Assignment 2 – Part 2

Total : 44 Points

1. Due Date & Time: Tuesday, June 11th, 2013 by 23:59 hrs.
2. You MUST upload this part of the Assignment on MyStatLab in "pdf" format.
3. You MUST also attach the Printed/ Signed "Integrity Statement".
4. Use MiniTab if or Excel if you have to. However you should cut and paste your result and sufficient explanation.
5. You should provide detailed explanation for each of your solution. You must show your complete manual calculation if the question involves using formula or any kind of calculations.
6. This Assignment covers weeks 3-5 for chapters 9.1, 9.2, 9.4, 9.6, 9.7, 9.8, 9.9, 9.10 .The questions listed below are also in the same order.

Question 1. (Show you calculation in detail for each part)

The following table shows the number of spam e-mails that arrive in my inbox each day.

Number of Spam E-Mails	Frequency
3	10
4	29
5	28
6	42
7	16

a) What is the mean number of spam e-mails received per day? **(2 points)**

Answer:

Number of Spam E-Mails	Frequency	Probability
3	10	0.080
4	29	0.232
5	28	0.224
6	42	0.336
7	16	0.128

$\mu = (3)(0.080) + (4)(0.232) + (5)(0.224) + (6)(0.336) + (7)(0.128)$
$\mu = 5.20$

b) What is the standard deviation for the number of spam e-mails received per day? **(2 points)**

Answer:

x_i	x_i^2	$P(x_i)$	$x_i^2 P(x_i)$
3	9	0.080	0.720
4	16	0.232	3.712
5	25	0.224	5.600
6	36	0.336	12.096
7	49	0.128	6.272
			Total = 28.400

$\mu = (3)(0.080) + (4)(0.232) + (5)(0.224) + (6)(0.336) + (7)(0.128)$
$\mu = 5.20$

$$\sum_{i=1}^{n} x_i^2 P(x_i) = 28.4$$

$$\sigma^2 = 28.4 - (5.20)^2 = 1.36$$

$$\sigma = \sqrt{1.36} = 1.17$$

c) What is the probability of receiving five or more spam e-mails today? **(2 points)**

Answer:

x_i	$P(x_i)$
3	0.080
4	0.232
5	0.224
6	0.336
7	0.128

$P(x \geq 5) = 0.224 + 0.336 + 0.128 + 0.688$

Question 2. (Show you calculation in detail for each part)

Kobe Bryant, a professional basketball player in the NBA, has made 84% of his free throws during his career with the Los Angeles Lakers.

a) Calculate the probability that Bryant will make exactly three of his next five free throws. **(2 points)**

Answer:

$$P(3,5)=\frac{5!}{(5-3)!3!}(0.84)^3(0.16)^{5-3}$$

$$P(3,5)=\frac{(5)(4)(3)(2)(1)}{(2)(1)(3)(2)(1)}(0.84)^3(0.16)^2$$

$$P(3,5)=(10)(0.5927)(0.0256)=0.1517$$

b) Calculate the probability that Bryant will make less than five of his next six free throws. **(2 points)**

Answer:

$$P(5,6)=\frac{6!}{(6-5)!5!}(0.84)^5(0.16)^{6-5}$$

$$P(5,6)=\frac{(6)(5)(4)(3)(2)(1)}{(1)(5)(4)(3)(2)(1)}(0.84)^5(0.16)^1$$

$$P(5,6)=(6)(0.4182)(0.16)=0.4015$$

$$P(6,6)=\frac{6!}{(6-6)!6!}(0.84)^6(0.16)^{6-6}$$

$$P(6,6)=\frac{(6)(5)(4)(3)(2)(1)}{(1)(6)(5)(4)(3)(2)(1)}(0.84)^6(0.16)^0$$

$$P(6,6)=(1)(0.3513)(1)=0.3513$$

$$P(x<5)=1.0-P(5,6)-P(6,6)=1.0-0.4015-0.3513=0.2472$$

c) Calculate the probability that Bryant will make four or five of his next six free throws. **(2 points)**

Answer:

$$P(4,6)=\frac{6!}{(6-4)!4!}(0.84)^4(0.16)^{6-4}$$

$$P(4,6)=\frac{(6)(5)(4)(3)(2)(1)}{(2)(1)(4)(3)(2)(1)}(0.84)^4(0.16)^2$$

$$P(4,6)=(15)(0.4979)(0.0256)=0.1912$$

$$P(5,6)=\frac{6!}{(6-5)!5!}(0.84)^5(0.16)^{6-5}$$

$$P(5,6)=\frac{(6)(5)(4)(3)(2)(1)}{(1)(5)(4)(3)(2)(1)}(0.84)^5(0.16)^1$$

$$P(5,6)=(6)(0.4182)(0.16)=0.4015$$

$$P(x=4 \text{ or } 5)=0.1912+0.4015=0.5927$$

d) Do part (a) in MiniTab (or Excel) and show your result. **(1 point)**

Question 3. (Show you calculation in detail for each part)

