UNIVERSITY OF OTTAWA

ENGINEERING ECONOMICS
ECO 1192

TYPICAL PROBLEMS & SOLUTIONS

PROBLEM SET #1

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These problems and solutions are designed to enhance the understanding of engineering students of key engineering economy subject-matter areas and to assist them in preparing for examinations.

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Problem Set #1

- ***10% discrete interest rate factors***
- ***General Interest Rate Formulas: Discrete & Continuous Cash Flows and Interest Compounding***

Sections 1 to 6 (this problem set)

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Problem Set #2 -- Sections 7 to 13

DISCRETE CASH FLOW AND COMPOUNDING							
10,0% DISCRETE RATE OF INTEREST							
N	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)	(A/G,i%,n)
1	1,1000	0,9091	1,1000	0,9091	1,0000	1,0000	0,0000
2	1,2100	0,8264	0,5762	1,7355	0,4762	2,1000	0,4762
3	1,3310	0,7513	0,4021	2,4869	0,3021	3,3100	0,9366
4	1,4641	0,6830	0,3155	3,1699	0,2155	4,6410	1,3812
5	1,6105	0,6209	0,2638	3,7908	0,1638	6,1051	1,8101
6	1,7716	0,5645	0,2296	4,3553	0,1296	7,7156	2,2236
7	1,9487	0,5132	0,2054	4,8684	0,1054	9,4872	2,6216
8	2,1436	0,4665	0,1874	5,3349	0,0874	11,4359	3,0045
9	2,3579	0,4241	0,1736	5,7590	0,0736	13,5795	3,3724
10	2,5937	0,3855	0,1627	6,1446	0,0627	15,9374	3,7255
11	2,8531	0,3505	0,1540	6,4951	0,0540	18,5312	4,0641
12	3,1384	0,3186	0,1468	6,8137	0,0468	21,3843	4,3884
13	3,4523	0,2897	0,1408	7,1034	0,0408	24,5227	4,6988
14	3,7975	0,2633	0,1357	7,3667	0,0357	27,9750	4,9955
15	4,1772	0,2394	0,1315	7,6061	0,0315	31,7725	5,2789
16	4,5950	0,2176	0,1278	7,8237	0,0278	35,9497	5,5493
17	5,0545	0,1978	0,1247	8,0216	0,0247	40,5447	5,8071
18	5,5599	0,1799	0,1219	8,2014	0,0219	45,5992	6,0526
19	6,1159	0,1635	0,1195	8,3649	0,0195	51,1591	6,2861
20	6,7275	0,1486	0,1175	8,5136	0,0175	57,2750	6,5081
21	7,4002	0,1351	0,1156	8,6487	0,0156	64,0025	6,7189
22	8,1403	0,1228	0,1140	8,7715	0,0140	71,4027	6,9189
23	8,9543	0,1117	0,1126	8,8832	0,0126	79,5430	7,1085
24	9,8497	0,1015	0,1113	8,9847	0,0113	88,4973	7,2881
25	10,8347	0,0923	0,1102	9,0770	0,0102	98,3471	7,4580
26	11,9182	0,0839	0,1092	9,1609	0,0092	109,1818	7,6186
27	13,1100	0,0763	0,1083	9,2372	0,0083	121,0999	7,7704
28	14,4210	0,0693	0,1075	9,3066	0,0075	134,2099	7,9137
29	15,8631	0,0630	0,1067	9,3696	0,0067	148,6309	8,0489
30	17,4494	0,0573	0,1061	9,4269	0,0061	164,4940	8,1762

INTEREST RATE FORMULAS

SUMMARY MEASURES	Cash Flow	Discrete	Discrete	Continuous
	Compounding	Discrete	Continuous	Continuous
Single Sum	***Compound Amount***	**F = P(1+i)^n = P(F/P,i,n)**	**F = Pe^{rn} = P(F/P,r,n)**	**F = Pe^{rn} = P(F/P,r,n)**
	Discount Amount	**P = F(1+i)$^{-n}$ = F(P/F,i,n)**	**P = Fe^{-rn} = F(P/F,r,n)**	**P = Fe^{-rn} = F(P/F,I,n)**
Annuities	***Compound Amount***	$F = A[\frac{(1+i)^n - 1}{i}]$ **F = A(F/A,r,n)**	$F = A[\frac{e^{rn} - 1}{e^r - 1}]$	$F = \overline{A}[\frac{e^{rn} - 1}{r}]$
	Sinking Fund	$A = F[\frac{i}{(1+i)^n - 1}]$ **A = F(A/F,r,n)**	$A = F[\frac{e^r - 1}{e^{rn} - 1}]$	$\overline{A} = F[\frac{r}{e^{rn} - 1}]$
	Discount Amount	$P = A[\frac{(1+i)^n - 1}{i(1+i)^n}]$ **P = A(P/A,r,n)**	$P = A[\frac{1 - e^{-rn}}{e^r - 1}]$	$P = \overline{A}[\frac{e^{rn} - 1}{re^{rn}}]$
	Capital Recovery	$A = P[\frac{i(1+i)^n}{(1+i)^n - 1}]$ **A = P(A/P,r,n)**	$A = P[\frac{e^r - 1}{1 - e^{-rn}}]$	$\overline{A} = P[\frac{re^{rn}}{e^{rn} - 1}]$

SUMMARY MEASURES	Cash Flow / Compounding	Discrete / Discrete	Discrete / Continuous	Continuous / Continuous
Gradient Series	**Uniform Gradient Series (Conversion to a Uniform Series)**	$A = G[\frac{1}{i} - \frac{n}{(1+i)^n - 1}]$ **A = G(A/G,r,n)**	$A = G[\frac{1}{e^r - 1} - \frac{n}{e^{rn} - 1}]$	**Not defined.**
	Geometric Gradient (Conversion to a Single Sum i.e. Present Worth)	***If i% ≠ k%:*** P = C(P/C,i,k,N) $P = [\frac{C}{(i-k)}](1 - [\frac{(1+k)}{(1+i)}]^n$ ***If i% = k%*** $P = \frac{CN}{(1+i)}$ C ≡ First (≠0) term of series	**Not defined**	**Not defined**

1.1 You borrowed $100 from a friend and promised to repay him $110 in 1 month.

a) What interest rate are you paying over the 1-month period?

From $F = P(1+i)^n$, $110 = 100(1+i)$
Solve for i; i = 0.1 or 10%/month

b) What is the nominal interest rate (annual rate)?

(12)10% = 120% annually

c) What is the effective interest rate (annual rate)?

From $(1+r/m)^m - 1$, $(1+1.2/12)^{12} - 1$
= 3.1384 - 1 = 2.1384 or 213.8% compounded annually

1.2 A bank loan is made at an interest rate of 0.75% per month on the unpaid balance. Determine the effective interest rate.

Effective rate = $[1 +(r/m)]^m - 1 = [1+0.0075]^{12} - 1$
= 0.0938 or 9.38% compounded annually

1.3 A savings institution advertises the interest paid on savings accounts as 8% interest compounded quarterly. Determine the effective interest rate.

Effective rate = $[1 +(r/m)]^m - 1 = [1+ (0.08/4)]^4 - 1$
= 0.0824 or 8.24% compounded annually

1.4 How much time (years) will it take for the balance in a bank account to increase from $500 to $900 if interest is given as 10% compounded semi-annually?

From F = P(F/P,i%,N),
900 = 500(F/P,(10%/2),N) = $500(1+0.05)^N$
where N is given as 6-month periods

Therefore log(900/500) = log 1.8 = N log 1.05

Solve for N; N = 12.05 6-month periods or about 6 years.

1.5 An investor purchased property for $75,000. He paid $20,000 cash and was given a loan for $55,000 payable at he rate of

$650 per month at a interest rate of 12 percent compounded monthly on the unpaid balance.

a) What is the effective annual interest rate?

Effective rate = $[1 +(r/m)]^m - 1 = [1+0.01^{12} - 1$
= 0.127 or 12.7%

b) How long will it take to pay off the loan?

