Lecture 2

ECOR1101 Mechanics I

Force Vectors

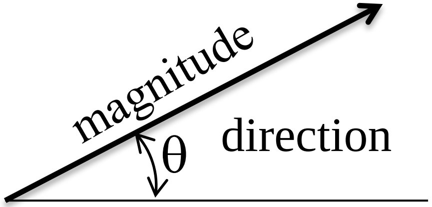
Objectives

- Among other things, learn to:
  - Vector operations (addition, subtraction of forces)
  - Resolve forces into components
  - Use the parallelogram law of vector addition
  - Express force and position vectors in Cartesian vector form
  - Determine vector magnitude and direction
  - Use the dot product of vectors to determine angle between two vectors or project one vector onto another

Scalar vs. Vectors

Engineering quantities such as length, speed, velocity, force, time are measured using either vectors or scalars

- Scalar
  - Any positive or negative physical quantity that can be fully specified by its magnitude only
  - Examples:
  - Mass
  - Length
  - Time
  - Volume
  - energy
- Vector
  - Any physical quantity that requires both a magnitude and a direction to be fully defined.
  - Examples
    - Force
    - Position
    - Moment
    - Velocity



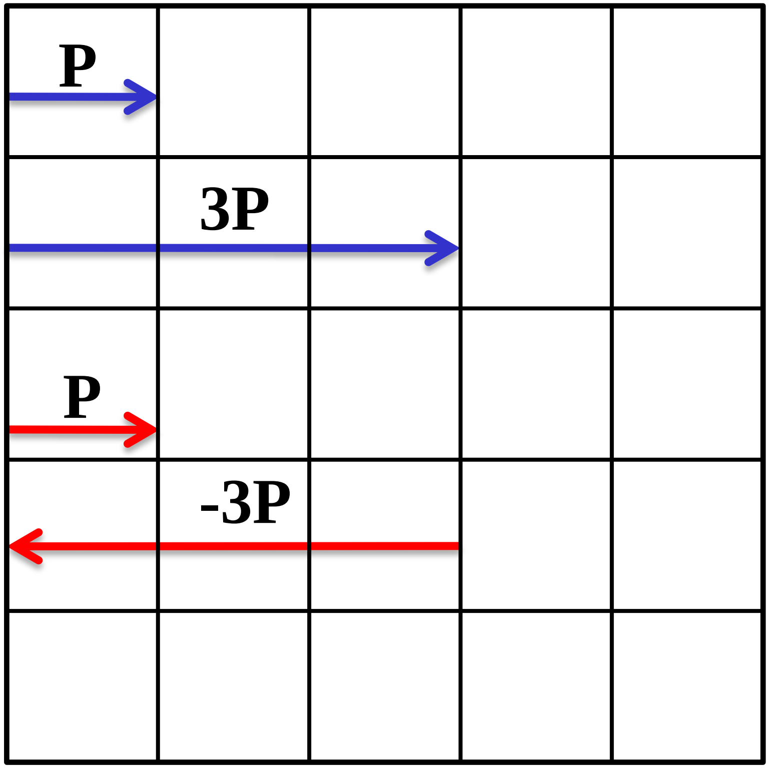
Vector Notation

- Vectors can be represented by a variety of notations:
  - Bold face type lettering (**A**)
  - Letter with an arrow over top ($\vec{A}$)
  - Underlined letters ($\underline{A}$)
  - Two letters denoting origin and end and an arrow over top ($\overrightarrow{AB}$)

Vector Operations

Multiplication of a vector by a scalar

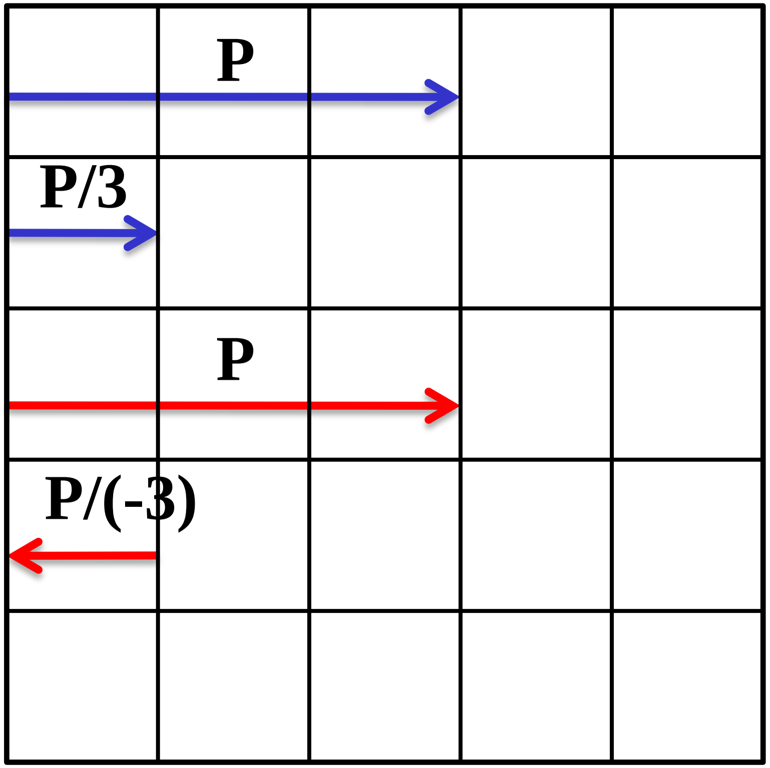
- Multiplication by +ve scalar increases magnitude by scalar value and direction remains unchanged
- Multiplication by –ve scalar increases magnitude by scalar value and changes directional sense of vector



