Concordia UNIVERSITY

Faculty of Engineering and Computer Science

ENGR 242/2 X Statics Fall 2005

Class Test 2 (Nov 4, 2005)

Instructions:

(a) Write on both sides of the test. There is an extra sheet at the end. If you need extra papers to write, please ask the test supervisor.
(b) Answer all three questions.
(c) Scientific calculator is permitted
(d) Books, notes and programmable calculator are not permitted
(e) Keep your Student ID available.
(f) There are 5 (five) pages in this test.
(g) If there is any data missing, make a reasonable assumption with sufficient explanation

Name (print) _____

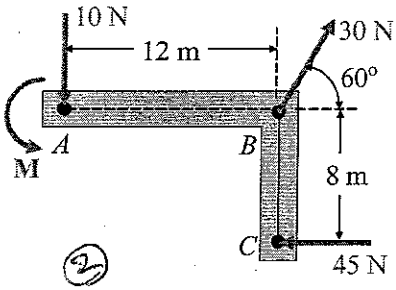
Student ID _____

Signature _____

Problem 1 (30 marks)

A couple of magnitude $M = 54$ N.m and the three forces shown are applied to an angle bracket.

(a) Find the resultant of this system of forces.
(b) Locate the points where the line of action of the resultant intersects line AB and line BC.



(a) $R_x = \Sigma F_x = 30\cos 60 - 45 = -30$ N (2)

$R_y = \Sigma F_y = 30 \sin 60 - 10 = 15.98$ N (2)

$\rightarrow R = R_x\vec{i} + R_y\vec{j} = -(30\text{ N})\vec{i} + (15.98)\vec{j}$ (2)

$\rightarrow |R| = \sqrt{30^2 + 15.98^2} = 34.0$ N (2), $\tan\theta = \frac{R_y}{R_x} = \frac{15.98}{-30} = -0.533$ (2)

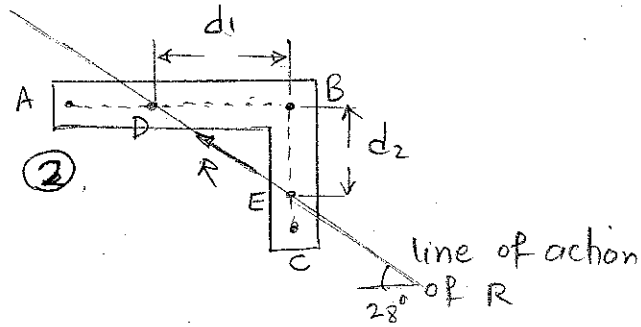
$\rightarrow$ $R = 34.0$ N $\theta \angle 28.0^\circ$ (2)

(b) First replace the given forces and couple with an equivalent force-couple at B:

$\Sigma M_B \overset{+}{\circlearrowleft} = M + (12)F_A - 8F_C$ (3)

$\rightarrow \Sigma M_B \overset{+}{\circlearrowleft} = 54 + (12)(10) - 8(45)$ (3)

$= -186$ N.m or 186 N.m $\circlearrowright$ (1)



(2) with R at D: $\Sigma M_B = 186$ N.m $= d_1 R_y = d_1(15.98)$

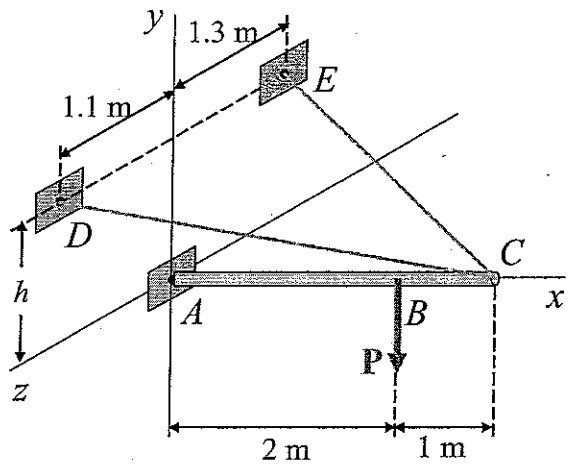
(1) $\rightarrow \boxed{d_1 = 11.64 \text{ m}}$ (To the left of B)

(3) with R at E: $\Sigma M_B = 186$ N.m $= d_2 R_x = d_2(30) \rightarrow \boxed{d_2 = 6.2 \text{ m}}$ (1)