From P = A(P/A,i%,N), 55000 = 650(P/A,1%,N)
[55000/650] = (P/A,1%,N) = 84.615
Therefore, N = 190.4 months

1.6 You borrow $100 on Tuesday and pay off the loan with $120 the next Tuesday.

a) What is the nominal interest rate charged?

From F = P(F/P,i%,N), 120 = 100(F/P,i%,1) = 1.2
or $(1+i)^1 = 1.2$; therefore i = 52x20% = 1040%

b) What is the effective interest rate charged?

Effective rate = $[1 +(r/m)]^m - 1 = [1+0.2]^{52} - 1$
= 1,310,363%

1.7 A car dealer advertises two options for the purchase of a car:

S cash price: $3575 or
S $375 down with 45 monthly payments of $93.41

What is the effective rate of interest charged by the car dealer?

From A = P(A/P,i%,N),
93.41 = (3575 - 375)(A/P,i%,N) or (A/P,i%,45) = 0.0291905

From the financial tables:
(A/P,1.5%,45) = 0.03072
(A/P,1%,45) = 0.02771

By interpolation:

(A/P,i%,45) =
1% + {[0.0291075-0.02771]/[0.03072-0.02771]}0.5%

= 1% + 0.24593 = 1.24593
Therefore, the effective interest rate is
$[1+0.0124593]^{45}$ - 1 = 16.019% compounded annually

1.8 Bank A pays 10% compounded monthly on its Guaranteed Investment Certificates. Determine

a) the nominal rate (annual): **10%**

b) the effective rate (annual)

From $(1+r/m)^m - 1$, $(1+0.1/12)^{12} - 1 = 0.1047$ or 10.47%

c) the actual rate: **r/m = 0.1/12 = 0.00833 or 8.33%**

1.9 Bank B pays 10% compounded continuously on its Guaranteed Investment Certificates. Determine

a) the nominal rate (annual): **10%**

b) the effective rate (annual)
From $e^r - 1$, $e^{0.1} - 1 = 0.105$ or 10.5%

c) the actual rate: **tends to zero**

1.10 You are asked to choose between investing $1,000 in an account that pays interest at 15% compounded quarterly and an account that pays interest at 15.865% (rounded) annually. Assuming that you want to maximize the return on your investment, which investment is better?

Determine the effective annual rate equivalent to 15% compounded quarterly.
From $(1+r/m)^m - 1$, $(1+0.15/4)^4-1 = 0.15865$ or 15.865%
You are indifferent between the two investments because they are equivalent.

1.11 The shares of the XYZ Company that you purchased 10 years ago for $40,000 are now worth $100,000. Disregarding taxes, determine the rate of return on your investment.

Find i from $F = P(1+i)^N$;
$100,000 = 40,000(1+I)^{10}$; Solving,
i = 0.096 or 9.6%

1.12 You must decide between depositing $2,000 in a bank account that pays interest at 6% compounded monthly and buying a preferred stock that yields 6.2% compounded annually. Assuming that you want to maximize the return on your investment, which investment is better?

The simplest way to determine the better investment is to compare their respective annual yield. The annual yield on the preferred stock is given as 6.2% per year. The annual yield on the bank account is its effective annual interest rate.
From $(1+r/m)^m - 1$, $(1 + 0.06/12)^{12} - 1$
$= 0.0617$ or 6.17%
Because the preferred stock has the higher annual yield (6.2%), it is the better investment.

1.13 A friend received her property tax bill from the City of Ottawa last week. Her first tax payment ($500) was due today (i.e., October 22); the second tax payment (also $500) is due in exactly two months (i.e., December 22). She was given the choice of making the tax payments on the dates given above OR making ONE payment today of $990.

If the annual interest rate is 4 per cent compounded annually, would you advise her to make ONE payment of $990 on October 22 OR two payments of $500 each on October 22 and December 22.

Find the monthly interest rate (r) equivalent to 4% per year:
$(1+r)^{12} = 1.04$; solve for r;
r = 0.003273726 or about 0.327% per month

The Present Worth on October 22 of the two $500 installments is:
500 + 500(P/F,r,2 months) = $996.74
Decision: To minimize costs, the taxpayer is better off paying $990 now.

1.14 How long will it take any sum (P)to double itself

a) with a 10% simple interest rate;

PiN = 2P; 0.1N = 2 Solve for N; N = 20 years

b) with a 10% interest rate compounded annually;

$2P = P(1+I)^{N;}$ log 2 = N log 1.1; solve for N; N = 7.27 years

c) with a 10% interest rate compounded continuously.

$2P = Pe^{rN} = 2 = e^{0.1N}$; solve N; N = 6.93 years

1.15 What nominal rate of return is required to ensure an effective annual rate of 10% if compounding is quarterly?

$0.10 = (1 + r/4)^4 - 1$; therefore, r = 0.0964 or 9.64%

1.16 Assume that the interest rate charged by various finance companies is 2% per month.

a) What is the annual nominal rate? **2(12) = 24%**

b) What is the annual effective rate?

$(1+0.02)^{12}-1 = 0.268$ or 26.8%

1.17 Convert a 19.56% annual effective rate to a monthly rate.

$0.1956 = (1 + r/m)^{12} - 1$; therefore, r/m=0.015 or 1.5%/month

1.18 Convert a 16% annual effective rate to a quarterly rate.

$0.16 = (1 + r/m)^4 - 1$; therefore, r/m = 0.0378 or 3.78%

1.19 What simple (annual) rate of interest, during a one-year period, is equivalent to a 10% rate of interest compounded semi-annually?

$PiN = P(F/P,r\%,N)$;therefore, $I = (F/P,r\%,1) = (1+0.1/2)^2 - 1 = 0.1025$ or 10.25%

1.20 What is the equivalent interest rate per quarter if compounding is 12% compounded monthly and the cash flow is on a quarterly basis?

$i = (1+r/m)^{(m/k)} -1] = (1+0.12/12)^{(12/4)} - 1]$
$= 0.0303$ or 3.03% per quarter

1.21 You deposit $2000 for 3 years in a bank account that pays 10% simple interest each year. Complete the following table.

	Amount		***Amount in***

End of year	deposited in account at end of year	Interest paid during year	account at end of year
0	$2000	$0	$2000
1	0	200	2200
2	0	200	2400
3	0	200	2600

1.22 You deposit $2000 for 3 years in a bank account that pays interest at 10% compounded annually. Complete the following table.

End of year	Amount Deposited in Account	Interest Paid During Year	Amount in Account at End of Year
0	$2000	$0	$2000
1	0	200	2200
2	0	220	2420
3	0	242	2662

1.23 You deposit $2000 for 3 years in a bank account that pays interest at 10% compounded semi-annually. Complete the following table.

End of year	Amount Deposited in Account	Interest Paid During Year	Amount in Account at End of Year
0	$2000	$0	$2000

1	0	100+105 = 205	2205
2	0	110.25+115.76 = 226.01	2431.01
3	0	121.55+127.63 = 249.18	2680.19

1.24 Capital rationing - This situation covers investment decisions with budget constraints that prevent the funding (implementation) of all projects with a positive net single worth (present and future values) or internal rates of return that exceed the MARR.

A business person is considering the three (3) following investments. Determine each project=s net present worth assuming that MARR=10%.

Year	Project A	Project B	Project C
0	-$2000	-$9000	-$15000
1	1600	4000	6500
2	1600	4000	6500
3	1600	4000	6500
Net Present Worth	**$1979**	**$948**	**$1165**

Suppose that this business person has $15,000 to spend on the projects. Indicate the feasible and the optimal investments.

Possible Decisions	Individual or Project Combinations (A,B,C)	Funds Required ($)	Residual Budget ($)	Net Present Worth ($)

1*	0 0 0	0	15000	0
2*	1 0 0	2000	13000	1979
3*	0 1 0	9000	6000	948
4*	0 0 1	15000	0	1165
5*,*	***1 1 0***	***11000***	***4000***	***2927***
6**	1 0 1	17000	-2000	3144
7**	0 1 1	24000	-9000	2113
8**	1 1 1	26000	-11000	4092
* = feasible; ** = not feasible; *** = optimal				

2.1 Consider the following investment decision:

Details	**Machine A**	**Machine B**
First Cost ($)	15,000	20,000
Economic Life (Years)	5	10
Annual Revenues ($)	12,000	14,000
Annual Operating Costs ($)	6,000	8,000
Salvage Value ($)	1,500	5,000
MARR (%)	10	10

a) Use the Present Worth Method to determine the economic viability of each machine at time t=0.

