Modification of Example 6 in the note:

Given

Section is W530x83 - Fy=300 MPa
Height is increased to 8.5m
Member is braced normal to the plane of the page at column mid-height
Internal forces:

> Axial force Cf =700 kN (length = 7 m),
> Maximum bending moment about strong axis Mfx = 410 kNm (under UDL)
> Weak axis bending linear 20 kNm at top and 10 kNm at bottom (double curvature)

Required

Determine whether the proposed section meets in-Plane stability requirements.

Solution:

The presence of the braces normal to the plane of the problem transforms the column into two segments (top and bottom). Separate design checks need to be performed for each segment.

CHECK FOR SEGMENT 1

1. Section Class

$\frac{b}{2t} = \frac{200}{2(13.3)} = 7.86 \qquad \frac{145}{\sqrt{F_y}} = \frac{145}{\sqrt{350}} = 5.37$

Flange meets Class 1 Requirements

$\frac{h}{w} = \frac{502}{9.5} = 52.84 \qquad C_y = AF_y = 10{,}500 * 300 = 3150 \text{ kN}$

$\frac{C_f}{C_y} = \frac{700}{3150} = 0.22 \qquad \frac{1100}{\sqrt{F_y}}\left[1-0.391(C_f/C_y)\right] = \frac{1100}{\sqrt{350}}\left[1-0.391(0.22)\right] = 58.05$

Web meets Class 1 Requirements

Section meets Class 1 requirements

2. Compressive Resistance

Based on weak axis buckling

$$\frac{K_y L}{r_y} = \frac{1\times 0.5\times 8{,}500}{44} = 96.5 \leq 200$$

$$\lambda_y = \sqrt{\frac{F_y}{F_{ey}}} = \frac{K_y L}{\pi r_y}\sqrt{\frac{F_y}{E}} = \frac{96.5}{\pi}\sqrt{\frac{300}{200{,}000}} = 1.19$$

$$C_{ry} = C_{ry}\left(K_y = 1\right) = \phi A F_y \left(1+\lambda_y{}^{2n}\right)^{-1/n}$$

$$C_{ry} = 0.9\times 10{,}500\times 0.30\times\left[1+1.19^{2\times 1.34}\right]^{-1/1.34} = 1{,}392 kN$$

3. Flexural Resistances

$$M_{rx} = \phi Z_x F_y = 0.9(2,070)(300)\times 10^{-6} = 559kNm$$

$$M_{ry} = \phi Z_y F_y = (0.9)(303\times 10^3)(0.3)\times 10^{-3} = 81.8kNm$$

4. Calculate U1x

$$C_{ex} = \frac{\pi^2 EI_x}{L^2} = \frac{\pi^2 (200,000)(479\times 10^6)}{8,500^2}\times 10^{-3} = 13,087kN$$

Note the braces at column mid-height prevent mid-height point from moving in the out-of-plane direction but the point remains free to move in plane. Thus L remains 8.5m when calculating C_{ex}

$$\frac{1}{1-\frac{C_f}{C_{ex}}} = \frac{1}{1-\frac{700}{13,087}} = 1.057$$

$\omega_{1x} = 1.0$ for uniformly distributed load

$$U_{1x} = \frac{\omega_{1x}}{1-\frac{C_f}{C_{ex}}} = 1.0\times 1.057 = 1.057$$

5. Calculate U1y

From similarity of triangles, moment at mid-height is 5 kNm (same direction as the 20 kNm moment)

For double curvature $\kappa_y = -\frac{5}{20} = -0.25$

$\omega_{1y} = 0.6 - 0.4\kappa_y = 0.6 - 0.4(-0.25) = 0.7 \geq 0.40$ - take $\omega_{1y} = 0.7$

$$\left(\frac{L}{r_y}\right) = \frac{0.5\times 8,500}{44} = 96.5 \leq 200$$

Note: Unlike Cex, when Calculating Cey, one needs to use half of the height

$$C_{ey} = \frac{\pi^2 EI_y}{L^2} = \frac{\pi^2 EA}{(L/r_y)^2} = \frac{\pi^2 (200,000)(10,500)}{96.5^2}\times 10^{-3} = 2,221\,kN$$

$$\frac{1}{1-\frac{C_f}{C_{ey}}} = \frac{1}{1-\frac{700}{2,221}} = 1.460$$

$$U_{1y} = \frac{\omega_{1y}}{1-\frac{C_f}{C_{ey}}} = 0.7\times 1.46 = 1.022$$

6. Interaction coefficient beta

$$\beta = 0.6 + 0.4\lambda_y = 0.6 + 0.4(1.314) \geq 0.85$$

$$\beta = 0.85$$

7. In-Plane Stability Interaction Requirement

$$\frac{C_f}{C_r} + 0.85\frac{U_{1x}M_{fx}}{M_{rx}} + \beta\frac{U_{1y}M_{fy}}{M_{ry}}$$

$$= \frac{700}{1,392} + 0.85\frac{1.057 \times 410}{559} + 0.85\frac{1.022 \times 20}{81.8}$$

$$= 0.503 + 0.659 + 0.212 > 1.0$$

Based on the code conservative procedure $C_r = C_{ry}$, the interaction equation is not met.

Note 1: recalculate the axial resistance based on weighted average method and re-check. (we will see how to do this in the last solved example in the notes)

Note 2: If section passes need to check second interaction requirement.

CHECK FOR SEGMENT 2

In a design situation, because Segment 1 failed the check, one would need to change the design (cross-section, bracing arrangement, etc.) before checking Segment 2. However, the steps are provided here for illustrative purposes.

Steps 1-4 and Step 6 are identical to those of segment 1. For step 5 and 7, the differences are highlighted in red.

5. Calculate U1y

For double curvature $\kappa_y = +\frac{5}{10} = +0.50$

$$\omega_{1y} = 0.6 - 0.4\kappa_y = 0.6 - 0.4(0.50) = 0.40 \geq 0.40 \text{ - take } \omega_{1y} = 0.4$$

$$\left(\frac{L}{r_y}\right) = \frac{0.5 \times 8,500}{44} = 96.5 \leq 200$$

Note: Unlike Cex, when Calculating Cey, one needs to use half of the height

$$C_{ey} = \frac{\pi^2 EI_y}{L^2} = \frac{\pi^2 EA}{(L/r_y)^2} = \frac{\pi^2 (200,000)(10,500)}{96.5^2} \times 10^{-3} = 2,221\, kN$$

$$\frac{1}{1 - \frac{C_f}{C_{ey}}} = \frac{1}{1 - \frac{700}{2,221}} = 1.460$$

$$U_{1y} = \frac{\omega_{1y}}{1 - \frac{C_f}{C_{ey}}} = 0.40 \times 1.46 = 0.584$$

7. In-Plane Stability Interaction Requirement

$$\frac{C_f}{C_r} + 0.85\frac{U_{1x}M_{fx}}{M_{rx}} + \beta\frac{U_{1y}M_{fy}}{M_{ry}}$$

$$= \frac{700}{1,392} + 0.85\frac{1.057 \times 410}{559} + 0.85\frac{0.584 \times 10}{81.8}$$

$$= 0.503 + 0.659 + 0.061 > 1.0$$

Segment 2 fails in-plane stability check. (Again, need to recheck things based on the weighted average method and re-check).