QUESTION II **(weight= 10% of total mark)**

In composite beam design, two plastic neutral axes can occur in certain situations.

(a) Describe under which conditions this is the case and (3%)

(b) Provide a clear rationale for this phenomenon (7%)

(a) Two plastic neutral axes occur in the case of partial interaction, i.e., when the shear stud capacity Q_r is smaller than the concrete maximum compressive capacity $\max C'_r$. Also, Q_r is smaller than the maximum tensile capacity $\max T_r$.

(b) When this is the case, the shear force in the studs $V_h = Q_r$ has to be equal to the compressive force of the concrete (from equilibrium of concrete slab alone- and/or from the weakest link analogy). Thus,

$$C_r = Q_r < \max C_r$$

$$\phi abf'_c < \phi tbf'_c \Rightarrow \boxed{a < t}$$

i.e, there must be a neutral axis within the concrete.

Also, from equilibrium of the steel section alone (or from weakest link analogy), the <u>net</u> tensile force $(T_r - C_r)$ in the steel must equal the shear force in the studs, i.e.,

$$T_r - C_r = Q_r \Rightarrow T_r < Q_r$$

$$Q_r < \max T_r$$

$$\Rightarrow T_r < Q_r < \max T_r \Rightarrow T_r < \max T_r \Rightarrow \boxed{NA \quad \text{lies in steel}}$$

QUESTION III **(weight= 60% of total mark)**

A simply supported beam is subject to a distributed load. The beam consists of a W410x60 structural steel section (Fy=350 MPa) and a concrete slab $(f'_c = 30MPa)$ with an effective width of 1,500 mm and thickness of 120 mm. The slab is connected to the top of steel section through 18 mm diameter studs (Fu=414 MPa). It is required to:

a) Determine the minimum number of studs to ensure full interaction. On a clear sketch, show the location and distribution of the studs. (10%)

$$A_s = 7580mm^2, b_{fl} = 178mm, t_{fl} = 12.8mm, w = 7.7mm, d = 407mm$$

$$\max T_r = \phi A_s F_y = 0.90 \times 7,580 \times 350 \times 10^{-3} = 2,388kN$$

$$\max C_r' = \phi_c \alpha_1 f_c' bt = 0.65 \times (0.85 - 0.0015 \times 30) \times 30 \times 1,500 \times 120 \times 10^{-3} = 2,826kN$$

$$V_{hf} = \min(2,388kN, 2,826kN) = 2,388kN$$

$$q_r^{I} = \phi_{sc} A_s F_u = 0.80 \times \left(\frac{\pi}{4} \times 18^2\right) \times \left(414 \times 10^{-3}\right) = 84.3kN$$

$$q_r^{II} = 0.5\phi_{sc} A_s \sqrt{f_c' E_c} = 0.5 \times 0.80 \times \left(\frac{\pi}{4} \times 18^2\right) \sqrt{30 \times 4,500\sqrt{30}} \times 10^{-3} = 87.5kN$$

$$q_r = \min(84.3, 87.5) = 84.3kN$$

$$n = \frac{V_{hf}}{q_r} = \frac{2,388}{84.3} = 28.3kN \Rightarrow n = 29$$

Need to place 29 studs between midspan and each support

b) Determine the section positive bending resistance based on the number of studs determined in step (a) (25%)

$\max(T_r)$ is the weakest link- Case is 1

$$a = \frac{\phi_s A_s F_y}{\phi_c \alpha_1 f_c ' b} = \frac{0.90 \times 7,580 \times 350}{0.65 \times 0.805 \times 30 \times 1,500} = 101mm$$

$$e' = \frac{d}{2} + t - \frac{a}{2} = \frac{407}{2} + 120 - \frac{101}{2} = 273mm$$

$$M_{rc} = T_r e' = \max T_r \times e' = (2,388kN) \times (273mm) \times 10^{-3} = 652kNm$$

c) If an interaction ratio of 65% is targeted, re-calculate the number of studs needed. (10%)

$$V_{h-req} = 0.65 V_{hf} = 0.65 \times 2,388kN = 1,552kN$$

$$n = \frac{V_h}{q_r} = \frac{1,552}{84.3} = 18.4 \geq n = 19 studs$$

Need 19 studs X two sides = 38 studs

d) Determine the location of the plastic neutral axes based on the number of studs computed in (c) (15%)

$$V_h = 19 \times 84.3 = 1,602kN$$

1. NA in concrete

$$C'_r = V_h$$

$$\phi_c \alpha_1 f_c ' b a = V_h \Rightarrow a = \frac{V_h}{\phi_c \alpha_1 f_c ' b} = \frac{1,602 \times 10^3}{0.65 \times 0.805 \times 30 \times 1,500} = 68mm$$

The depth of the equivalent stress block is 68mm from top of concrete

The neutral axis location c is located at

$$c = \frac{a}{\beta} = \frac{68}{0.97 - 0.0025 \times f_c '} = \frac{68}{0.895} = \boxed{76mm}$$

2. NA in steel section

$$C_r = \frac{\phi_s A_s F_y - C_r '}{2} = \frac{2,388 - 1,602}{2} = 393kN$$

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$$\max(C_{fl}) = \phi_s A_{fl} F_y = 0.9 \times (178 \times 12.8) \times 350 \times 10^{-3} = 718kN > C_r$$

Thus, PNA lies in flange (Case 3a)

$$d_{NA} = \frac{C_r}{\phi b_{fl} F_y} = \frac{393 \times 10^3}{0.9 \times 178 \times 350} = \boxed{7.0mm}$$