Present Worth (Machine A) = -15,000 + (12,000-6,000)(P/A,10%,5) + 1,500(P/F,10%,5) = $8,676.10

Present Worth (Machine B) = -20,000 + (14,000-8,000)(P/A,10%,10) + 5,000(P/F,10%,10) = $18,795.12

NOTE: Because you are asked to calculate the Present or Future worth of the two machines and NOT to determine which machine is better, there is no need to use a common period of analysis (e.g., 10 years).

b) Use the Future Worth Method to determine to determine the economic viability of each machine.

Future Worth (Machine A) = -15,000(F/P,10%,5) + (12,000-6,000)(F/A,10%,5) + 1,500 = $13,972.95

Future Worth (Machine B) = -20,000(F/P,10%,10) + (14,000-8,000)(F/A,10%,10) + 5,000 = $48,749.10

NOTE: Because you are asked to calculate the Present or Future worth of the two machines and NOT to

determine which machine is better, there is no need to use a common period of analysis (e.g., 10 years).

c) Using the Present Worth method, determine which machine is better (at time t=0).

Note: Because the Present Worth Method is a single sum transformation, you must use a common period of analysis to determine the better (best) mutually exclusive project.

Using a 10-year period:
Present Worth (Machine A) = -15,000[1+(P/F,10%,5)] + (12,000-6,000)(P/A,10%,10) + 1,500[(P/F,10%,5)+(P/F,10%,10)]
= $14,063.28

Present Worth (Machine B) = -20,000 + (14,000-8,000)(P/A,10%,10) + 5,000(P/F,10%,10)
= $18,795.12

Decision: Machine B is better than Machine A because it has a larger Present Worth.

d) Using the Future Worth method, determine which machine is better (at time t=0).

Note: Because the Present Worth Method is a single sum transformation, you must use a common period of analysis to determine the better (best) mutually exclusive project.

Using a 10-year period
Future Worth (Machine A) = -15,000[(F/P,10%,10) + (F/P,10%,5)]+(12,000-6,000)(F/A,10%,10)+1,500[1+(F/P,10%,5)]
= $36,477

Future Worth (Machine B)
= -20,000 + (14,000-8,000)(F/A,10%,10) + 5,000
= $48,749.10

Decision: Machine B is better than Machine A because it has a lower Present Worth.

2.2 What initial expenditure is justified (now) for a new piece of equipment that is expected to reduce annual operating costs as follows during the next 5 years:

Year	Annual Reduction in Operating Costs ($) (end-of-year)
1	4,000
2	3,500
3	3,000
4	2,500
5	2,000
Assume that the 10% interest rate is compounded semi-annually	

Find the annual interest rate (r) equivalent to 10% compounded semi-annually:
$(1+0.10/2)^2 - 1 = 0.1025$ or 10.25% per year

PW = 4000(P/A,10.25%,5) - 500(P/G,10.25%,5)
= 4000(P/A,10.25%,5) - 500(P/A,10.25%,5)(A/G,10.25%,5)
= $11,666.22

2.3 If you deposit $1,000 today, $1,000 3 years from today, and $1,000 6 years from today, how much could you withdraw 9 years from today if i=10% compounded annually?

There are several equivalent approaches to this problem. One method consists of finding the present worth of each deposit (at t=0) and subsequently converting this present

amount to future dollars (at t=9).
[1,000 + 1000(P/F,10%,3) + 1000(P/F,10%,6)](F/P,10%,9)
= $5,460

A second way consists of converting each deposit directly into future dollars (at t=9).

1,000(F/P,10%,9) + 1000(F/P,10%,6) + 1000(F/P,10%,3)
= $5,460

2.4 Find the PRESENT WORTH of the three cash flow scenarios below when the interest rate is 10% per year

a) a uniform series of 20 end-of-year payments of $100.

Present Worth = A(P/A,i%,N)
= 100(P/A,10%,20)
= $851.36

b) an arithmetic series of 20 end-of-year payments, the first being $100, the second $105, the third $110, and each succeeding one being $5 more than its predecessor.

Present Worth = A(P/A,i%,N) + G(P/G,i%,N)
= 100(P/A,10%,20) + 5(P/G,10%,20)
= $1,128.39

PLEASE NOTE THAT (P/G,i%,N) = (P/A,i%,N)(A/G,i%,N)

c) an arithmetic series of 20 end-of-year payments, the first being $100, the second $98, the third $96, and each succeeding one being $2 less than its predecessor.

Present Worth = A(P/A,i%,N) + G(P/G,i%,N)
= 100(P/A,10%,20) - 2(P/G,10%,20)
= $740.55

PLEASE NOTE THAT (P/G,i%,N) = (P/A,i%,N)(A/G,i%,N)

d) a geometric series of 20 end-of-year payments, the first one being $100 and each succeeding one being 5% **LESS** than its predecessor.

From the geometric series discount factor (g% □ i%):
(P/C,i%,g%,N) = {C/(i-g)}{1-[(1+g)/(1+i)]^{N}}
= **{100/(0.1+0.05}{1-[(1-.05)/(1+0.1)]^{20}}**
= $631.14

e) a geometric series of 20 end-of-year payments, the first one being $100 and each succeeding one being 10% **GREATER** than its predecessor.

From the geometric series discount factor (g% = i%):
(P/C,i%,g%,N) = {CN/(1+i)} = {CN/(1+g)}
= **{100*20}/(1+0.1} = $1,818.20**

2.5 Find the FUTURE WORTH of the three cash flow scenarios below when the interest rate is 10% per year

a) a uniform series of 20 end-of-year payments of $100.

Future Worth = A(F/A,i%,N)
= 100(F/A,10%,20)
= $5,727.50

b) an arithmetic series of 20 end-of-year payments, the first being $100, the second $105, the third $110, and each succeeding one being $5 more than its predecessor. Please note that (A/G,10%,20) = 6.5081

Future Worth = A(F/A,i%,N) + G(F/G,i%,N)
= 100(F/A,10%,20) + 5(F/G,10%,20)
= $7,591.25

NOTE THAT (F/G,i%,N) = (F/A,i%,N)(A/G,i%,N)

c) a geometric series of 20 end-of-year payments, the first one being $100 and each succeeding one being 5% **LESS** than its predecessor.

From the geometric series discount factor:

$(P/C,i\%,g\%,N) = \{C/(i-g)\}\{1-[(1+g)/(1+i)]^N\}$, we obtain
$FW = (P/C,i\%,g\%,N)(F/P,i\%,N)$
$= C(P/100,10\%,-5\%,20)(F/P,10\%,20)$
$= \{100/(0.1+0.05\}\{1-[(1-.05)/(1+0.1)]^{20}\}(F/P,10\%,20)$
= $4,246

2.6 Find the Present Worth of the following cash flow (i=10%).

(End of) YEAR	CASH FLOW ($)
0	0
1	+1,000
2	+2,000
3	+3,000
4	+2,500
5	+2,000
6	+1,500
7	+1,000
8	+500

There are several ways to obtain the Present Worth of this cash flow pattern. Only one method is presented here.

First, find the Present Worth of the first 3 terms:

$\text{Present Worth}_1 = A(P/A,i\%,N) + G(P/G,i\%,N)$
$= 1{,}000(P/A,10\%,3) + 1{,}000(P/G,10\%,3) = \$\ 4{,}815.80$

Then find the Present Worth PW_2 of the last five terms:

$PW_2 = A(P/A,i\%,N) + G(P/G,i\%,N)$
$= [2{,}500(P/A,10\%,5) - 500(P/G,10\%,5)](P/F,10\%,3)$
$= \$\ 6{,}899.28$

Note that it is necessary to move the Present Worth$_2$ from t=3 to the present (t=0)

$4,815.80 + $6,899.28 = $11,715.08

2.7 What is the present worth or value today (at n=0) of a series of ten (10) end-of-year deposits of $1,000 each if the first deposit is made one year from now and the account pays interest at a rate of 10% compounded annually?

