0.19	No
Inventory-turnover ratio	2.2	25	Yes
Return-on-total-assets ratio	0.08 (8%)	274%	Yes