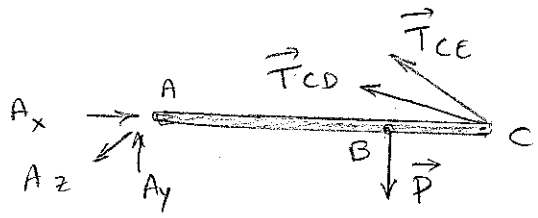
(Below B)

check for the angle $\theta \rightarrow \tan\theta = \frac{d_2}{d_1} = \frac{6.2}{11.64} = 0.533 \rightarrow \theta = 28^\circ$ ✓ (1)

Problem 2 (40 marks)
A 3-m pole is supported by a ball-and-socket joint at A and by the cables CD and CE. Knowing $\mathbf{P} = 5$ kN, and $h = 2$ m determine:
(a) the tension in cables CD and CE,
(b) the reaction at A.



$A(0,0,0)$ ①
$B(2,0,0)$ ①
$C(3,0,0)$ ①
$D(0,2,1.1)$ ①
$E(0,2,-1.3)$ ①



(FBD) ②

(a)

$\vec{T}_{CD} = T_1 \frac{\vec{CD}}{|\vec{CD}|}$, $\vec{T}_{CE} = T_2 \frac{\vec{CE}}{|\vec{CE}|}$ ②

$\vec{CD} = -3\vec{i} + 2\vec{j} + 1.1\vec{k}$

$|\vec{CD}| = \sqrt{3^2 + 2^2 + 1.1^2} = 3.77$ → $\vec{T}_{CD} = T_1 \frac{-3\vec{i} + 2\vec{j} + 1.1\vec{k}}{3.77}$ ②

$\vec{CE} = -3\vec{i} + 2\vec{j} - 1.3\vec{k}$ ①

$|\vec{CE}| = \sqrt{3^2 + 2^2 + 1.3^2} = 3.83$ → $\vec{T}_{CE} = T_2 \frac{-3\vec{i} + 2\vec{j} - 1.3\vec{k}}{3.83}$ ②

$\vec{P} = -P\vec{j} = -5\vec{j}$ ①

$\Sigma \vec{M}_A = 0$ → $\vec{r}_{C/A} \times \vec{T}_{CD} + \vec{r}_{C/A} \times \vec{T}_{CE} + \vec{r}_{B/A} \times \vec{P} = 0$ ②

$\vec{r}_{C/A} = \vec{AC} = 3\vec{i}$ ① $\vec{r}_{B/A} = \vec{AB} = 2\vec{i}$ ①

$$\frac{T_1}{3.77}\begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ -3 & 2 & -1.1 \end{vmatrix} + \frac{T_2}{3.83}\begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ -3 & 2 & -1.3 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 2 & 0 & 0 \\ 0 & -5 & 0 \end{vmatrix} = 0 \quad (3)$$

$$\left(\frac{-3(1.1)T_1}{3.77} + \frac{3(1.3)T_2}{3.83}\right)\vec{j} + \left(\frac{6T_1}{3.77} + \frac{6T_2}{3.83} - 10\right)\vec{k} = 0 \quad (2)$$

$$\rightarrow \begin{cases} -\frac{1.1T_1}{3.77} + \frac{1.3T_2}{3.83} = 0 & (*) \\ \frac{T_1}{3.77} + \frac{T_2}{3.83} = \frac{5}{3} & (2) \end{cases}$$

$$\rightarrow \boxed{T_1 = 3.4\ kN} \quad (1)$$

$$\rightarrow \boxed{T_2 = 2.92\ kN} \quad (1)$$

(b) $\sum F = 0 \rightarrow \vec{A} + \vec{T}_{CE} + \vec{T}_{CD} + \vec{P} = 0$ (1)

$\rightarrow$ x Component: $A_x - \frac{3T_1}{3.77} - \frac{3T_2}{3.83} = 0 \rightarrow A_x = \frac{3(3.4)}{3.77} + \frac{3(2.92)}{3.83}$ (2)

$$\rightarrow \underline{A_x = 5\ kN} \quad (1)$$

$\rightarrow$ y Component: $A_y + \frac{2T_1}{3.77} + \frac{2T_2}{3.83} - 5 = 0 \rightarrow A_y = 5 - \frac{2(3.4)}{3.77} - \frac{2(2.92)}{3.83}$ (2)

