<u>CVG4143-Major Assignment-Composite Sections</u>

A simply supported composite beam is subject to a distributed load. The beam consists of a W410x54 structural steel section (Fy=300 MPa) and a concrete slab $\left(f'_c = 30MPa\right)$ with an effective width of 1,500 mm and a thickness of 150 mm. The slab is connected to the top of steel section through 20 mm diameter studs (Fu=414 MPa). It is required to:

a) Determine the minimum number of studs to ensure full interaction for positive bending. On a clear sketch, show the location and distribution of the studs.

$$A_s = 6,810mm^2$$

$$b_{fl} = 177mm, t_{fl} = 10.5mm, w = 7.5mm, d = 403mm$$

$$\max T_r = \phi A_s F_y = 0.90 \times 6,810 \times 300 \times 10^{-3} = 1,839kN$$

$$\alpha_1 = 0.85 - 0.0015 f_c' = 0.85 - 0.0015 \times 30 = 0.805$$

$$\max C_r' = \phi_c \alpha_1 f_c' bt = 0.65 \times 0.805 \times 30 \times 1,500 \times 150 \times 10^{-3} = 3,532kN$$

$$V_{hf} = \min\left(1,839kN, 3,532kN\right) = 1,839kN$$

$$q_r^{I} = \phi_{sc} A_s F_u = 0.80 \times \left(\frac{\pi}{4} \times 20^2\right) \times \left(414 \times 10^{-3}\right) = 104kN$$

$$q_r^{II} = 0.5\phi_{sc} A_s \sqrt{f_c' E_c} = 0.5 \times 0.80 \times \left(\frac{\pi}{4} \times 20^2\right) \sqrt{30 \times 4,500\sqrt{30}} \times 10^{-3} = 108kN$$

$$q_r = \min\left(104, 108\right) = 104kN$$

In order to use the expression for q_r^{II}, $h/d \geq 4$. For this, concrete thickness has to be greater than 20x4=80mm. The given concrete thickness of 150mm is larger than the required thickness of 120mm.

$$n = \frac{V_{hf}}{q_r} = \frac{1,838.7}{104} = 17.7kN \Rightarrow n = 18$$

Need to place 18 studs in each of the two half spans (i.e., the two regions of zero moment to maximum positive moment). Thus a total of 36 studs are needed.

b) Determine the section positive resistance based on the number of studs determined in step (a)

$$\max\left(C_r'\right) > \max\left(T_r\right)$$

$$Q_r > \max\left(T_r\right)$$

Thus, weakest link is steel and Case is 1

$$a = \frac{\phi_s A_s F_y}{\phi_c \alpha_1 f_c' b} = \frac{0.90 \times 6,810 \times 300}{0.65 \times 0.805 \times 30 \times 1,500} = 78.1mm$$

$$e' = \frac{d}{2} + t - \frac{a}{2} = \frac{403}{2} + 150 - \frac{78.1}{2} = 312.5mm$$

$$M_{rc} = T_r e' = \max T_r \times e' = \left(1,839kN\right) \times \left(312.5mm\right) \times 10^{-3} = 575kNm$$

c) If an interaction ratio of 75% is targeted, re-calculate the number of studs needed.

$$V_h = 0.75V_{hf} = 0.75 \times 1,839kN = 1,379kN$$

$$n = \frac{V_h}{q_r} = \frac{1,379}{104} = 13.3 \geq n = 14studs$$

Use 14 studs x 2 sides =28 studs

d) Determine the section resistance based on the number of studs determined in (c)

$$Q_r = nq_r = 14 \times 104kN = 1,456kN$$

$$\max T_r = 1,839kN$$

$$\max C_r' = 3,532kN$$

As expected, the weakest link is Q_r and we should design the section as a Case 3

Unlike Case 1, for Case 3, need to ensure that section meets Class 2 requirements

$$\left(\frac{b_{fl}}{2t_{fl}}\right) = \frac{177}{2(10.9)} = 8.11 < \frac{170}{\sqrt{F_y}} = \frac{170}{\sqrt{300}} = 9.81$$

$$\left(\frac{h}{w}\right) = \frac{403 - 2(10.9)}{7.5} = 50.8 < \frac{1,900}{\sqrt{F_y}} = \frac{1,900}{\sqrt{300}} = 109.7$$

Classification requirement is met.

