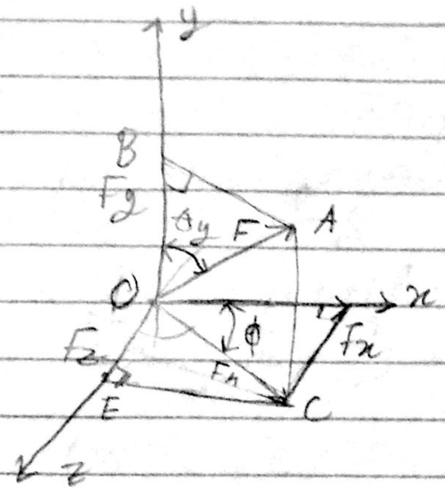
Sept 17th 2013

Forces in space



$F_y = F\cos\theta_y$; $F_h = F\sin\theta_y$

$F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$

$F_z = F_h\sin\phi = F\sin\theta_y\sin\phi$

ΔOAB , ΔOCD

By pythagoras:

$F^2 = (OA)^2 = (OB)^2 + (BA^2) = F_y^2 + F_h^2$

$F_h^2 = (OC)^2 = (OD)^2 + (DC)^2 = F_x^2 + F_z^2$

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

$$\therefore F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$F_x = F\cos\theta_x$; $F_y = F\cos\theta_y$; $F_z = F\cos\theta_z$

$\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ = cosine directions of a force.

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$\vec{F} = F\cos\theta_x \hat{i} + F\cos\theta_y \hat{j} + F\cos\theta_z \hat{k}$$
$$= F(\cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k})$$

assume that $\cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k} = \vec{\lambda}$

$\therefore \vec{F} = F\vec{\lambda}$

λ is a unit vector along $\vec{F}$

$$\vec{\lambda}^2 = \lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1$$

$$\therefore \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos\theta_x = \frac{F_x}{F} \; ; \; \cos\theta_y = \frac{F_y}{F} \; ; \; \cos\theta_z = \frac{F_z}{F}$$

$$\vec{MN} = d_x \hat{i} + d_y \hat{j} + d_z \hat{k}$$

($d_{x,y,z}$ ... are components of MN along the x, y, z axes)

$$\vec{MN} = MN\vec{\lambda}$$

$$\vec{\lambda} = \frac{\vec{MN}}{MN} = \frac{\vec{MN}}{d} = \frac{1}{d}(d_x \hat{i} + d_y \hat{j} + d_z \hat{k})$$

$$\vec{F} = F\vec{\lambda} = \frac{F}{d}(d_x \hat{i} + d_y \hat{j} + d_z \hat{k})$$

$$= F\left(\frac{d_x}{d} i + \frac{d_y}{d} \hat{j} + \frac{d_z}{d} \hat{k}\right)$$

$$\therefore F_x = \frac{F d_x}{d} \; , \; F_y = \frac{F d_y}{d} \; , \; F_z = \frac{F d_z}{d}$$

$$\therefore d = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\cos\theta_x = \frac{d_x}{d} \; ; \; \cos\theta_y = \frac{d_y}{d}$$

$$\cos\theta_z = \frac{d_z}{d}$$

addtion of concurrent forces in space

$$\vec{R} = \sum \vec{F}$$

$$R_x\hat{i} + R_y\hat{j} + R_z\hat{k} = \sum(F_x\hat{i} + F_y\hat{j} + F_z\hat{k})$$
$$= (\sum F_x)\hat{i} + (\sum F_y)\hat{j} + (\sum F_z)\hat{k})$$

$$\therefore R_x = \sum F_x \; ; \; R_y = \sum F_y \; ; \; R_z = \sum F_z$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$\cos\theta_x = \frac{R_x}{R} \; ; \; \cos\theta_y = \frac{R_y}{R} \; ; \; \cos\theta_z = \frac{R_z}{R}$$

Equil of a particular in space

$$R = \sum F = 0$$
$$\sum F_x = 0 \; ; \; \sum F_y = 0 \; ; \; \sum F_z = 0$$