Vector Operations

Division of a vector by a scalar

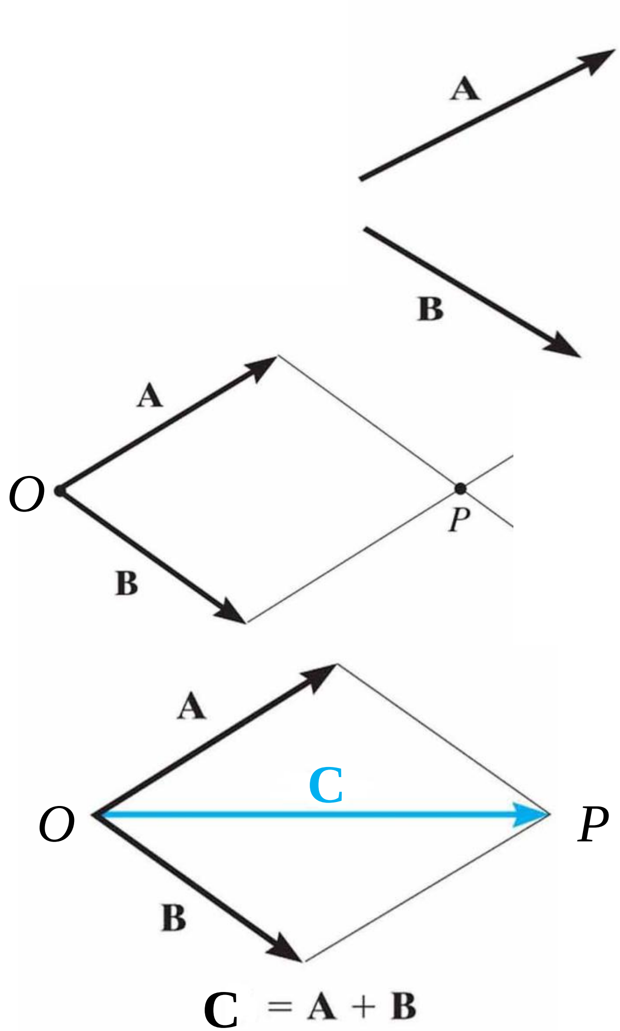
- Division by +ve scalar decreases magnitude by scalar value and direction remains unchanged
- Division by –ve scalar decreases magnitude by scalar value and changes directional sense of vector



Vector Operations

Vector Addition

- Vectors obey the parallelogram law of vector addition.
- When two vectors, **A** and **B** are added they form a resultant vector **C**.
- The general rule is:
  - To join the tails of **A** and **B** at a point (*O*) to make them concurrent
  - From head of **B** draw line parallel to **A**. Then draw a line from head of **A** parallel to **B** so they intersect at point (*P*) and form a parallelogram
  - The diagonal of the parallelogram (O-P) represents the resultant vector **C**