PRESENT WORTH = A(P/A,i%,N) = 1000(P/A,10%,10)
= 1000(6.1446)
= $6,145

2.8 What is the present worth or value today (at n=0) of a series of ten (10) deposits of $1,000 each if the first deposit is made NOW (with the remaining 9 deposits made one year apart) and the account pays interest at a rate of 10% compounded annually?

PRESENT WORTH = A + A(P/A,i%,N)
= 1000 + 1000(P/A,10%,9)
= 1000 + 1000(5.7590)
= $6,759

2.9 What initial expenditure is justified for a new piece of equipment that is expected to reduce annual operating costs by $3,000 over each of the next five years? Assume that the savings represent a uniform series of continuous cash flows with continuous compounding at 10%.

Present Worth = $3000(e^{0.1(5)} - 1)$ = 3.9346(3000) = $11,804

2.10 How much money would you have in an account fifteen (15) years from now if you made fifteen (15) end-of-year deposits of $1,000 each? Assume that the first $1,000 deposit is made one year from now and the account pays interest at a rate of 12% compounded quarterly.

12% quarterly is $(1+r/m)^M - 1 = (1+0.03)^4 - 1 = 0.1255$
Future Worth = 1000(F/A,12.55%,15) = $38,971

2.11 How much money would you have in an account fifteen (15) years from now if you made thirty (30) semi-annual deposits of $500 each. Assume that the first deposit is made in 6 months from now and the account pays interest at a rate of 10% compounded semi-annually.

Future Worth = A(F/A,i%,N) = 500(F/A,5%,30) = $33,218

2.12 How much money would you have in an account fifteen (15) years from now if you made thirty (30) semi-annual deposits of $500 each. Assume that the first deposit is made **NOW** and the account pays interest at a rate of 10% compounded semi-annually.

Future Worth = 500(F/P,5%,30) + 500(F/A,5%,29)(F/P,5%,1)
= 500(4.3218) + 500(6232)(1.05)
= $34,878.9

2.13 How much money would you have in an account fifteen (15) years from now if you made fifteen (15) end-of-year deposits of $1,000 each? Assume that the first $1,000 deposit is made one year from now and the account pays interest at a rate of 10% compounded continuously.

Future Worth = A(F/A,r,N) = 1000(e^{rn} - 1)/(e^{r} - 1) = $33,105

2.14 How much money would you have in an account fifteen (15) years from now if you made fifteen (15) continuous deposits amounting to $1,000 per year? Assume that the account pays interest at a rate of 10% compounded semi-annually.

Note that a continuous, uniform cash flow totalling $1,000 per year is identical (in this problem) to a $500 deposit every six months.

Therefore, A = A(F/A,i%,N) = 500(F/A,5%,30) = $33,220

2.15 How much money would you have in an account fifteen (15) years from now if you made fifteen (15) **continuous deposits** amounting to $1,000 per year? Assume that the account pays interest at a rate of 10% compounded continuously.

Future Worth = A(F/A,r%,N) = 1000(F/A,10%,15)
= 1000($e^{0.1(15)}$ - 1)/0.1 = $34,817

2.16 For how many years can an investment of $40,000 provide a continuous flow of funds amounting to $8,000 per year at an interest rate of 15% compounded continuously?

Present Worth = A(P/A,r%,N)
Therefore 40,000 = 8000(e^{rN} - 1)/(re^{rn})
= 8000($e^{0.15N}$ -1)/($0.15e^{0.15N}$); solve for n

2.17 Using the Present Worth and the Future Worth Methods, determine which projects are economically valid if A, B and C are independent.

Using the Present Worth and the Future Worth Methods, determine which project is best if A, B and C are mutually exclusive.

Parameters	Project A	Project B	Project C
First Cost ($)	10,000	18,000	30,000
Salvage Value ($)	0	-1,000	3,000
Life (Years)	10	10	20
Net Annual Profits (Annual operating revenues minus annual operating costs)($)	2,000	3,850	$2,800 at the end of the first year followed by annual increases of $400 (i.e., $3,200 at the end of the 2nd year; $3,600 at the end of

			the 3rd year, etc.)
MARR (%)	10	10	10

Note: When projects are independent, there is no requirement to use a common period of analysis.

Project A:

PW = -P + A(P/A,i%,N) + SV(P/F,i%,N)
= -10,000 + 2,000(P/A,10%,10) + 0(P/F,10%,10)
= $2,289.13

FW = -P(F/P,i%,N) + A(F/A,i%,N) + SV
= -10,000(F/P,10%,10) + 2,000(F/A,10%,10) + 0
= $5,937.33

Project B:

PW = -P + A(P/A,i%,N) + SV(P/F,i%,N)
= -18,000 + 3,850(P/A,10%,10) - 1,000(P/F,10%,10)
= $5,271.04

FW = -P(F/P,i%,N)+ A(F/A,i%,N) + SV
= -18,000(F/P,10%,10)+3,850(F/A,10%,10)-1,000
= $13,671.50

Project C:

PW = -P + A(P/A,i%,N) + G(P/G,i%,N) + SV(P/F,i%,N)
= -30,000+2,800(P/A,10%,20)+400(P/G,10%,20) +3,000(P/F,10%,20)
= $16,446.67

FW = -P(F/P,i%,N) + A(F/A,i%,N) + G(F/G,i%,N) + SV
= -30,000 + 2,800(F/A,10%,20) + 400(F/G,10%,20) + 3,000
= $110,641.68

Decision: Select the three projects because their individual PWs and FWs are positive.

2.18 Using the Present Worth and the Future Worth Methods, determine which project is best if A, B and C are mutually exclusive projects.

Parameters	**Project A**	**Project B**	**Project C**
First Cost ($)	10,000	18,000	30,000
Salvage Value ($)	0	-1,000	3,000
Life (Years)	10	10	20
Net Annual Profits (Annual operating revenues minus annual operating costs)($)	2,000	3,850	$2,800 at the end of the first year followed by annual increases of $400 (i.e., $3,200 at the end of the 2nd year; $3,600 at the end of the 3rd year, etc.)
MARR (%)	10	10	10

Note: When projects are mutually exclusive and have unequal durations, project comparisons must be made on the basis of a (least) common periods of analysis.

Project A:

PW = -P + A(P/A,i%,10) + SV(P/F,i%,10)
= -10,000 + 2,000(P/A,10%,10) + 0
= $2,289.13

FW = -P(F/P,i%,10) + A(F/A,i%,10) + SV
= -10,000(F/P,10%,10)+2,000(F/A,10%,10)+ 0
= $5,937.33

Project B:

PW = -P + A(P/A,i%,10) + SV(P/F,i%,10)
= -18,000 + 3,850(P/A,10%,10) - 1,000(P/F,10%,10)
= $5,271.04

FW = -P(F/P,i%,10) + A(F/A,i%,10) + SV
= -18,000 + 3,850(P/A,10%,10) - 1,000
= $13,671.50

Decision: Project B is a better project than A; now compare Projects B and C using a 20-year period of analysis.

Project B

PW = -P[1 + P(P/F,i%,10)]+ A(P/A,i%,20) + SV[(P/F,i%,10) +(P/F,i%,20)]
= -18,000[1+(P/F,10%,10)]+ 3,850(P/A,10%,20) - 1,000[(P/F,10%,10)+(P/F,10%,20)]
= $7,303.29

FW = -P[(F/P,i%,20)+(F/P,i%,10)]+A(F/A,i%,20)+SV[1+(F/P,i%,N)]
= -18,000[(F/P,10%,20)+(F/P,10%,10)] + 3,850(F/A,10%,20) - 1,000[1+(F/P,10%,10)]
= $49,131.42

Project C

PW = -P + A(P/A,i%,N) + G(P/G,i%,N) + SV(P/F,i%,N)
= -30,000 + 2,800(P/A,10%,20) + 400(F/G,10%,20) + 3,000(P/F,10%,20)
= $16,446.67

FW = -P(F/P,i%,N) + A(F/A,i%,N) + G(F/G,i%,N) + SV
= -30,000(F/P,10%,20) + 2,800(F/A,10%,20) + 400(P/G,10%,20) + 3,000
= $110,641.68

Decision: Select Project C because it has the largest

positive PW and FW values.