Cars arrive at a toll booth on an exit ramp on Route 408 in Florida at an average rate of 3.4 cars every five minutes. Assume that the arrival of cars at this toll booth follows the Poisson distribution.

a) What is the probability that exactly three cars will arrive at this toll booth during the next five minutes? **(2 points)**

Answer:

$$\lambda=3.4$$

$$P(3)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{\left(3.4^3\right)\left(2.71828^{-3.4}\right)}{3!}$$

$$P(3)=\frac{(39.304)(0.033373)}{6}=0.2186$$

b) What is the probability that six or seven cars will arrive at this toll booth during the next five minutes? **(2 points)**

Answer:

$$\lambda = 3.4$$

$$P(6) = \frac{\left(3.4^{6}\right)\left(2.71828^{-3.4}\right)}{6!} = \frac{(1,544.804)(0.033373)}{720} = 0.0716$$

$$P(7) = \frac{\left(3.4^{7}\right)\left(2.71828^{-3.4}\right)}{7!} = \frac{(5,252.335)(0.033373)}{5,040} = 0.0348$$

$$P(6 \text{ or } 7) = 0.0716 + 0.0348 = 0.1064$$

c) What is the probability that two or more cars will arrive at this toll booth during the next five minutes? **(2 points)**

Answer:

$$\lambda = 3.4$$

$$P(0) = \frac{\left(3.4^{0}\right)\left(2.71828^{-3.4}\right)}{0!} = \frac{(1)(0.033373)}{1} = 0.0334$$

$$P(1) = \frac{\left(3.4^{1}\right)\left(2.71828^{-3.4}\right)}{1!} = \frac{(3.4)(0.033373)}{1} = 0.1135$$

$$P(x \geq 2) = 1.0 - P(x < 2) = 1.0 - 0.0334 - 0.1135 = 0.8531$$

d) Do part (a) in MiniTab (or Excel) and show your result. **(1 point)**

Question 4. (Show you calculation in detail for each part)

A local grocery store receives strawberries from suppliers in Florida and California. Currently there are 18 strawberry containers on the shelf and 11 of them are from Florida. A shopper selects three containers to purchase.

a) What is the probability that two or more containers are from the Florida supplier? **(2 points)**

Answer:

$N = 18$, $R = 11$, $n = 3$, $x = 2, 3$

$$P(2)=\frac{{}_{18-11}C_{3-2}\cdot {}_{11}C_2}{{}_{18}C_3}=\frac{{}_7C_1\cdot {}_{11}C_2}{{}_{18}C_3}$$

$${}_7C_1=\frac{7!}{(7-1)!1!}=7$$

$${}_{11}C_2=\frac{11!}{(11-2)!2!}$$

$$(n-x+1)=(11-2+1)=10$$

$${}_{11}C_2=\frac{(11)(10)}{(2)(1)}=55$$

$${}_{18}C_3=\frac{18!}{(18-3)!3!}$$

$$(n-x+1)=(18-3+1)=16$$

$${}_{13}C_3=\frac{(18)(17)(16)}{(3)(2)(1)}=816$$

$$P(2)=\frac{{}_7C_1\cdot {}_{11}C_2}{{}_{18}C_3}=\frac{(7)(55)}{816}=0.4718$$

$$P(3)=\frac{{}_{18-11}C_{3-3}\cdot {}_{11}C_3}{{}_{18}C_3}=\frac{{}_7C_0\cdot {}_{11}C_3}{{}_{18}C_3}$$

$${}_7C_0=\frac{7!}{(7-0)!0!}=1$$

$${}_{11}C_3=\frac{11!}{(11-3)!3!}$$

$$(n-x+1)=(11-3+1)=9$$

$${}_{11}C_3=\frac{(11)(10)(9)}{(3)(2)(1)}=165$$

$$P(3)=\frac{{}_7C_0\cdot {}_{11}C_3}{{}_{18}C_3}=\frac{(1)(165)}{816}=0.2022$$

b) What is the mean of this distribution? **(2 points)**

Answer:

$N=18, R=11, n=3$

$$\mu=\frac{nR}{N}=\frac{(3)(11)}{18}=1.83$$

c) What is the standard deviation of this distribution? **(2 points)**

Answer:

$N = 18, R = 11, n = 3$

$$\sigma = \sqrt{\frac{nR(N-R)}{N^2}}\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{(3)(11)(18-11)}{(18)^2}}\sqrt{\frac{18-3}{18-1}} = 0.793$$

Question 5. (Show you calculation in detail for each part)

The average age of women filing for Social Security in 2010 was 63.8 years. Assume that the age for filing for Social Security follows the normal probability distribution and that the standard deviation for this population is 1.5 years.

a) What is the probability that a randomly selected woman who filed for Social Security was less than 65 years old? **(2 points)**

Answer:

$$z65 = \frac{x - \mu}{\sigma} = \frac{65 - 63.9}{1.5} = 0.80$$

$P(x < 65) = P(z < 0.80) = 0.7881$

b) do part (a) in MiniTab (or Excel) and show your result. **(1 point)**

Question 6. (Show you calculation in detail for each part)

The average cost to rent a full-size car is \$60.25 per day. Assume that this rental cost follows the normal probability distribution and that the standard deviation for this population is \$4.60. What is the probability that a randomly selected full-size rental car cost between \$50 and \$65 per day? **(2 points)**

Answer:

$$z65 = \frac{x - \mu}{\sigma} = \frac{\$65 - \$60.25}{\$4.60} = 1.03$$

$$z50 = \frac{x - \mu}{\sigma} = \frac{\$50 - \$60.25}{\$4.60} = -2.23$$

$P(\$40 \le x \le \$65) = P(-2.23 \le z \le 1.03) = 0.8485 - 0.0129 = 0.8356$

Question 7. (Show you calculation in detail for each part)

According to College Board, the average room and board costs for a private four-year college for the 2011-2012 academic year was $10,090. Assume that this cost follows the normal distribution with a standard deviation of $1,125.
a) Determine the interval of cost around the mean that includes 95% of the sample means. **(2 points)**

Hint : Refer to the empirical rule of the normal distribution

Answer:

$x = \mu \pm z\sigma$
x = $10,090 + (2)($1,125) = $12,340
x = $10,090 - (2)($1,125) = $7,840

b) Do part (a) in MiniTab (or Excel) and show your result. **(1 point)**

Question 8. (Show you calculation in detail for each part)

According to Nielsen, men account for 53% of tablet owners in 2011. A random sample of 75 tablet owners was selected.

a) Using the normal approximation to the binomial distribution, what is the probability that exactly 36 people from this sample were men? **(2 points)**

Answer:

$$\mu = np = (75)(0.53) = 39.75$$
$$\sigma = \sqrt{npq} = \sqrt{(75)(0.53)(1-0.53)} = 4.322$$
$$z_{36.5} = \frac{x-\mu}{\sigma} = \frac{36.5-39.75}{4.322} = -0.75$$
$$z_{35.5} = \frac{x-\mu}{\sigma} = \frac{35.5-39.75}{4.322} = -0.98$$
$$P(35.5 \le x \le 36.5) = P(-0.98 \le z \le -0.75) = 0.2266 - 0.1635 = 0.0631$$

b) Using the normal approximation to the binomial distribution, what is the probability that more than 34 people from this sample were men? **(2 points)**

Answer:

$$z_{43.5} = \frac{x-\mu}{\sigma} = \frac{34.5-39.75}{4.322} = -1.214$$
$$P(x \geq 34.5) = P(z \geq -1.214) = 1 - P(z \leq -1.214) = 0.887$$

Question 9. (Show you calculation in detail for each part)

The time customers spend on the phone with technical support from their Internet provider follows the uniform probability distribution between 8 and 43 minutes. What is the probability that a randomly selected student will spend more than 16 minutes on the phone with technical support? **(2 points)**

Answer:

$$P(x_1 \leq x \leq x_2) = \frac{x_2 - x_1}{b-a}$$
$$P(x \geq 16) = \frac{43-16}{43-8} = 0.7714$$

Question 10. (Show you calculation in detail for each part)

Monthly sales have a mean of $25,000 and a standard deviation of $4,000. Profits are calculated by multiplying sales by 30% and subtracting fixed costs of $6,000.

a) Find the mean monthly profit. **(2 points)**

Answer:

Profit = .30(Sales) – 6,000
E(Profit) = 30(25,000) – 6,000 = 1,500

b) Find the standard deviation of monthly profits. **(2 points)**

Answer:

Profit = .30(Sales) – 6,000

V(profit)=V[.30(Sales) – 6,000] = V[.30(Sales)] = $(.30)^2$V(Sales)=
$(30)^2$(16,000,000) = 1,440,000