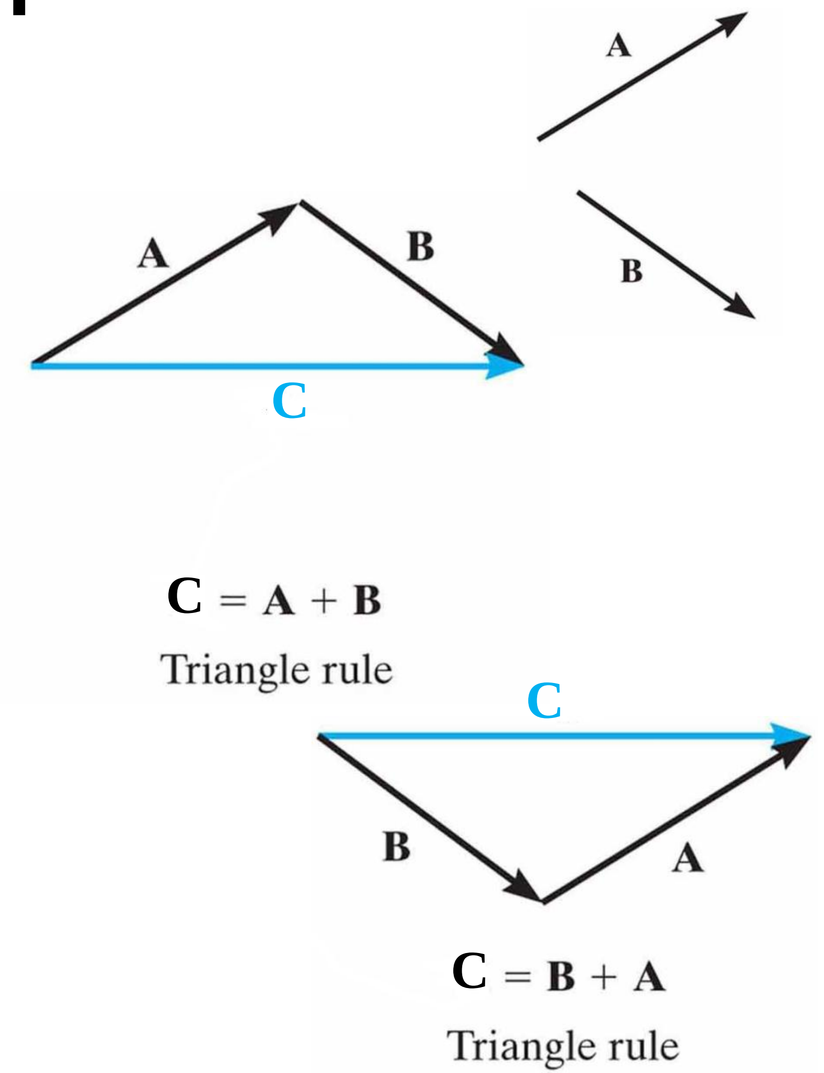


Parallelogram law

Vector Operations

Vector Addition

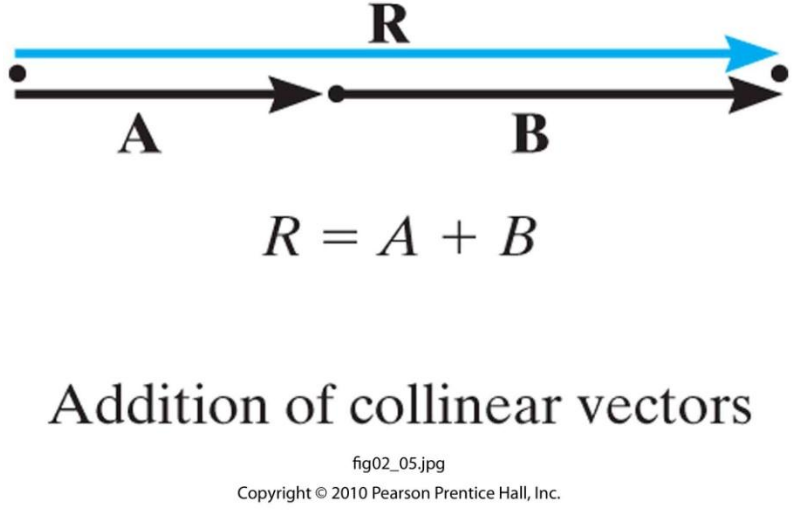
- Vectors can also be added by using the triangle rule - a special case of the parallelogram law of vector addition.
- For the two vectors, **A** and **B**, the general rule is:
  - Draw vector **A** and then from the head of **A** draw vector **B** in a "head-to-tail" fashion
  - The resultant vector **C** extends from the tail of **A** to the head of **B**.



Vector Operations

Collinear vectors

- If two vectors **A** and **B** are collinear, i.e. both have the same line of action, the parallelogram law of vector addition reduces to algebraic sum (or scalar sum) of their magnitudes.

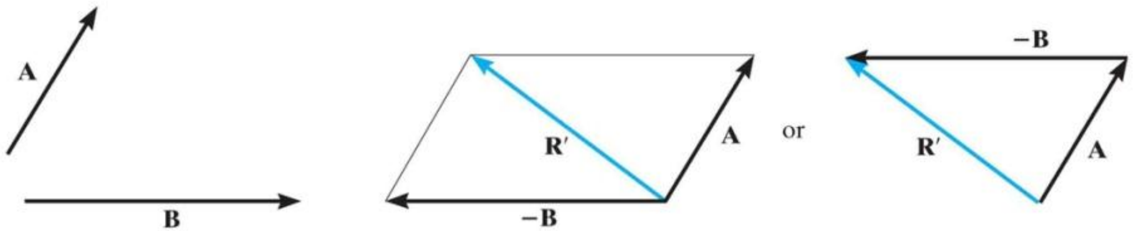


Vector Operations

Vector Subtraction

- The resultant **R′** or difference between two vectors **A** and **B** can be expressed as:

$$\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$



Parallelogram law

Triangle construction

Vector subtraction

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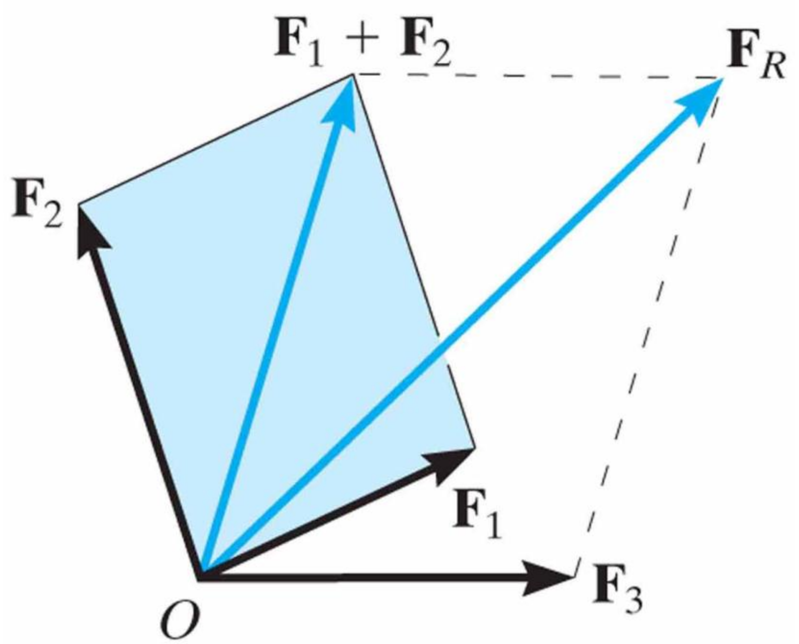
Forces

- A Force is a vector quantity.
  - It has a specified magnitude and direction as do vectors.
- In statics, we often have two **forces** (or components of a force) and are required to find their **resultant (force)**
- Or we have a **resultant force** and are required to resolve the force into its **component forces**

Addition of Several Forces

- Forces can be added and subtracted in the same manner as vector by the parallelogram law or triangle rule
- If more than two forces (vectors) are to be added, successive applications of the parallelogram law can be used.