2.19 Two alternatives for processing hamburger are under consideration by a firm. The key parameters of these alternatives are shown below.

Parameters	Project A	Project B
First Cost ($)	50,000	120,000
Salvage Value ($)	10,000	20,000
Life (Years)	20	40
Annual operating costs)($)	9,000	6,000
MARR (%)	6	6

a) Using the annual equivalent method, determine which alternative is preferred?

EAC(A) = (50000-10000)(A/P,6%,20) +10000(0.06) +9000
= $13,088

EAC(B) = (120000-20000)(A/P,6%,40) + 20000(0.06) + 6000
= $13,846

Project A is preferred because of its lower annual equivalent cost.

b) Determine which alternative is preferred if a perpetual service life is assumed?

PW(A) = 13,088/0.06 = $218,133

PW(B) = 13,846/0.06 = $230,767

Therefore project A is the preferred alternative.

2.20 Johnny Paycheck has $10,000 to invest for a 5-year period.. His financial planner develops four investment opportunities for him.

Option 1: Johnny would deposit the $10,000 in a bank account that pays interest at 6% compounded monthly.

Option 2: Johnny would lend the money to his brother and receive $2,300 per year for 5 years.

Option 3: Johnny would buy shares of the GetRich Company which are expected to grow at 12% per year for 5 years.

Option 4: Johnny would purchase Canada Savings Bonds that have a face value of $10,000. The bonds would pay $450 in interest every 6 months for 5 years and Johnny would get his $10,000 back (no risk) in 5 years.

a) Using the present worth method (with a 10% MARR), help Johnny to select the best option (i.e., to maximize the return on his $10,000 investment).

PW(Option 1) = 10,000[F/P,(6%/12),60](P/F,10%,5) = $8375
PW(Option 2) = 2,300(P/A,10%,5) = $8719
PW(Option 3) = 10,000(P/C,i%,g%,N)
= 10,000(P/C,10%,12%,5) = $47,138
PW(Option 4) = -10,000 + 900(P/A,10,5) +10,000(P/F,10%,5)
= $9621

b) Using the future worth method (with a 10% MARR), help Johnny to select the best option (to maximize the return on his $10,000 investment).

FW(Option 1) = 10,000(F/P,(6%/12),60) = $13,488
FW(Option 2) = 2,300(F/A,10%,5) = $14,042
FW(Option 3) = 10,000(P/C,i%,g%,N)(F/P,10%,5) =

10,000(P/C,10%, 12%,5)(F/P,10%,5) = $75,916
FW(Option 4) = -10,000(F/P,10%,5) + 450(F/A,%,10) + 10,000
= $15,495

2.21 You will be renovating a major downtown office building over the next three years. Due to our cold Ottawa winters, you will a heating system in a temporary building used as your Aoffice and storage during the renovating contract. You have two options: install i) electric baseboard heating or 2)a natural gas furnace. The costs are as follows:

Cost	Electric Heating	Natural Gas Heating
Investment Cost ($)	12,000	10,000
Service Life (Years)	3	3
Salvage value ($)	1,000	-500
Annual Operating Cost ($)	4,000	3,500
Miscellaneous Expenses ($)	500	900

a) Compare the alternatives using the present worth method if MARR=10%

Note: Assume that costs are positive (to avoid the excessive use of negative values).

PW(Elec) = 12,000-1000(P/F,10%,3)+[4000+500](P/A,10%,3)
= $22,440

PW(Gas) = 10,000+500(P/F,10%,3)+[3,500+900](P/A,10%,3)
= $21,318

b) Compare the alternatives using the future worth method if MARR=10%

Note: Assume that costs are positive (to avoid the excessive use of negative values).

```
FW(Elec)= -12,000(F/P,10%,3)+1000+[4000+500](F/A,10%,3)
          =$29,868
FW(Gas) = -10,000(F/P,10%,3)-500+[3,500+900](F/A,10%,3)
          =$28,374
```

3.1 Consider the following investment decision.

Details	Machine A	Machine B
First Cost ($)	15,000	20,000
Economic Life (Years)	5	10
Annual Revenues ($)	12,000	14,000
Annual Operating Costs ($)	6,000	8,000
Salvage Value ($)	1,500	5,000
MARR (%)	10	10

Please note: Unequal lives (durations) are not important when applying the Annual Equivalent Method as long as the projects can be repeated indefinitely with their initial parameters.

Give the equations required to calculate the Annual Equivalent of each machine.

Annual Worth (A):
= -15,000(A/P,10%,5)+12,000 - 6,000 + 1,500(A/F,10%,5)
= $2,288.73

Annual Worth (B):
= -20,000(A/P,10%,10)+14,000-8,000+ 5,000(A/F,10%,10)
= $3,058.82

3.2 If you deposit $2,000 today, $3,000 4 years from today, and $5,000 8 years from today, how much could you withdraw each year starting 12 years from today and ending in 20 years if i=10% compounded annually?

Find the present worth at t=0 (as your reference point)
PW = 2000 + 3000(P/F,10%,4) + 5000(P/F,10%,8)
PW = 2000 + 3000(0.68302) + 5000(0.46651) = $ 6,381.61
Therefore the annuity is given by:
A = PW(F/P,10%,11)(A/P,10%,9) = 6381.61(2.853)(0.17364)
= $3,161.42

3.3 Find the Annual Equivalent Worth of the following cash flow (i=10%).

(End of) YEAR	CASH FLOW ($)
0	0
1	+1,000
2	+2,000
3	+3,000
4	+2,500
5	+2,000
6	+1,500
7	+1,000
8	+500

There are several ways to obtain the Present Worth of this cash flow. Only one method is presented here.

First, find the Present Worth of the first 3 terms:

Present $Worth_1$ = A(P/A,i%,N) + G(P/G,i%,N)
= 1,000(P/A,10%,3) + 1,000(P/G,10%,3) = $ 4,815.80

Spread this present worth across ALL eight terms:

AE_1 = Present $Worth_1$(A/P,10%,8) = $ 902.72

Then find the Present Worth of the last five terms:

Present $Worth_2$ = A(P/A,i%,N) + G(P/G,i%,N)
= [2,500(P/A,10%,5) - 500(P/G,10%,5)](P/F,10%,3)
= $ 6,899.28

Note that it is necessary to move the Present $Worth_2$ from

t=3 to the present (t=0)
Spread this present worth across ALL eight terms:

AE_2 = Present $Worth_2$(A/P,10%,8) = $ 1,293.27

Therefore, the Annual Worth is $AE_1 + AE_2$ = $ 2,195.99

3.4 Find the Annual Equivalent Worth of the following cash flows (i=10%). Select the best (if any) project if A, B and C are mutually exclusive. Repeat the selection process if A, B and C are independent projects.

Parameters	Project A	Project B	Project C
First Cost ($)	10,000	18,000	30,000
Salvage Value ($)	0	-1,000	3,000
Life (Years)	10	10	20
Net Annual Profits (Annual operating revenues minus annual operating costs)($)	2,000	3,850	$2,800 at the end of the first year followed by annual increases of $400 (i.e., $3,200 at the end of the 2nd year; $3,600 at the end of the 3rd year, etc.)
MARR (%)	10	10	10

Please note: Unequal lives (durations) are not important

when applying the Annual Equivalent Method as long as the projects can be repeated with the same parameters.

ANNUAL EQUIVALENT (AE) METHOD

Project A:

AE = -P(A/P,i%,N) + A + SV(A/F,i%,N)
= -10,000(A/P,10%,10) + 2,000 + 0(A/F,10%,10)
= $372.55

Project B:

AE = -P(A/P,i%,N) + A + SV(A/F,i%,N)
= -18,000(A/P,10%,10) + 3,850 - 1,000(A/F,10%,10)
= $857.84

Project C:

AE = -P(A/P,i%,N) + A + G(A/G,i%,N) + SV(A/F,i%,N)
= -30,000(A/P,10%,20) + 2,800 + 400(A/G,10%,20) + + 3,000(A/F,10%,20)
= $1,931.82

Decision: If Projects A, B and C are independent, select the three projects because their non-negative AEs.