From the weakest link analogy $C_r' = Q_r = 1,456kN$

$$C_r = \frac{\phi_s A_s F_y - C_r'}{2} = \frac{0.90 \times 6,810 \times 300 \times 10^{-3} - 1,456}{2} = 191kN$$

$$\max(C_{fl}) = \phi_s A_{fl} F_y = 0.9 \times (177 \times 10.9) \times 300 \times 10^{-3} = 521kN > C_r$$

Thus, PNA lies in flange (Case 3a)

$$d_{NA} = \frac{C_r}{\phi b_{fl} F_y} = \frac{191 \times 10^3}{0.9 \times 177 \times 300} = 4.00mm$$

$$\bar{y}_c = \frac{d_{NA}}{2} = \frac{4mm}{2} = 2mm$$

$$\bar{y}_t = \frac{A_s\left(\frac{d}{2}\right) - A_s^c\left(d - \bar{y}_c\right)}{A_s - A_s^c} = \frac{6,810\left(\frac{403}{2}\right) - (4.00 \times 177)(403 - 2)}{6,810 - (4.00 \times 177)} = 178mm$$

Determine the depth of the equivalent stress block

$$Q_r = nq_r = \phi_c \alpha f_c' ba$$

$$14 \times 104 \times 10^3 = 0.65 \times 0.805 \times 30 \times 1,500a$$

$$a = 61.6mm$$

Determine the moment arms

$$e = d - \bar{y}_c - \bar{y}_t = 403 - 2 - 178 = 223mm$$

$$e' = d + t - \frac{a}{2} - \bar{y}_t = 403 + 150 - \frac{61.6}{2} - 178 = 344mm$$

$$M_{rc} = C_r e + C_r' e' = 191 \times 223 \times 10^{-3} + 1,456 \times 344 \times 10^{-3} = 543kNm$$

Note: By reducing the interaction ratio from 100% to 75%, we lost only about 5% of flexural resistance of the section.

e) If the designer decides to replace the steel section by a heavier W410X114 section, re-calculate the number of studs required to ensure full interaction

$$A_s = 14,800mm^2$$

$$b_{fl} = 261mm, t_{fl} = 19.3mm, w = 11.6mm, d = 420mm$$

$$\max T_r = \phi A_s F_y = 0.90 \times 14,800 \times 300 \times 10^{-3} = 3,996kN$$

$$\max C_r' = \phi_c \alpha_1 f_c' bt = 0.65 \times 0.805 \times 30 \times 1,500 \times 150 \times 10^{-3} = 3,532.5kN$$

$$V_{hf} = \min(3,996kN, 3,532kN) = 3,532kN$$

As shown under item (a)

$$q_r = \min(104,108) = 104kN$$

$$n = \frac{V_{hf}}{q_r} = \frac{3,532}{104} = 33.96 \Rightarrow n = 34$$

Use 34x2 studs.

f) Recalculate the section capacity of the composite section given in (e).

1. Given that there is full interaction at the interface, one can conclude Q_r is larger than V_{hf} and the weakest link becomes the compressive strength of the concrete. This is a <u>case 2.</u>

2. Need to show that Section meets Class 2 requirements.

3. Determine whether the steel neutral axis lies in the flange or the web

$$C'_r = \phi_c \alpha_1 f'_c bt = 0.65 \times 0.805 \times 30 \times 1,500 \times 150 \times 10^{-3} = 3,532kN$$

$$C_r = \frac{\phi A_s F_y - \alpha_1 \phi_c f'_c bt}{2} = \frac{3,996 - 3,532}{2} = 232kN$$

$$\max C_{fl} = \phi b_{fl} t_{fl} F_y = 0.90 \times 261 \times 19.3 \times 0.30 = 1,360kN$$

The maximum compressive resistance provided by the flange is larger than the internal compressive force C_r to be provided by the steel. Thus, neutral axis lies in flange and this is a Case 2a.

Depth of the neutral axis

$$d_{NA} = \frac{C_r}{\phi b_{fl} F_y} = \frac{232 \times 10^3}{0.9 \times 261 \times 300} = 3.29mm$$

Centroidal distance of the compressive resistance in the steel

$$\bar{y}_c = \frac{d_{NA}}{2} = \frac{3.29mm}{2} = 1.65mm$$

Centroidal distance of the compressive resistance in the steel

$$\bar{y}_t = \frac{A_s\left(\frac{d}{2}\right) - A_s^c\left(d - \bar{y}_c\right)}{A_s - A_s^c} = \frac{14,800\left(\frac{420}{2}\right) - (3.29 \times 261)\left(420 - 1.65\right)}{14,800 - (3.29 \times 261)} = 197.1mm$$

$$e = d - \bar{y}_c - \bar{y}_t = 420 - 1.65 - 197.1 = 221.3mm$$

$$e' = d + \frac{t}{2} - \bar{y}_t = 420 + \frac{150}{2} - 192.2 = 302.8mm$$

$$M_{rc} = C_r e + C_r' e' = 232 \times 221.3 \times 10^{-3} + 3,532 \times 302.8 \times 10^{-3} = 1,121kNm$$