ADM 2304 **Winter 2010**

Solutions to Final Exam

Question 1. [8 marks]

(a) We can use a two sample t-test despite the extreme skewness of the wait times data simply because the sample sizes are so large. *Also accept the observation that the nonparametric test is safer because of the extreme skewness of the two samples—however, it is not enough to say the data may not be normally distributed.*

(b) Parametric test: $H_0: \mu_1 - \mu_2 = 0 \quad H_a: \mu_1 - \mu_2 \neq 0$

First, if we do not assume equal variance, we get:

$$t = \frac{124 - 115 - 0}{\sqrt{\frac{270^2}{1374} + \frac{217^2}{783}}} = 0.8459$$

If we assume equal variances (which is not necessary), the pooled standard deviation is:

$$s_p = \sqrt{\frac{(1374 - 1) \times 270^2 + (783 - 1) \times 217^2}{783 + 1374 - 2}} = 252.0591$$

Then the test statistic is …

Regardless, the *difference* is non-significant (critical value = 1.96) and thus there is no evidence that either hospital is getting preferential treatment.

Marks: 1 for hypotheses, 1.5 for t-statistic, .5 for decision rule with critical value, 1 for decision and conclusion (.5 for each)

(c) Non-parametric test:

1 for hypotheses (deduct .5 if they do not mention medians)

1 for p-value = .2061

1 for decision and conclusion (.5 for each)

Question 2. [5 marks]

This is a chi-square test of independence.

H0: hospitals have same breakdown of client type or client type and hospital are independent
Ha: at least one hospital has a different breakdown

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

       Private    Ward  Either  Total
    1      216     175      83    474
        146.95  187.65  139.40
        32.450   0.853  22.821

    2      275     241     147    663
        205.54  262.47  194.99
        23.474   1.757  11.810

    3      444     778     657   1879
        582.51  743.87  552.61
        32.937   1.566  19.719

Total      935    1194     887   3016

Chi-Sq = 147.387, DF = 4, P-Value = 0.000
```

Therefore, the hospitals do differ in the breakdown of client type.

Marks:

1 for hypotheses

1 for calculation of expected values and contributions to chi-square

1 for chi-square statistic and critical value of 9.49 (or p-value < .0000005)

1 for decision and conclusion

1 for giving p-value < .0000005

<u>Question 3.</u> [6 marks]

This is a Chi-square goodness of fit test so:

$H_0: p_1 = .11, p_2 = .08, p_3 = .08, p_4 = .06, p_5 = .06$ could add p(other) = .61

$H_a: at\ least\ one\ differs$

Not as good but acceptable (barely):

Ho: Preferences are consistent with capacities; Ha: Not consistent

The expected values and contributions are:

	Obs.Freq	Exp.Val	Cont.
PR	422	387.97	2.984872
GR	304	282.16	1.690479
SA	293	282.16	0.41645
PC	239	211.62	3.542503
GA	233	211.62	2.160025
Other	2036	2151.47	6.197307
	3527		16.99164

Thus the test statistic is 17. This is again significant with a critical value (5 degrees of freedom) of 11.07. Thus, there appears to be evidence that the preferences are inconsistent with the capacities.

Marks:

1 for hypotheses

1 for intermediate calculations

1 for chi-square value of 17

1 for decision rule based on critical value of 11.07 using 5 d.f. (deduct .5 if d.f. are incorrect)

1 for decision and conclusion

Part b) The capacity data are clearly rounded to the nearest decimal and thus may not be accurate enough. Because the sample size is very large even small differences of less than half a percentage point may be detected as significant. It appears that we have a classic example of something that is statistically significant but not practically significant.

Half a mark if they mention the lumping of the 23 smaller facilities into a single category does not allow the assessment of matching among the smaller facilities. However, this would not affect the conclusion.

<u>Question 4.</u> [13 Marks]

Test and CI for One Proportion