$\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3 = ((\mathbf{F}_1+\mathbf{F}_2)+\mathbf{F}_3)$



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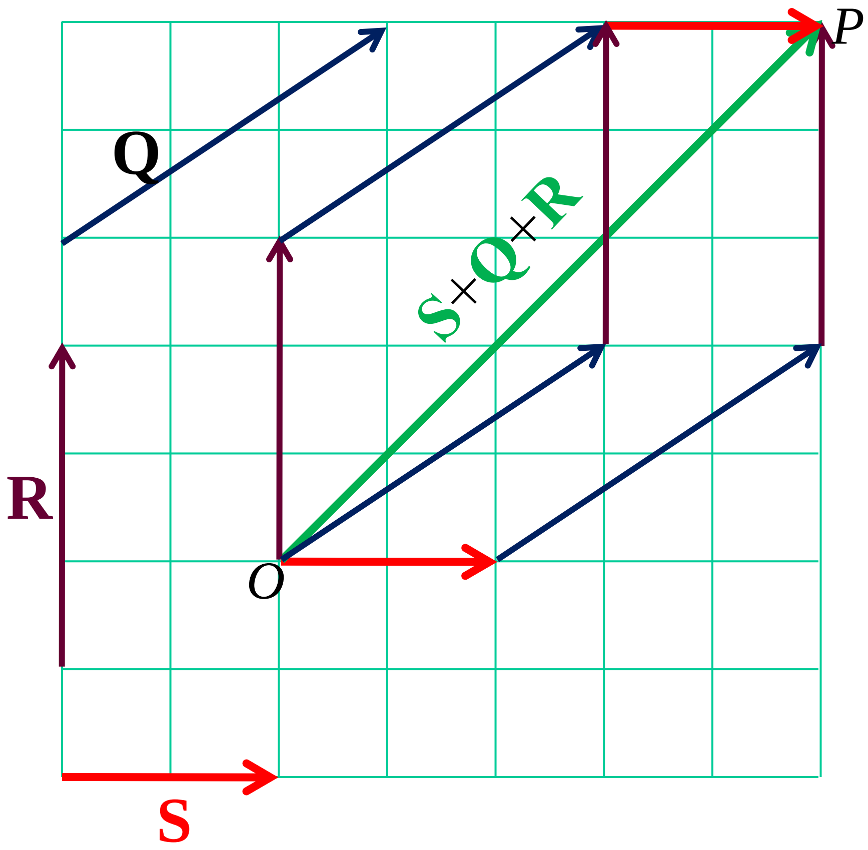
Sample Problem

Given vectors Q, R, and S. Show that:

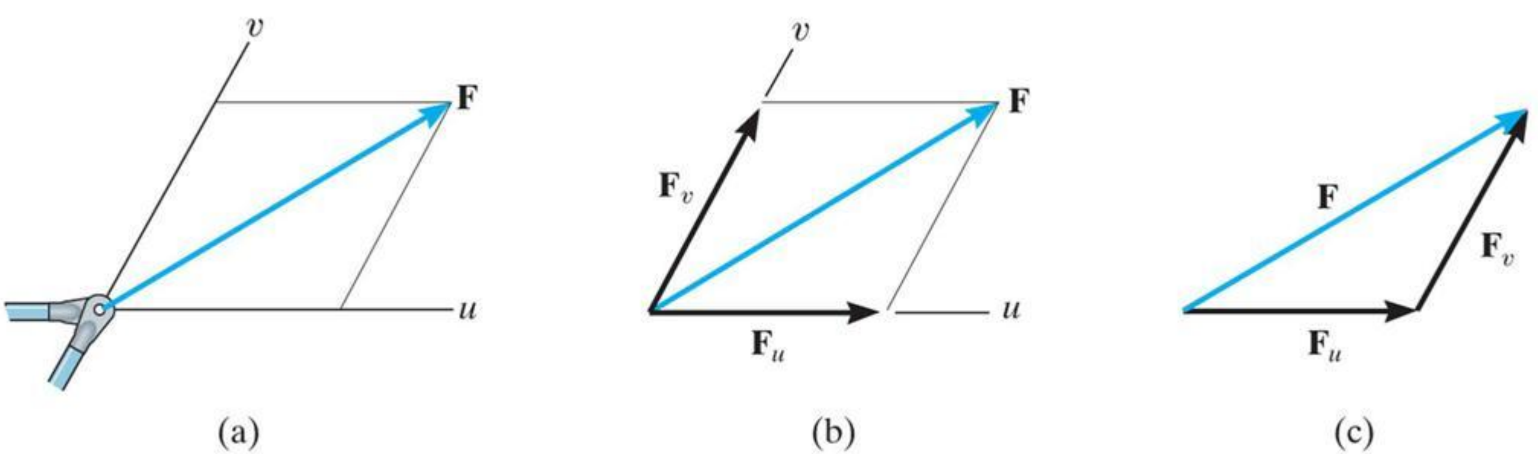
(S + Q + R) = (R + Q + S) = (Q + R + S)



The problem can be solved graphically as shown

System of Coplanar Forces

An arbitrary force, **F**, can be resolved into two coplanar components, $\mathbf{F}_u$ and $\mathbf{F}_v$, along any two axes *u* and *v* by the parallelogram law of vector addition or the triangle rule



$$\mathbf{F} = \mathbf{F}_u + \mathbf{F}_v$$

Cosine and Sine Laws

Using the triangle rule of vector addition, the sine and cosine laws can be used to determine the direction and magnitude of the resultant force, respectively.

