Midterm Solutions EECE 364

NAME: NUMBER:

November-06-09

1. Plot the function $\vec{A} = \cos[2\pi(x-t)]\hat{z}$ indicating wavelength, direction of travel, and amplitude.

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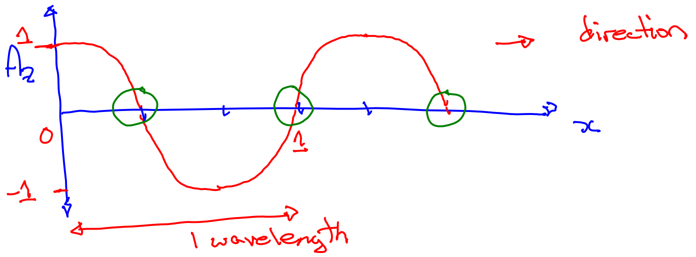

/1 Circle where is the curl maximum on your plot.

/1 What is the wave speed? $\frac{\omega}{k} = 1 \frac{\text{DISTANCE UNIT}}{\text{TIME UNIT.}}$

2. Peak solar radiation intensity at the surface of the earth is approximately 1000 Watts per square meter. What is the approximate equivalent strength of the E field associated with solar radiation?

$$\vec{I} = \frac{\vec{E}\times\vec{B}}{\mu_0} \qquad I = \frac{E^2}{\mu_0 c} = E^2\sqrt{\frac{\epsilon_0}{\mu_0}}$$

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$$E = \sqrt{\frac{1000w}{m^2}\cdot\sqrt{\frac{\mu_0}{\epsilon_0}}} \simeq 30\cdot 20\frac{V}{m} = 6\cdot 10^2\ V/m$$

3. In order for a B-field to exist there must be:

/2 Current (moving charge) and/or changing E field.

4. Given Snell's Law ($n_1 \sin \Theta_1 = n_2 \sin \Theta_2$), describe the conditions under which total internal reflection occurs.

/2 TIR occurs when then angle of incidence is beyond a critical angle, θ_c, where light is refracted by 90° ($\theta_2 = \pi/2$), with θ_c described by

$$\sin\theta_c = \frac{n_2}{n_1}\sin\pi/2 = \frac{n_2}{n_1}$$

$$\boxed{\theta_I > \arcsin\left(\frac{n_2}{n_1}\right)} \quad \text{with} \quad \boxed{n_1 > n_2}$$

5. A constant field E_1 is incident from air onto a dielectric at an angle, Θ_1. If the new medium has a relative permittivity, ε_r, find equations relating Θ_1 and Θ_2. Combine these to form 1 equation relating the two angles.

B.C.'s $\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}$, $E_1^{\parallel} = E_2^{\parallel}$

$E_1^{\parallel}$ ε_r

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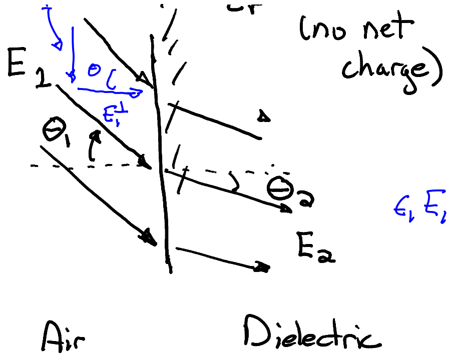

$E_1^\perp = E_1 \cos\theta_1$ $E_2^\perp = E_2 \cos\theta_2$

$E_1^\parallel = E_1 \sin\theta_1$ $E_2^\parallel = E_2 \sin\theta_2$

$\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2$, $E_1 \sin\theta_1 = E_2 \sin\theta_2$

$E_2 = \frac{E_1 \sin\theta_1}{\sin\theta_2}$

$\therefore \epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_1 \cos\theta_2 \sin\theta_1 / \sin\theta_2$

$\cot\theta_2 = \frac{\epsilon_1}{\epsilon_2} \cot\theta_1$

Can you explain why the direction changes (relating it to the effect of polarization)?

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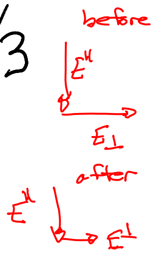

$E_\parallel$ is unchanged by the interface, but $E^\perp$ is reduced because of the reduction in field due to the induced surface charge.

$\therefore$ The field is deflected away from normal.