If projects A, B and C are mutually exclusive, select Project C (the largest Project) because it has the largest non-negative AE.

3.5 What are the different expressions of the annual equivalent cost measure?

a) AE = P(A/P,i%,N)-SV(A/F,i%,N)+ annual operating costs

From (A/P,i%,N) = (A/F,i%,N) + i%

b) AE = (P-SV)(A/P,i%,N)+i%SV + annual operating costs

c) AE = (P-SV)(A/F,i%,N)+i%P + annual operating costs

3.6 <u>Capital rationing</u> - This situation covers investment decisions with budget constraints that prevent the funding (implementation) of all projects with a positive net single worth (present and future values) or internal rates of return that exceed the MARR.

A business person is considering the three (3) following investments. Determine each project=s net annual equivalent worth assuming that MARR=10%.

Year	Project A	Project B	Project C
0	-$2000	-$9000	-$15000
1	1600	4000	6500
2	1600	4000	6500
3	1600	4000	6500
Net Annual Worth	**$796**	**$381**	**$468**

Suppose that this business person has $15,000 to spend on the projects. Indicate the feasible and the optimal investments.

Possible Decisions	Individual or Project Combinations (A,B,C)	Funds Required ($)	Residual Budget ($)	Net Annual Worth ($)
1*	0 0 0	0	15000	0
2*	1 0 0	2000	13000	796
3*	0 1 0	9000	6000	381
4*	0 0 1	15000	0	468
5*=*	***1 1 0***	***11000***	***4000***	***1177***
6**	1 0 1	17000	-2000	1264

7**	0 1 1	24000	-9000	850
8**	1 1 1	26000	-11000	1646
* = feasible; ** = not feasible; *** = optimal				

3.7 Mary, an engineer, is planning for a $100,000 retirement fund in 10 years from now. If she finds a fund that pays 8% compounded annually, what must be her annual deposit for 10 years (at the end of each year beginning one year from now) to reach her retirement goal?

The annual amount is
A = 100000(A/F,8%,10) = 100000(0.069) = $6,900

3.8 Mary, an engineer, is planning for a $100,000 retirement fund in 10 years from now. If she finds a fund that pays 8% compounded annually, what must be the value of each of ten annual deposits if the first deposit is made NOW?

The annual amount is
100000 = A(F/A,8%,9) + A(F/P,8,10)
= A(12.487+2.1589) = A(14.6459)
Therefore A = $6,827.85

Please note that the annual deposit is smaller in problem 3.8 ($6,827.85) than it is in problem 3.7 ($6,900) because of the cumulative interest income generated by the earlier deposit (i.e., at the beginning instead of at the end of the first year).

3.9 You will be renovating a major downtown office building over the next three years. Due to our cold Ottawa winters, you will a heating system in a temporary building used as your Aoffice and storage during the renovating contract. You have two options: install electric baseboard heating or a use natural gas furnace. The costs are as follows:

Cost	Electric Baseboards	Natural Gas Heating
Investment Cost ($)	12,000	10,000
Service Life (Years)	3	3
Salvage value ($)	1,000	-500
Annual Operating Cost ($)	4,000	3,500
Miscellaneous Expenses ($)	500	900

Compare the alternatives using the annual equivalent worth method if MARR=10%

Note: Assume that costs are positive (to avoid the excessive use of negative values).

AEW(Elec) = 12,000(A/P,10%,3)- 1000(A/F,10%,3)+[4000+500]
= $9,023

AEW(Gas) = 10,000(A/P,10%,3)+500(A/F,10%,3)+[3,500+900]
= $8,572

3.10 A couple bought furniture worth $40,000 on the day of their wedding. Ten years later (exactly), the couple split. The furniture was now worth only $2,000. If MARR=10%, determine the annual equivalent cost of the furniture using three different approaches.

EAC(1) = P(A/P,i%,N) - SV(A/F,i%,N)
EAC(1) = 40,000(A/P,10%,10) - 2,000(A/F,10%,10)
= $6,383

Note: The two following approaches are obtained by substituting

(A/P,i%,N) = (A/F,i%,N) + i% in method 1

```
EAC (2) = (P - SV)(A/P,i%,N) + SV*i
EAC(2) = [40,000 - 2,000](A/P,10%,10) + 2,000(0.10)
          =  $6,383

EAC (3) = (P-SV)(A/F,i%,N) + P*i
EAC(3) = [40,000 - 2,000](A/F,10%,10) + 40,000(0.10)
          =  $6,383
```

4.1 The following projects are ranked in ascending order of their initial cost, and have identical lives and negligible salvage values.

Projects A to F

- **are ranked in ascending order of their first cost (P)**
- **have identical lives or duration (N)**
- **have negligible salvage values (SV=0)**

PROJECTS	**RATES OF RETURN**					
	A	**B**	**C**	**D**	**E**	**F**
A	12	-	-	-	-	-
B	20	20	-	-	-	-
C	25	18	25	-	-	-
D	22	19	15	22	-	-
E	19	16	17	12	19	-
F	14	18	16	9	6	14

a) Select projects B, C, D and E if MARR = 18% and A, B, C, D, E and F are independent.
b) Select none of the projects if MARR = 30% and A, B, C, D, E and F are independent.
c) Select all the projects if MARR = 10% and A, B, C, D, E and F are independent.
d) Select project E if MARR = 17% and A, B, C, D, E and F are mutually exclusive.
e) Select project C if MARR = 23% and A, B, C, D, E and F are mutually exclusive.
f) Select project E if MARR = 10% and A, B, C, D, E and F are mutually exclusive.
g) Select project F if MARR = 5% and A, B, C, D, E and F are mutually exclusive.

4.2 Peter purchased a 10-year bond with a market value of $700 this morning. He expects to sell this bond for $1100 in five years from now. Annual interest income of $130 is to be paid at the end of each year for 10 years.

a) Give the equation (no calculations necessary) to determine the rate of return on Peter's investment in the absence of price changes and income taxes.

Find i* such that
Present Worth of cash outflows = Present Worth of cash inflows
700 = 130(P/A,i*,5) + 1100(P/F,i*,5)
Solve for i*

b) Give the equation (no calculations necessary) to determine the rate of return on Peter's investment if there was NO INFLATION but interest income and capital gains were taxed at a rate of 50%.

Find i* such that
Present Worth of cash outflows after taxes = Present Worth of cash inflows
700 = 130(1-0.5)(P/A,i*,5)+[1100-(1100-700)(1-0.5)](P/F,i*,5)
Solve for i*

c) Give the equation (no calculation necessary) to determine the rate of return on Peter's investment if the tax rate was 50% on interest income and on capital gains and the inflation rate was 5% per cent annually.

Find i* such that
Present Worth of the real after-tax cash inflows = Present Worth of the real after-tax cash outflows
700 = 130(1-0.5)(P/F,i*,1)(P/F,5%,1)
+ 130(1-0.5)(P/F,i*,2)(P/F,5%,2)
+ 130(1-0.5)(P/F,i*,3)(P/F,5%,3)
+ 130(1-0.5)(P/F,i*,4)(P/F,5%,4)
+ {130(1-0.5)+[1100-(1100-700)(1-0.5)]}(P/F,i*,5)(P/F,5%,5)

Solve for i*

4.3 Consider the following investment decision.

Details	Machine #1	Machine #2
First Cost ($)	15,000	20,000

Economic Life (Years)	5	10
Annual Revenues ($)	12,000	14,000
Annual Operating Costs ($)	6,000	8,000
Salvage Value ($)	1,500	5,000
MARR (%)	10	10

a) Give the equation required to calculate the individual rate of return for machine #1.

Find i* such that the $PW_{(cash\ inflows)} = PW_{(cash\ outflows)}$
(12,000 - 6,000)(P/A,i*%,5) + 1,500(P/F,i*%,5) = 15,000
Solve for i* by the trial and error method.