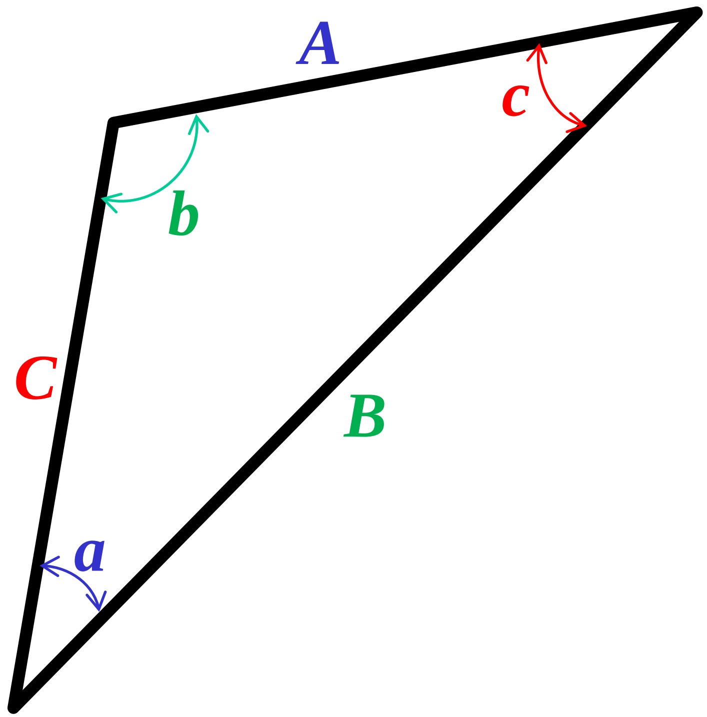
- Sine Law

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

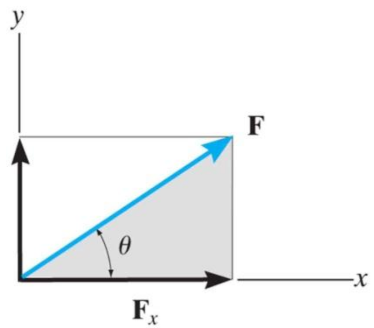
- Cosine Law

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

$$A = \sqrt{B^2 + C^2 - 2BC\cos a}$$



Scalar Notation



(a)

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$$F_x = F\cos\theta$$

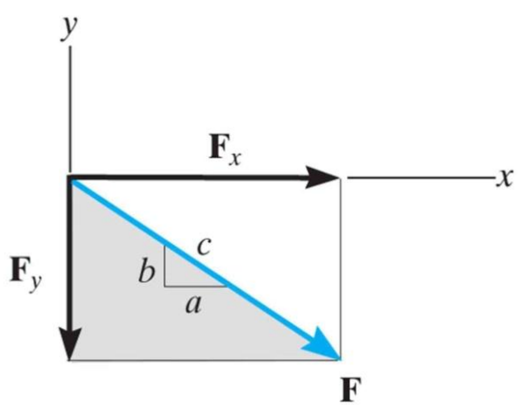
$$F_y = F\sin\theta$$

- When the forces are resolved along the *x* and *y* axes the components are called rectangular
- **Scalar Notation**

The components of the force can be expressed in Scalar Notation using the parallelogram law.

F can be resolved along *x* and *y* axes.

The magnitudes of the components can be calculated with the angle θ or by using the slope of the force **F**



(b)

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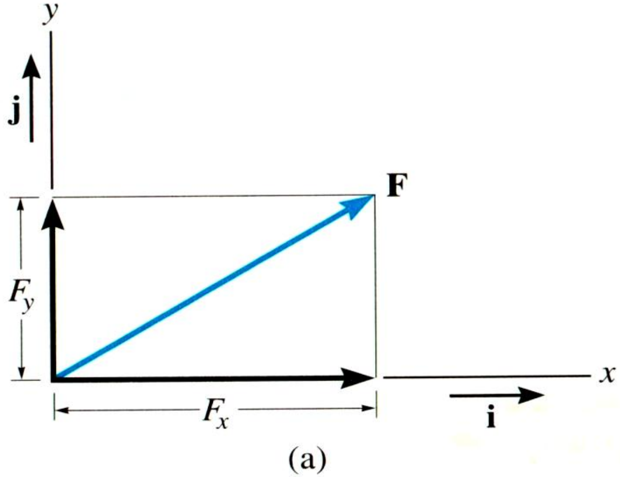
$$F_x = F\left(\frac{a}{c}\right)$$

$$F_y = -F\left(\frac{b}{c}\right)$$

Cartesian Vector Notation



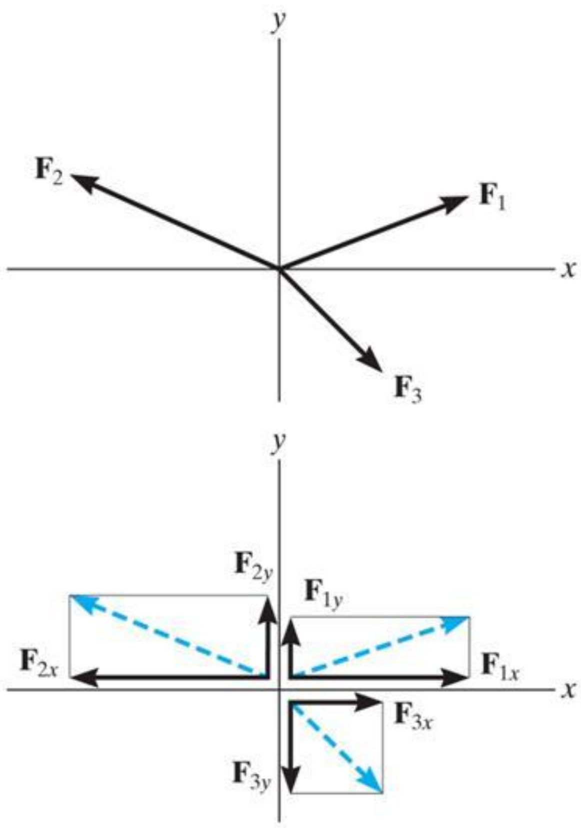
- Components of a Forces can also be expressed in Cartesian Vector Notation using unit vectors **i** and **j** in the *x* and *y* axes, respectively.
- In Cartesian Vector Notation, **F** can be expressed as a vector as ff:

  $\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$

  F_x and F_y are scalar quantities representing the magnitude of the components of the force, **F.**

  The unit vectors **i** and **j** designate the directions along *x* and *y* axes

Resultant of Coplanar Forces



- Using either the Scalar or Cartesian Vector notation, the resultant of forces can be determined
- First resolve forces into their components (rectangular)
- Using scalar algebra, sum collinear vectors

$$\mathbf{F}_R = (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} = (F_{Rx})\,\mathbf{i} + (F_{Ry})\,\mathbf{j}$$

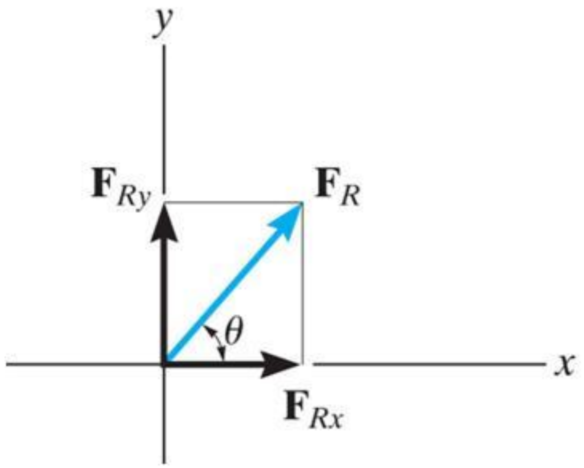
$(\overrightarrow{+})$ $\quad F_{Rx} = F_{1x} - F_{2x} + F_{3x}$

$(+\uparrow)$ $\quad F_{Ry} = F_{1y} + F_{2y} - F_{3y}$

Resultant of Coplanar Forces

- The magnitude of the resultant force is given as:

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

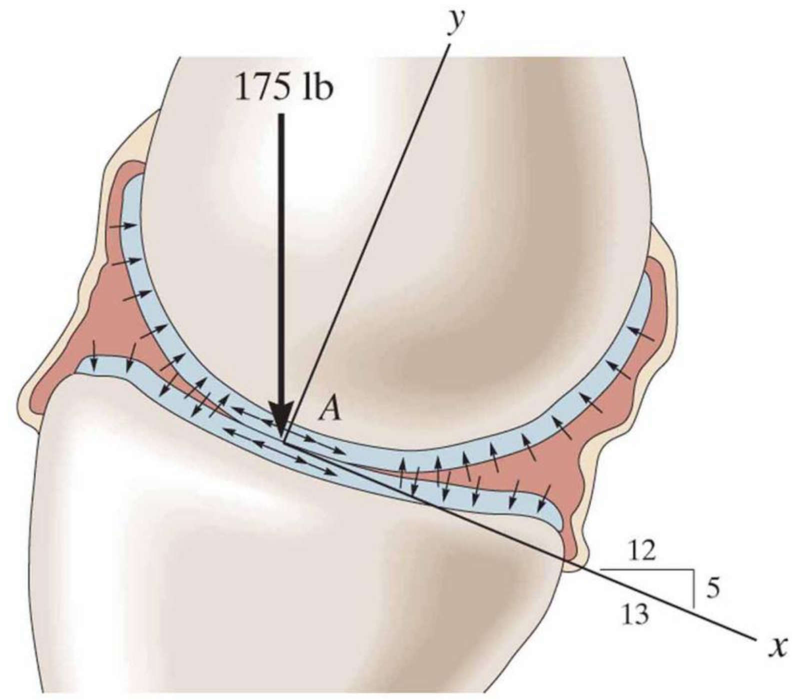


- While the direction of the resultant force (measured from +ve *x*-axis) is given as:

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right)$$

Sample Problem

2-35 The contact point between the femur and tibia bones of the leg is at A. If the vertical force of 175 lb is applied at this point, determine the components along the *x*- and y-axes