```
Test of p = 0.115 vs p > 0.115

                                 95% Lower
Sample   X    N  Sample p      Bound  Z-Value  P-Value
1       60  450  0.133333   0.106975     1.22    0.111
```

a. S1: H0: p = (p0 = 0.115) S2: α = 0.05

Ha: p > (p0 = 0.115) ➔ Right Tail Z-Crit = Z $_{\alpha = 0.05}$ = 1.645

S3: Z-Calc = $\frac{\hat{p}-p_0}{SD(\hat{p})} = \frac{0.1333-0.115}{0.01504} = 1.2168$

S4: Since {Z-Calc = 1.2168}<{Z-Crit = 1.645}➔ Do not Reject H0.

There is insufficient statistical evidence to claim that the population proportion is more than 11.5%.

1 for hypotheses, 1 for z-statistic, 1 for decision and conclusion.

b. For the above test, we assume that ***the sample proportion*** is normally distributed. To check if this is valid, we check if ***n x p0 = 450 x 0.115 = 51.75 is more than 10 (or 5)***.

c. One-sided Confidence interval for a Right Tail test is the Lower Bound, and it is:

$$\hat{p} - z_{\alpha=0.05} SE(\hat{p}) = 0.1333 - 1.645\sqrt{\frac{0.1333(1-.1333)}{450}} = 0.1333 - 1.645(0.0160)$$

Lower Bound: 0.1069 ➔ **p > 0.1069**

2 marks: 1 for approach, 1 for final lower bound. Deduct .5 for using critical value of 1.96. No more than total of 1 mark for an upper bound and deduct .5 for using 1.96 here. 1 mark for a 2-sided CI.

d. This can be done in two ways:

1. Since true "p" is unknown, p can be assumed to be 0.5 and.

$$n_{Max} = \frac{z_{\alpha/2}^2 (0.5)(0.5)}{ME^2} = \left(\frac{z_{\alpha/2}}{2ME}\right)^2 = \left(\frac{1.96}{2(0.01)}\right)^2 = 9604$$

2. Because of the Hypothesis Test above, 'p' could be assumed to be 0.115, and

$$n=\frac{z_{\alpha/2}^2(0.115)(1-0.115)}{ME^2}=\left(\frac{1.96}{0.01}\right)^2 x0.1018=3910.7488\approx 3911$$

*3. Should accept the sample proportion of **.06** as basis for another calculation of n=2213*

e.

Test and CI for Two Proportions

```
Sample   X    N  Sample p
1       60  450  0.133333
2       30  500  0.060000
Difference = p (1) - p (2)
Estimate for difference:  0.0733333
95% CI for difference:  (0.0356536, 0.111013)
Test for difference = 0 (vs not = 0):  Z = 3.85  P-Value = 0.000
Fisher's exact test: P-Value = 0.000
```

S1:

H0: p1 – p2 = (p0 = 0)

H1: p1 – p2 ≠ (p0 = 0)

S2: α = 0.05

z-Crit = $z_{\alpha/2}$ = z(0.025) = 1.96

S3: $\overline{p}=\frac{60+30}{450+500}=0.0947$ and

$$SE(\hat{p}_1-\hat{p}_2)=\sqrt{\overline{pq}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}=\sqrt{0.0947(0.9053\left(\frac{1}{450}+\frac{1}{500}\right)}=0.0190$$

z-Calc = $\frac{(\hat{p}_1-\hat{p}_2)-\Delta_0}{SE(\hat{p}_1-\hat{p}_2)}=\frac{(0.1333-0.06)-0}{0.0190}=3.8528$

S4: Since {|z-Calc| = 3.8528}>{z-Crit = 1.96}➔ Reject H0.

There is sufficient evidence to claim that the two population proportions are different.

Marking: 1 for hypotheses, .5 for pooled proportion and .5 for standard error, .5 for z-statistic, .5 for critical value, 1 for decision and conclusion.

<u>Question 5.</u> [14 Marks]

Source	DF	SS	MS	F	P
Training	2	593.06	296.528	4.23	0.027
EquipType	3	2396.53	798.843	11.39	0.000
Interaction	6	418.06	69.676	0.99	0.452
Error	24	1683.33	70.139		
Total	35	5090.97			