OR

Find i* such that the $AE_{(cash\ inflows)} = AE_{(cash\ outflows)}$
12,000 - 6,000 + 1,500(A/F,i*%,5) = 15,000(A/P,i*,5)
Solve for i* by the trial and error method.

b) Give the equation required to calculate the individual rate of return for machine #2.

Find i* such that the $PW_{(cash\ inflows)} = PW_{(cash\ outflows)}$
(14,000 - 8,000)(P/A,i*%,10) + 5,000(P/F,i*%,10) = 20,000
Solve for i* by the trial and error method.

OR

Find i* such that the $AE_{(cash\ inflows)} = AE_{(cash\ outflows)}$
14,000 - 8,000 + 5,000(A/F,i*%,10) = 20,000(A/P,i*,10)
Solve for i* by the trial and error method.

c) Give the equation required to calculate the incremental Rate of Return between machines #1 and machine #2.

NOTE: Because machines #1 and #2 have unequal lives or durations, it is easier to use the Annual Equivalent (AE) approach.

Find i* such that the $AE_{(machine\ \#1)} = AE_{(machine\ \#2)}$

12,000 - 6,000 + 1,500(A/F,i*%,5) - 15,000 (A/P,i*,5)
= 14,000 - 8,000 + 5,000(A/F,i*%,10) - 20,000(A/P,i*,10)

Solve for i* by the trial and error method.

4.4 Assume the following cash flow:

Year	***Cash Flow***
0	-100
1	+20
2	+30
3	+20
4	+40
5	+40

Try i% = 10%

Present Worth (COSTS) = 100

Present Worth (REVENUES) = 20(P/F,10%,1) +30(P/F,10%,2) + 20(P/F,10%,3) + 40(P/F,10%,4)+ 40(P/F,10%,5)
= 110.16

DECISION: Wrong interest rate; PW(REVENUES) too low; try a higher interest rate

Try i*% = 15%

Present Worth (COSTS) = 100

Present Worth (REVENUES) = 20(P/F,15%,1) +30(P/F,15%,2) + 20(P/F,15%,3) +40(P/F,15%,4)+ 40(P/F,15%,5)
= -4.02

By linear interpolation i*% = 13.5%

GRAPH: Plot net present worth against the project's internal rate of return (i%)

4.5 A corporate bond was initially sold by a stockbroker to an investor for $1000. The issuing corporation promised to pay the bond holder $40 interest on the $1000 face value of the bond every 6 months, and to repay the $1000 at the end of nine years. The bond was purchased for $950.

What rate of return did the buyer expect to receive if she keeps the bond for the full nine years?

The PW of cash outflows = PW of cash inflows
950 = 40(P/A,i*,18) + 1000(P/F,i*,18)
Let i*=5%: 950 □ 883; therefore i*=5% is too high

Let i*=4%: 950 □ 999.96; therefore i*=4% is too small

By linear interpolation,
i* = 4% +(5%-4%)[49.96/(49.96+66.9)]

i* = 0.0443 or 4.43%

4.6 a) Determine the individual rate of return for projects A, B and C.

Parameters	Project A	Project B	Project C
First Cost ($)	10,000	18,000	30,000
Salvage Value ($)	0	-1,000	3,000
Life (Years)	10	10	20
Net Annual Profits (Annual operating revenues minus annual operating costs)($)	2,000	3,850	$2,800 at the end of the first year followed by annual increases of $400 (i.e., $3,200 at the end of the 2nd year; $3,600

			at the end of the 3rd year, etc.)
MARR (%)	10	10	10

For independent projects

From the Present Worth (and the Annual Equivalent) method), we can conclude that the three projects have individual rates of return greater than 10% because their PW, FW and AE exceed $0.

The actual rate for each project is shown in the following table:

Project	**Internal Rate (%)**
A	**15.1**
B	**16.6**
C	**15.3**

For mutually exclusive projects: Use the incremental method

	INTERNAL RATES OF RETURN (%)		
	PROJECTS		
Projects	**A**	**B**	**C**
A	15.1	--	--
B	18.5	16.6	--
C	15.6	14.9	15.5

Decision: With a 10% MARR, the best project is "C" (which is the same decision from the use of the Present Worth and Annual Equivalent Methods in earlier sections).

4.7 You plan on purchasing a new car worth $35,000. You will make a $10,000 down payment and use the dealer=s offer to finance the remainder of the purchase price. The car salesperson calculates your monthly payments to be $1,100 for 48 months. Give three ways to calculate the rate on the loan from the car manufacturer.

a) Use the Present Worth (PW) Approach

Let i* be the interest rate that equates the PW (Loan Received) to the PW (Cash Outflows).

$$25{,}000 = 1100(P/A,i^*,48)$$

Solve for i*, where i* is a monthly rate. To obtain the nominal (annual) rate on the loan, multiply i* by 12.

b) Use the Future Worth (FW) Approach

Let i* be the interest rate that equates the FW (Loan Received) to the FW (Cash Outflows).

$$25{,}000\,(F/P,i^*,48) = 1100(F/A,i^*,48)$$

Solve for i*, where i* is a monthly rate. To obtain the nominal (annual) rate on the loan, multiply i* by 12.

c) Use the Equivalent Annual Worth (EAW) Approach
Let i* be the interest rate that equates the EAW(Loan Received) to the EAW (Cash Outflows).

$$25{,}000(A/P,i^*,48) = 1100$$

Solve for i*, where i* is a monthly rate. To obtain the nominal (annual) rate on the loan, multiply i* by 12. Please note that the three methods will generate the same rate of interest (i*).

5.1 Consider the following investment decision.

DETAILS	***PROJECT A***	***PROJECT B***
First Cost($)	30,000	40,000
Economic Life (years)	5	10
Annual Revenues ($)	20,000	12,000
Annual operating cost ($)	3,000 the first year followed by annual increases of 5% (i.e., 3,000 in year 1, $3,150 in year 2 ..)	3,000 in the first year followed by annual decreases of $100 (e.g., 3,000 in year 1; 2,900 in year 2 ..)
Salvage Value ($)	0	-5,000
MARR (%)	10	10

a) Project A's external rate of return is found from

a) -30,000(F/P,i*,5)-20,000(F/A,i*,5)+[{3,000/(0.1-0.05)}{(1-(1.1/1.05)^5 }] = 0
b) -30,000(F/P,i*,5)+20,000(F/A,10%,5)-[{3,000/(0.1-0.05)}{(1-(1.1/1.05)^5 }] (F/P,10%,5) = 0
c) [-30,000 + 20,000- [{3,000/(0.1- i*)}{(1-(1.1/1+i*)^5 }]](F/P,10%,5) = 0
d) -30,000+20,000-[{3,000/(0.1-0.05)}{(1-(1.1/1.05)^5 }] (A/F,105,5) = 0
e) None of the above answers.

b) Project B's external rate of return is found from

a) -40,000(A/P,i*,10)+12,000-3,000-100(A/G,i*,10)-5,000(A/P,10%,10) = 0
b) -40,000(A/F,10%,10)+12,000-3,000+100(A/F,10%,10)-5,000(A/F,10%,10) = 0

c) -40,000+12,000-3,000+100(A/F,i*,10)-5,000(A/P,i*,10) = 0
d) -40,000(F/P,i*,10)+[12,000-3,000](F/A,10%,10)+100(F/G,10%,10)-5,000 = 0
e) None of the above answers.

5.2 Given:

DETAILS	***PROJECT A***	***PROJECT B***
First Cost($)	15,000	20,000
Economic Life (years)	5	10
Annual Revenues ($)	12,000	14,000
Annual operating cost ($)	6,000	8,000
Salvage Value ($)	1,500	5,000
MARR (%)	10	10

a) Find the external rate of return for machine #1.

Find i* such that the $FW_{(cash\ inflows)} = FW_{(cash\ outflows)}$
(12,000 - 6,000)(F/A,10%,5) + 1,500 = 15,000 (F/P,i*,5)
Solve directly for i*; i* = 0.205 or 20.5%
Decision: Machine #1 is a valid investment because its individual external rate exceeds the MARR.

b) Find the external rate of return for machine #2.