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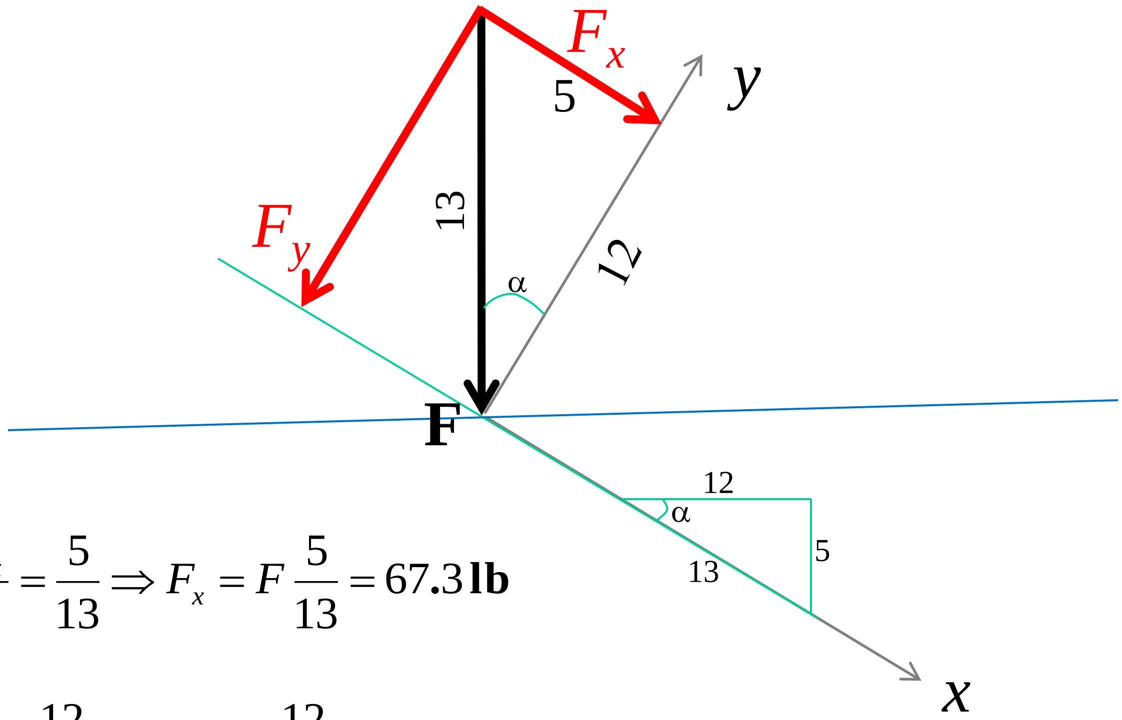
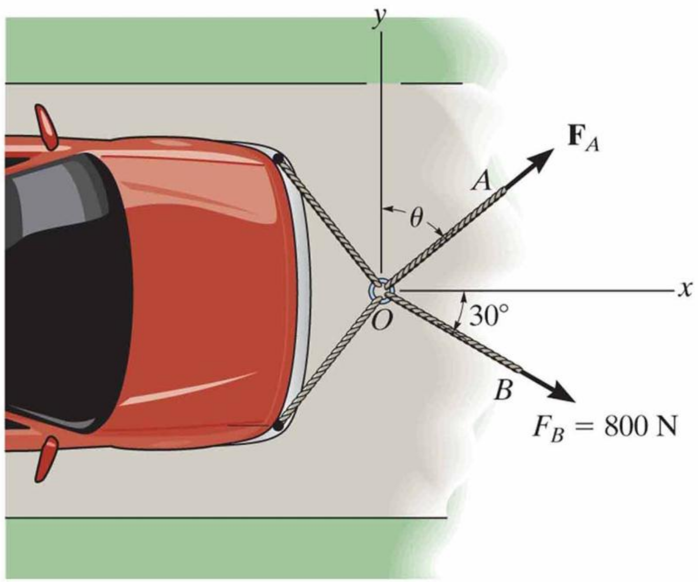


$$\frac{F_x}{F} = \frac{5}{13} \Rightarrow F_x = F\,\frac{5}{13} = 67.3\,\mathbf{lb}$$

$$\frac{F_y}{F} = \frac{12}{13} \Rightarrow F_y = F\,\frac{12}{13} = -162\ \mathbf{lb}$$

Sample Problem

2-48 Determine the magnitude and direction measured counterclockwise from the positive *x* axis of the resultant force acting on the ring at *O* if $F_A = 750$ N and $\theta = 45^{o}$.



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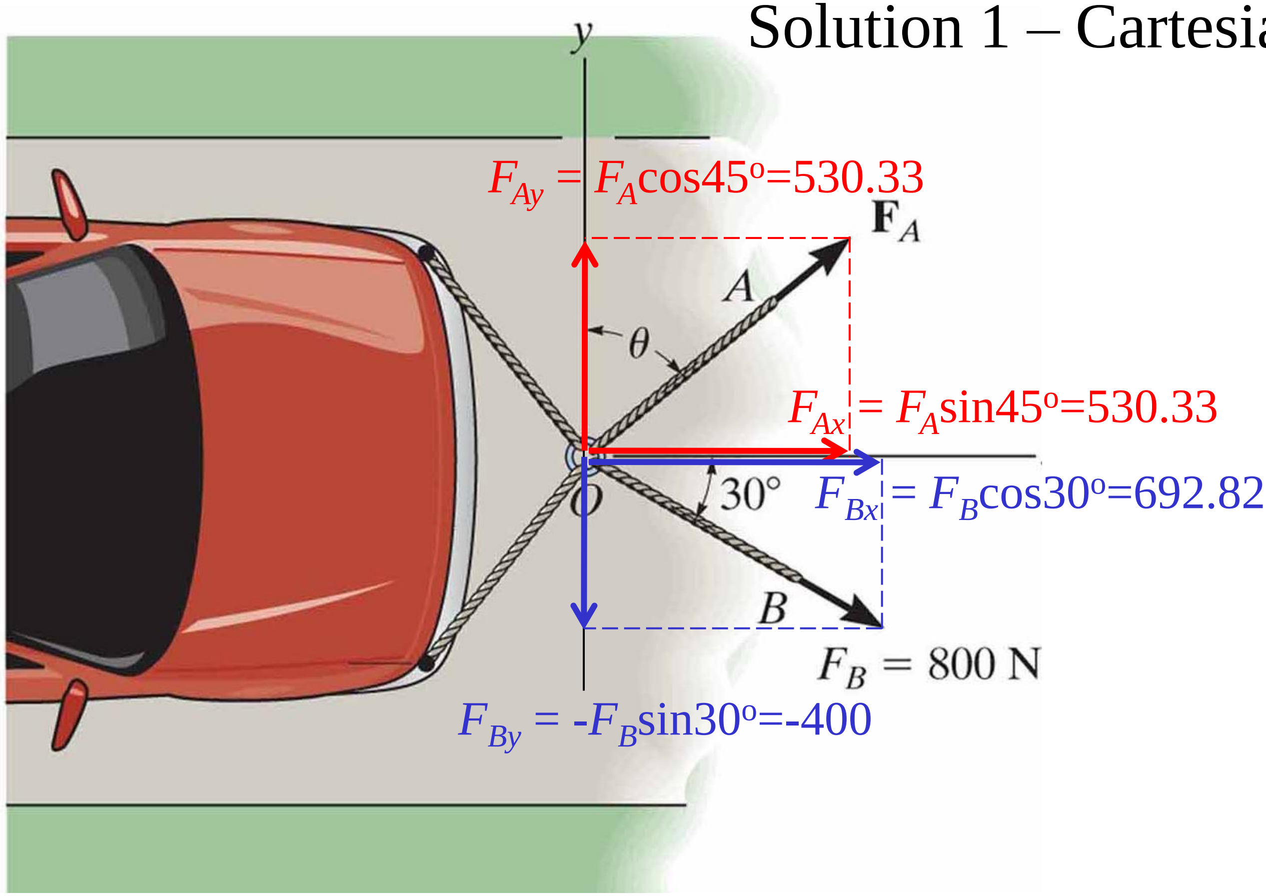
Solution 1 – Cartesian Vector Notation



