

Answer keys –these are not complete solutions

1 Domain of  $e^x$  is  $\mathbf{R}$ , range of  $e^x$  is  $y > 0$ , x intercept is none, y intercept is  $(0, 1)$

Domain of  $\ln x$  is  $x > 0$ , range of  $\ln x$  is  $\mathbf{R}$ , x intercept is  $(1, 0)$ , y intercept is none.

Their graphs can be found on the textbook. They are symmetric about the line  $y = x$  because they are inverse to each other.

$$\begin{aligned}(x-1)(x+1) &= e^2 \\ x^2 - 1 &= e^2 \\ x^2 &= 1 + e^2 \\ x &= \pm\sqrt{1 + e^2}\end{aligned}$$

However,  $x - 1 > 0$  so the only solution is

$$x = \sqrt{1 + e^2}$$

2. 1)

$$2e^{\tan 2x} \sec^2 2x$$

2)

$$\frac{\cos x}{\sin x} - \sin x \cos x$$

3)

$$\begin{aligned}&\frac{8x(x^2 + 1)^3(2x + 1)^3(3x - 1)^5 - 6(x^2 + 1)^4(2x + 1)^2(3x - 1)^5 - 15(x^2 + 1)^4(2x + 1)^3(3x - 1)^4}{(2x + 1)^6(3x - 1)^{10}} \\ &= \frac{8x(x^2 + 1)^3(2x + 1)(3x - 1) - 6(x^2 + 1)^4(3x - 1) - 15(x^2 + 1)^4(2x + 1)}{(2x + 1)^4(3x - 1)^6}\end{aligned}$$

4)

$$\begin{aligned}\arcsin y + \frac{xdy}{dx\sqrt{1-y^2}} &= \frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{\arcsin y}{1 - x/\sqrt{1-y^2}}\end{aligned}$$

31)

$$-\frac{1}{2}e^{-2x} + C$$

2)

$$\begin{aligned}u &= x, dv = \cos x dx, du = dx, v = \sin x \\ x \sin x - \int \sin x dx &= x \sin x + \cos x + C\end{aligned}$$

3)

$$4 \ln e^3 - 4 \ln 1 = 12 - 0 = 12$$

4)

$$\frac{x+2}{2x+3} = \frac{x+2}{2(x+1.5)} = \frac{x+1.5+0.5}{2(x+1.5)} = \frac{x+1.5}{2(x+1.5)} + \frac{0.5}{2x+3} = 0.5 + \frac{0.5}{2x+3}$$

$$\int 0.5 dx + \int \frac{0.5}{2x+3} dx = 0.5x + 0.5 \times 0.5 \ln |2x+3| + C = 0.5x + 0.25 \ln |2x+3| + C$$

5) one antiderivative is

$$\frac{(2t-1)^{101}}{202}$$

definite integral is

$$\frac{1^{101}}{202} - \frac{(-5)^{101}}{202} = \frac{1}{202} + \frac{5^{101}}{202}$$

4

$$v(t) = y'(t) = 6t - 5$$

units are centimeters per second.

$$a(t) = y'' = 6$$

units are centimeters per second squared

$$y = (t-2)(3t+1)$$

so  $t = 2$

5  $y$  is a parabola that opens downward. Its x intercepts are  $x = 5$  and  $x = -3$

$$y = -x^2 + 2x + 15 = -x^2 + 2x - 1 + 16 = -(x-1)^2 + 16$$

So  $y$  has vertex at  $(1, 16)$ . The vertex is the highest point on the parabola. For a general parabola

$$y = ax^2 + bx + c$$

its vertex is the point  $(-\frac{b}{2a}, c - \frac{b^2}{4a})$ . Maybe you should memorize this general formula if you are not comfortable with completing the squares method above.

The graph of the parabola in question can be obtained by reflecting the graph of  $y = x^2$  about the x axis to get  $y = -x^2$ , then shift the latter graph one unit to the right and then 16 units upward. Notice that the graph is always above the x axis on the interval  $[-3, 5]$ . So area is just equal to the definite integral.

$$\int_{-3}^5 (-x^2 + 2x + 15) dx = 85.33334$$

Average value is the integral divided by the length of the interval, so it is

$$\frac{85.3334}{8}$$

This means that definite integral gives another way of finding average. Arithmetic only allows one to compute the average of 2, 3, or finintely many numbers. 6 by the product rule

$$y' = e^{-x} - (2+x)e^{-x} = (1-2-x)e^{-x} = (-1-x)e^{-x}$$

when  $x = 0$

$$y'(0) = -1$$

so the tangent line is

$$y - 2 = -(x - 0)$$

In order for the derivative to be zero,  $-1 - x$  must be zero because the other factor,  $e^{-x}$  is positive. So the only horizontal tangent occurs when  $x = -1$ . Apparently this this related to local extrtrema or absolute extrema.