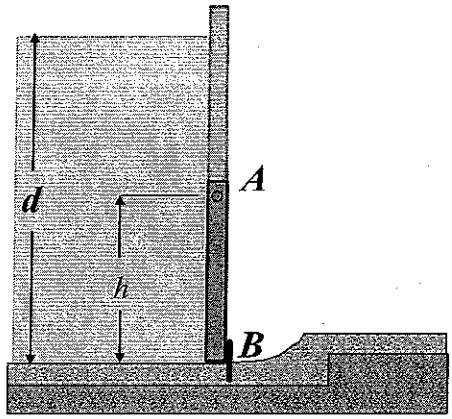
$$\rightarrow \underline{A_y = 1.67\ kN} \quad (1)$$

$\rightarrow$ z component: $A_z + \underbrace{\frac{1.1T_1}{3.77} - \frac{1.3T_2}{3.83}}_{\text{From } (*)} = 0 \rightarrow \underline{A_z = 0}$ (1)

$\rightarrow$ Reaction at A: $\boxed{A = (5\ kN)\vec{i} + (1.67\ kN)\vec{j}}$ (2)

Problem 3

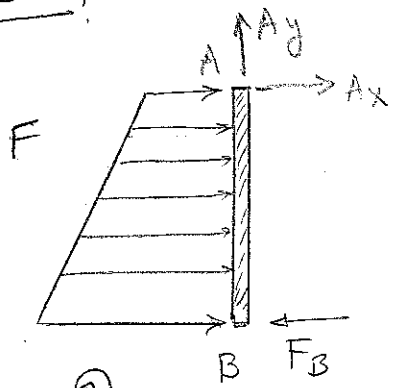
The square gate AB with $h = 1.5$ m is held in the position shown by hinges along its top edge A and by a shear pin at B. For a depth of water $d = 3$ m, determine the force exerted on the gate by the shear pin.



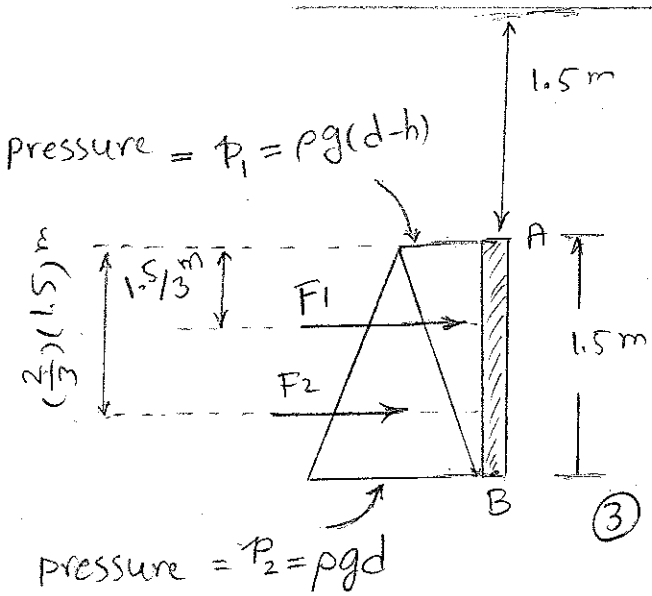
F = Force of the water on the gate

$F = F_1 + F_2$

FBD:



② ③



③ P_1 = Force intensity = $(P_1)(\text{width}) = (P_1)(1.5\ m)$

③ P_2 = " " = $(P_2)(\text{width}) = (P_2)(1.5\ m)$

③ $F_1 = \frac{1}{2} P_1 h = \frac{1}{2} P_1 (1.5) h = \frac{1}{2} \rho g (d-h)(1.5)(1.5)$

② $F_1 = \frac{1}{2}\left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(3\ m - 1.5\ m)(1.5\ m)(1.5\ m)$

① $F_1 = 16550\ N = 16.55\ kN$

(3) $F_2 = \frac{1}{2} P_2 h = \frac{1}{2} P_2 (1.5) h = \frac{1}{2} \rho g d (1.5)(1.5)$

(2) $F_2 = \frac{1}{2}(1000 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(3.0\ m)(1.5\ m)(1.5 m)$

(1) $F_2 = 33110\ N = 33.1\ kN$

(3) $\sum M_A \circlearrowleft^+ = 0 \rightarrow (\frac{1.5}{3}) F_1 + (\frac{2}{3})(1.5) F_2 - (1.5) F_B = 0$

(3) $F_B = \frac{1}{3} F_1 + \frac{2}{3} F_2 = \frac{1}{3}(16.55\ kN) + \frac{2}{3}(33.11\ kN)$

(1) $\boxed{F_B = 27.6\ kN}$