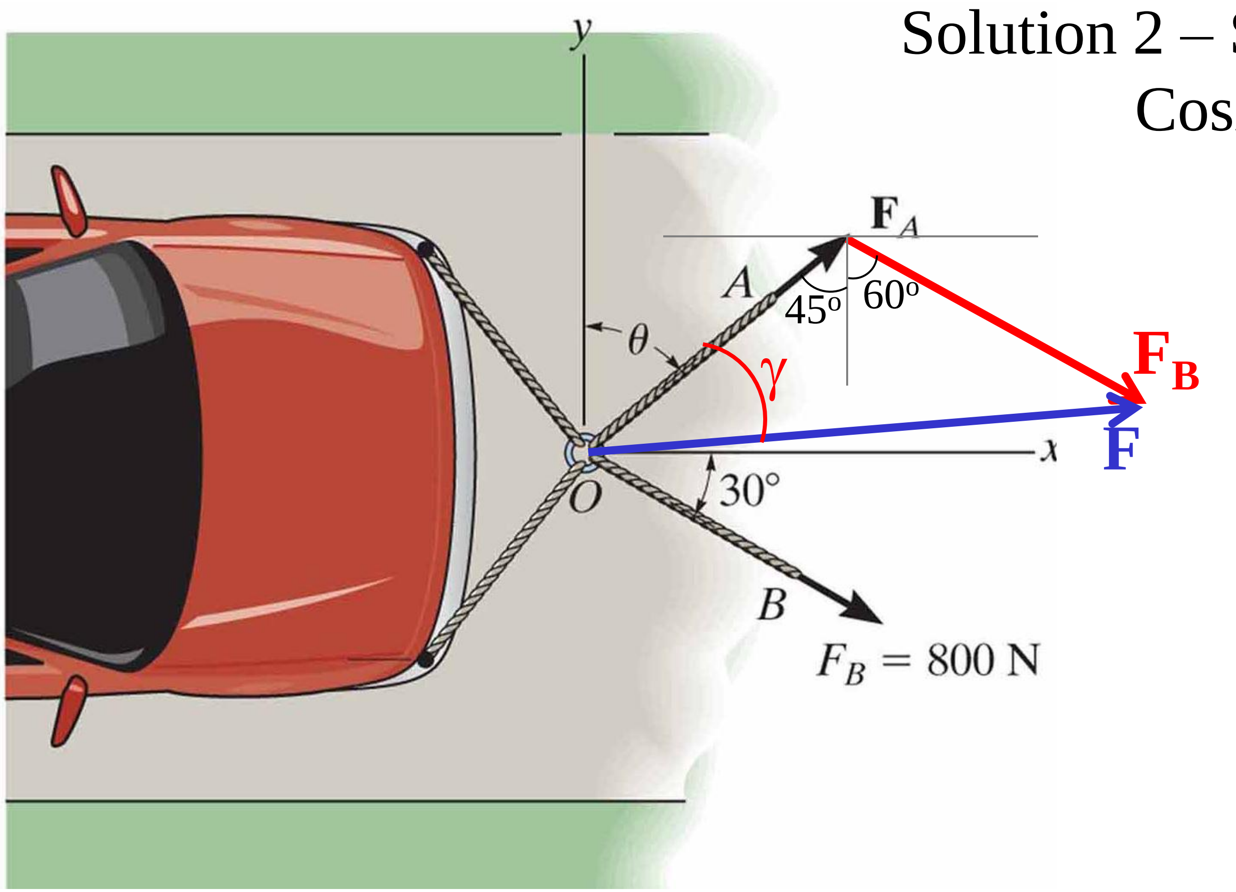
$$\mathbf{F} = \left(F_{Ax} + F_{Bx}\right)\mathbf{i} + \left(F_{Ay} + F_{By}\right)\mathbf{j}$$

$$\mathbf{F} = \left(530.33 + 692.82\right)\mathbf{i} + \left(530.33 - 400.00\right)\mathbf{j} = 1223.15\mathbf{i} + 130.33\mathbf{j}$$

$$F = \sqrt{1223.15^2 + 130.33^2} = 1230.1\ \text{N} = 1.23\times 10^3\ \text{N} = 1.23\ \text{kN}$$

$$\beta = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08^o$$

Solution 2 – Sine and Cosine Laws



$$F = \sqrt{F_A^2 + F_B^2 - 2 \cdot F_A \cdot F_B \cos 105^o} = 1230.1\,\mathbf{N} = 1.23\,\mathbf{kN}$$

$$\frac{F_B}{\sin\gamma} = \frac{F}{\sin 105^o} \Rightarrow \gamma = \arcsin\left(\frac{750 \times \sin 105^o}{1230.1}\right) = 38.92^o$$

$$\beta = 45 - 38.92 = 6.08^o$$