```
S = 8.375   R-Sq = 66.93%   R-Sq(adj) = 51.78%
```

a. Although MST is **not** given by Minitab, it is MST = 5090 / 35 = 145.43

Marking: Deduct .5 marks for each error out of the 7 numbers in the table. The MST does not have to be calculated.

b. s = s-e = sqrt(70.139) = 8.375

c. R-sq(adj) = 1 – 70.139/145.43 = 51.78%

Since many students will not get this R^2_{Adj},

R-sq = 1 – 1683.33/5090.97 = 66.9% can be accepted.

d. The interaction plot for the data means of Training Time have line segments which are not parallel. This indicates that interaction between the two factors, Equipment and Training Times could exist. The presence or absence of interaction should be checked by a formal test.

e.

S1: Ho: There is no Interaction between Training Time and Equipment Type

Ha: There is an Interaction between Training Time and Equipment Type

S2: F = .99

S3: With α = 0.05, F-Crit = $F_{\alpha=.05}(6, 24)$ = 2.5082

S4: {Since F-Calc = 0.99} < {F-Crit = 2.5082}➔ Do not Reject H0.

Conclude there is insufficient evidence to claim that the effect of training time on service time depends on equipment type.

Marking: 1 for hypotheses, 1 for F-stat and critical value, 1 for decision and conclusion.

A test for a main effect should be marked out of a total of 2 marks here.

f. $ME = t^{**} s_e \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ Here $m = \binom{a}{2} = \binom{4}{2} = \frac{4(3)2!}{2!2!} = 6$

$t^{**} = t_{\alpha/2m}(df_E) = t_{0.05/2x6}(24) = t_{0.0042}(24) \approx 2.89$ (exact value is 2.87169)

With s-e = 8.375, n1 = n2 = 9, ME = 2.89 * 8.375 * sqrt(1/9+1/9) = 11.4

*Marking: .5 for ME formula, 1 for t** value, 1 for correct s-e and n1 and n2, .5 for ME=11.4*

g. Now the mean service time for repairing TVs is 41.11 min, and

the mean service time for repairing Computers is 30.00 min.

Since the difference of 11.11 is not > 11.4, there is no evidence of real difference in mean service times.

The second part of this compares the mean service times for fax machines and photocopiers.

The observed difference is less than the calculated ME.

Question 6.

(a) There is an extreme outlying residual beyond 3 std errs.
This violates the assumption that the errors are normally distributed.
.5 for each point above

(b) Yes, removing the outlying observation in Model 2 gets rid of the outlier in the residual plot. This means that it is reasonable to assume the errors are normally distributed.

(c) Ho: beta(1)=0, … , beta(k)=0; Ha: some beta(j) is nonzero
F = 14.63 with critical value = 1.93

Reject Ho, conclude that the model is useful.

1 mark for each point above.

(d) Ho: beta(cultratio)=0; Ha: beta(cultratio) ≠ 0
T = .051/.01 = 5.1, critical value is approx. 2.0

Reject Ho, conclude the variable is useful, *given the other predictor variables in the model*.

1 mark for each point above (suggest not requiring the condition above).

(e) The value of the coefficient represents the estimated average difference in infection risk between hospitals in the West and those not in the West, assuming all the other predictor variables are kept constant.
1 mark for overall concept, .5 mark each for specifying "estimated" or "average" difference and "assuming …. constant".

(f) The number of beds is highly correlated with the number of nurses (see VIFs)
1 mark for above point.

This collinearity makes the coefficient unstable (high variance)—it also means that changing the number of beds without changing the number of nurses is not realistic.

(g) We want the 99% prediction interval
(i) Fit = 4.52, SE Fit = .3943 and MSE = .771. SE for PI is sqrt (.3943^2 + .771) = .96
99% PI is 4.52 ± 2.63 * = 4.52 ± .96 = 4.52 ± 2.53 = (2.0,7.1)
This interval does not include the actual value of 7.6
(ii) An alternative solution starts with the 95% PI `(2.6057, 6.4254)`
We have 4.52 ± 1.98 SE = (2.6, 6.4). Therefore SE = (6.4254- 4.52) / 1.98 = .96, etc.
1 mark for calculating the proper standard error of .96
1 mark for the 99% prediction interval
1 mark for noting the interval does not cover the actual value of 7.6

1 mark for calculating the 99% confidence interval instead plus .5 for noting this CI does not cover the value 7.6
.5 for the Minitab generated 95% prediction interval plus 1 for noting this PI does not cover the value 7.

There is a general confusion between a critical value and a p-value.

For example, if $z = 1.0$, then they find the critical value to be .8413, which is the cumulative probability $P(Z<1)=.8413$.   For a one-sided test, the p-value would be 1-.8413.