Find i* such that the $FW_{(cash\ inflows)} = FW_{(cash\ outflows)}$
(14,000 - 8,000)(F/A,10%,10) + 5,000 = 20,000
Solve directly for i*; i* = 0.175 or 17.5%
Decision: Machine #2 is a valid investment because its individual external rate exceeds the MARR.

c) Find the incremental external rate of return between machines #1 and machine #2.

NOTE: Beware that machines #1 and #2 have unequal lives and that you will be using a single sum approach.

Find i* such that the $FW_{(machine\ \#1)} = FW_{(machine\ \#2)}$

(12,000 - 6,000)(F/A,10%,10) + 1,500[1 + (F/P,10%,5)] - 15,000 [1 + (P/F,10%,5)] (F/P,i*,10)
= (14,000 - 8,000)(F/A,10%,10) + 5,000 - 20,000(F/P,i*,10)
Solve for i*: i* is a negative rate. Therefore, select the machine with the smaller rate of return i.e., Machine #1

5.2 Find the external rate of return for projects A, B and C, described below.

Parameters	Project A	Project B	Project C
First Cost ($)	10,000	18,000	30,000
Salvage Value ($)	0	-1,000	3,000
Life (Years)	10	10	20
Net Annual Profits ($)	2,000	3,850	$2,200 at the end of the first year followed by annual increases of $400 (i.e., $2,600 at the end of the 2nd year; $3,000 at the end of the 3rd year,

			etc.)
MARR (%)	10	10	10

Project A

Find i* such that the $FW_{(cash\ inflows)} = FW_{(cash\ outflows)}$
2,000(F/A,10%,10) + 0 = 10,000 (F/P,i*,10)
2,000(15.932) + 0 = 10,000 (F/P,i*,10)
Solve directly for i*; i* = 0.123 or 12.3%
Decision: Project A is a valid investment

Project B

Find i* such that the $FW_{(cash\ inflows)} = FW_{(cash\ outflows)}$
3,850(F/A,10%,10) - 1,000 = 18,000 (F/P,i*,10)
3,850(15.932) - 1,000 = 18,000 (F/P,i*,10)
Solve directly for i*; i* = 0.129 or 12.9%
Decision: Project B is a valid investment

Project C

Find i* such that the $FW_{(cash\ inflows)} = FW_{(cash\ outflows)}$
2,200(F/A,10%,20)+400(F/G,10%,20)+3,000 = 30,000(F/P,i*,20)
2,200(57.273)+400(372.7327)+3,000 = 30,000(F/P,i*,20)
Solve directly for i*; i* = 0.117 or 11.7%
Decision: Project C is a valid investment

DECISION: If projects A, B and C are independent, select all three projects.

b) If A, B and C are mutually exclusive projects, which project(if any) is best?

Compare the two smallest projects, A and B

Find i* such that the $FW_A = FW_B$

2,000(F/A,10%,10)+0 - 10,000(F/P,i*,10) = 3,850(F/A,10%,10)
-1,000 - 18,000 (F/P,i*,10)
Solve for the incremental external rate i*;
i* = 0.135 or 13.5%

Since the incremental external rate of return exceeds the MARR (13.% versus 10%), select B i.e., the bigger project between A and B.

Please that this step was not required as Project A=s (the smaller project) external rate was smaller than Project B=s (the bigger project) external rate; from this comparison, you could have concluded that the larger project (with the higher rate) was the better project.

Now compare B and C (over a 20-year period)

Find i* such that the $FW_B = FW_C$
3,850(F/A,10%,20) - 1,000[1+(F/P,10%,10)]-
18,000[1+(P/F,10%,10)](F/P,i*,20)
= 2,200(F/A,10%,20)+400(P/G,10%,20)+3,000-30,000(F/P,i*,20)

Solve for the incremental external rate i*; i* = 0.13 or 13%
Since the incremental external rate of return exceeds the MARR (13% versus 10%), select C i.e., the bigger project between C and B

6.1 Calculate the payback period (N_A*,N_B* and N_C*) for projects A, B and C using the simple payback method.

Parameters	Project A	Project B	Project C
First Cost ($)	10,000	18,000	30,000
Salvage Value ($)	0	-1,000	3,000
Life (Years)	10	10	20
Net Annual Profits (Annual operating revenues minus annual operating costs)($)	2,500	3,850	$2,800 at the end of the first year followed by annual increases of $400 (i.e., $3,200 at the end of the 2nd year; $3,600 at the end of the 3rd year, etc.)
MARR (%)	10	10	10
Please note: The industry threshold or standard for projects A, B and C is 4 years.			

SIMPLE PAYBACK

Project A

YEAR	CASH FLOW	PROJECT BALANCE

YEAR	CASH FLOW	PROJECT BALANCE
0	-10000	-10000
1	2500	-7500
2	2500	-5000
3	2500	-2500
4	2500	0
5	2500	2500
6	2500	5000
7	2500	7500

The recovery period is 10,000/2,500 = 4 years

Decision: Project A is valid because its recovery period is equal to the industry threshold of 4 years.

Project B

YEAR	CASH FLOW	PROJECT BALANCE
0	-18000	-18000
1	3850	-14150
2	3850	-10300
3	3850	-6450
4	3850	-2600
5	3850	1250
6	3850	5100
7	3850	8950
8	3850	12800
9	3850	16650

The recovery period is 4 + [2600/(2600+1250)] = 4.67 years

Decision: Project B is not valid because its recovery period is longer than the industry threshold of 4 years.

Project C

YEAR	CASH FLOW	PROJECT BALANCE
0	-30000	-30000
1	2800	-27200
2	3200	-24000
3	3600	-20400
4	4000	-16400
5	4400	-12000
6	4800	-7200
7	5200	-2000
8	5600	3600
9	6000	
10	6400	
11	6800	

The recovery period is 7 + [2000/(2000+3600)] = 7.4 years

Decision: Project C is not valid because its recovery period is longer than the industry threshold of 4 years.

DISCOUNTED PAYBACK

Project A

YEAR	CASH FLOW	OPPORTUNITY COST (10% discount rate)	PROJECT BALANCE
0	-10000	--	-10000
1	2500	-1000	-8500
2	2500	-850	-6850
3	2500	-685	-5035
4	2500	-503.5	-3038.5
5	**2500**	**-303.85**	**-842.35**
6	**2500**	**-84.24**	**1573.41**

The recovery period is 5 + 842.35/[842.35+1573.41] = 5.35 years (which is longer than the industry threshold of 4 years)

Decision: Project A is not valid because its recovery period is longer than the industry threshold of 4 years.

Project B

YEAR	CASH FLOW	OPPORTUNITY COST (10% discount rate)	PROJECT BALANCE
0	-18000	--	-18000
1	3850	-1800	-15950.00
2	3850	-1595	-13695.00
3	3850	-1369.50	-11214.50
4	3850	-1121.45	-8485.95
5	3850	-845.60	-5484.55

6	**3850**	**-548.60**	**-2183**
7	**3850**	**-218.30**	**1448.70**
8	3850	144.87	5443.57

The recovery period is 6 + [2183/(2183+1448.7)]
= 6.60 years

Decision: Project B is not valid because its recovery period is longer than the industry threshold of 4 years.

Project C

YEAR	CASH FLOW	OPPORTUNITY COST (10% discount rate)	PROJECT BALANCE
0	-30000	--	-30000
1	2800	-3000	-30200
2	3200	-3020	-30020
3	3600	-3002	-29422
4	4000	-2942.20	-28364.20
5	4400	-2836.42	-26800.62
6	4800	-2680.06	-24680.68
7	5200	-2468.07	-21948.75
8	5600	-2194.88	-18543.63
9	6000	-1854.36	-14397.99
10	6400	-1439.8	-9437.79
11	**6800**	**-943.78**	**-3581.57**

12	**7200**	**-358.16**	**3260.28**

The recovery period is 11 + [3581.6/(3260.28+3581.57)]
= 11.52 years

Decision: Project C is not valid because its recovery period is longer than the industry threshold of 4 years.

None of the projects is valid whether independent or mutually exclusive.