6. What is the wavevector, *k* of a wave of wavelength, λ, traveling at 45° in the positive X-Z plane?

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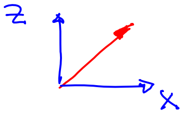
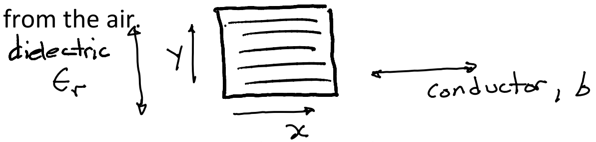


$|k| = \frac{2\pi}{\lambda}$ direction $\hat{k} = \frac{\hat{x}}{\sqrt{2}} + \frac{\hat{y}}{\sqrt{2}}$

$$\boxed{\vec{k} = \frac{\sqrt{2}\pi}{\lambda}(\hat{x} + \hat{y})}$$

7. A material is perfectly conductive in the *x*-direction and is a linear dielectric in the *y* axis. A wave with E field components in the *x* and *y* directions irradiates the material at normal incidence from the air.



a. The x-component of the E field will be (circle one):
   - i. Transmitted and reflected, with incident and reflected fields in phase.
   - ii. Transmitted and reflected, with incident and reflected fields out of phase.
   - iii. Totally reflected, with the incident and reflected E fields in phase at the interface.
   - [✓ iv. Totally reflected, with the incident and reflected E fields out of phase at the interface.]
   - v. The wave will be completely dissipated (converted to heat) due to the high conductivity of the material, and thus the phase will be imaginary.

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Will the x-directed field be larger or smaller in the material? Explain in terms

/2 of the material properties.

In a perfect conductor, E fields are zeroed immediately by conduction currents, so the E field will be zero (i.e. smaller).

/2 Explain how the material properties determine the phase of any reflection.

$E_{inside} = 0 \quad \therefore \quad E_I + E_R = 0 \Rightarrow E_I = -E_R.$

b. The y-component of the E field will be:

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   i. Transmitted and reflected, with incident and reflected fields in phase.
   [✓ ii.] Transmitted and reflected, with incident and reflected fields out of phase.
   iii. Totally reflected, with the incident and reflected E fields in phase at the interface.
   iv. Totally reflected, with the incident and reflected E fields out of phase at the interface.
   v. The wave will be completely dissipated (converted to heat) due to the high conductivity of the material.

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Will the y-directed field be larger or smaller in the material?

E field will be smaller inside the dielectric due to reflection ($E_R^{\parallel} + E_I^{\parallel} = E_T^{\parallel}$, since some reflection $E_T^{\parallel} < E_I^{\parallel}$, so $E_R^{\parallel}$ is opposite in sign to $E_I^{\parallel}$).

/2 Explain how the material properties determine the phase of any reflection.

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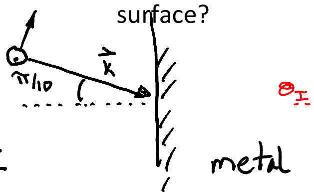
As above $E_R^{\parallel}$ opposite to $E_I^{\parallel}$ in direction (180° phase shift).

8. A *TM* radio wave is incident on a 'perfect' metal surface at a small angle of incidence, π/10 radians. (You may use sin(x) ~ x for small angles.)
   a. Given a wavelength 1 m, what is the apparent wavelength as seen along the surface?



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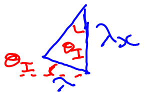

$$\sin\theta_I = \frac{\lambda}{\lambda_x}$$

$$\lambda_x = \frac{\lambda}{\sin\theta_I} = \frac{10\lambda}{\pi}$$

   a. If the field strength is (10/8.85) V/m, what is the peak magnitude of charge per unit area on the metal?

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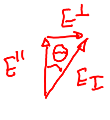
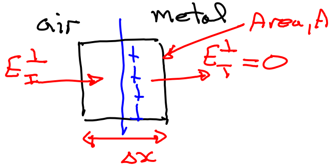
Surface charge is a result of perpendicular fields.



$$E^{\perp} = E_I \sin\theta_I \approx \frac{E_I \pi}{10}$$

Gauss' Law $\oint \vec{E}\cdot\hat{n}\,dA = \frac{Q_{enc}}{\epsilon}$



Let $\Delta x \to 0$ $\quad E_I^{\perp} A = \frac{\sigma A}{\epsilon_0} \quad E_I^{\perp} = \sigma/\epsilon_0 \quad \sigma = \frac{10}{8.85}\frac{V}{m} \cdot 8.85 \cdot 10^{-12} = 10^{-11} \frac{C}{m^2}$

1. The E-field within a metal resulting from a wave at normal incidence is found to be

$$\vec{E} \cong E_0 e^{-\sqrt{\pi f \sigma \mu}\, z} e^{-j\omega t} \hat{x}$$

a. Over what distance does the field strength drop by a factor of e^{-1}?

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$$\frac{1}{\sqrt{\pi f \sigma \mu}}$$

b. What is the current density as a function of z?

$$j = \sigma E = \mathrm{Re}\left[\sigma E_0 \exp\left(-\sqrt{\pi f \sigma \mu}\, z - j\omega t\right)\right)$$

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c. (BONUS) How would you figure out heat dissipation per unit area?