

## Solutions to Capital Structure Problems

### Problem 1

We are given that the firm currently has no debt, that is, it is all-equity capital financed firm. Therefore, the firm's existing equity shareholders get all  $(1 - .35)$  \$80,000. The cost of equity capital is 25%. This means that the existing shareholders require 25% return on their capital investment in the firm. Thus:

$$\begin{aligned} V_U &= \frac{(1 - .35) (\$80,000)}{.25} \\ &= \$208,000 \end{aligned}$$

To answer the second question note that the firm borrows \$50,000 and uses this amount to repurchase \$50,000 worth of shares. Thus, it makes its capital structure levered with debt. According to M&M, the value of the levered firm,  $V_L$ , will be:

$$\begin{aligned} V_L &= V_U + t_c D, \text{ where } t_c \text{ is the corporate tax rate} \\ &= \$208,000 + (.35) (\$50,000) \\ &= \$208,000 + \$17,500 \\ &= \$225,500 \end{aligned}$$

### Problem 2

The cost of equity after "recapitalization" means the rate of return required on the levered equity after the firm is recapitalized with \$50,000 debt. Thus, the problem asks for: What is  $k_e^l$ ?

The easiest way to find  $k_e^l$  is to use the formula:

$$k_e^l = \frac{(1 - t_c)(EBIT - I)}{E_L}$$

We know:  $t_c = .35$ ,  $EBIT = \$80,000$ ,  $I = 14\%$  of \$50,000

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$$\begin{aligned}
 \text{And } E_L &= V_L - D \\
 &= \$225,500 - \$50,000 \\
 &= \$175,500
 \end{aligned}$$

Plug in these values and get:

$$\begin{aligned}
 k_e^i &= \frac{(1-.35)(\$80,000 - \$7,000)}{\$175,500} \\
 &= .27037 \text{ or } 27.037\%
 \end{aligned}$$

$$\begin{aligned}
 \text{WACC} &= \left( \frac{E_L}{V_L} \right) k_e^e + \left( \frac{D}{V_L} \right) k_b \quad \text{where } k_b = \text{after-tax int. rate} \\
 &= (1-.35) 14\% = 9.1\% \\
 &= \frac{\$175,500}{\$225,500} (27.037\%) + \frac{\$50,000}{\$225,000} (9.1\%) \\
 &= .2306 \text{ or } 23.06\%
 \end{aligned}$$

Now remember, without the debt, the cost of capital was 25%. With debt, it has gone down to 23.06%. The lower the cost of capital, higher the value of the firm. So the implication is, the firm should lever itself with debt in presence of corporate tax to benefit from the int. tax shield  $t_c D$ .

### Problem 3

Notice that the problem gives us  $\text{WACC} = 12\%$ . So, we can find  $V_L$  by using the formula:

$$\begin{aligned}
 V_L &= \frac{(1-t_c) EBIT}{\text{WACC}} \\
 &= \frac{(1-.35) \$26,000}{.12} \\
 &= \$140,833.33
 \end{aligned}$$

This answers **only** the first question in the problem. To answer the second question, we have to find  $V_U$  to see whether or not  $V_L$  exceeds  $V_U$ . But how to find  $V_U$ ? For this we know:

$$\begin{aligned}
 V_L &= V_U + t_c D \\
 \$140,833 &= V_U + .35D \quad \dots(1)
 \end{aligned}$$

Now if we can find D, we will be able to find  $V_U$  from the above equation. For this, we use:

$$k_e^l = \frac{(1-t_c)(EBIT-I)}{E_L}$$

We know:  $I = .08D$ , and  $E_L = V_L - D = \$140,833.33 - D$  ... (2)

So:

$$k_e^l = \frac{(1-.35)(\$26,000 - .08D)}{\$140,833.33 - D}$$

We can solve this equation for D if we knew  $k_e^l$ . To find  $k_e^l$  we proceed as follows:

$$\begin{aligned} \text{WACC} &= k_e^l \left( \frac{E_L}{V_L} \right) + k_b \left( \frac{D}{V_L} \right) \text{ where } k_b = (1-t_c)i \\ &= (1-.35) 8\% = 5.2\% \end{aligned}$$

$$V_L = D + E_L$$

The problem gives: debt-equity

$$\text{Ratio } \frac{D}{E_L} = .60 \Rightarrow \frac{D}{E_L} = \frac{3}{5} \Rightarrow \frac{D}{D+E_L} = \frac{3}{3+5}$$

$$\Rightarrow \frac{D}{V_L} = \frac{3}{8}, \Rightarrow \frac{E_L}{V_L} = \frac{5}{8}$$

We plug these capital structure ratios in the above equation to get:

$$.12 = k_e^l \left( \frac{5}{8} \right) + 5.2\% \left( \frac{3}{8} \right)$$

$$\Rightarrow k_e^l = .1608$$

Now go back to Equation 2 for  $k_e^l$  set up previously and plug in  $k_e^l = .1608$  and solve that equation for D.

$$.1608 = \frac{(1-.35)(\$26,000 - .08D)}{\$140,833.33 - D}$$

Solving this expression for D, we get:

$$D = \$52,812$$

Now plug this value of D, in Equation 1 and get

$$\$140,833.33 = V_U + .35 (\$52,812), \text{ which gives:}$$

$$V_U = \$122,349.13$$

Since  $V_L$  of \$140,833.33 exceeds  $V_U$ , it means the firm's debt of \$52,812 will increase its value, and even more debt will further increase its value if bankruptcy costs are ignored.

#### Problem 4

$$\begin{aligned} V_U &= \frac{(1 - .35) \$6,000}{.16} \\ &= \$24,640.63 \end{aligned}$$

The firm is currently all-equity financed and the market value of the total equity capital invested in the firm = \$24,640.63.

If the firm converts 50% of the \$24,640.63 to debt, then debt  $D = 50\%$  of \$24,640.63 = \$12,320.18 and

$$\begin{aligned} V_L &= V_U + t_c D \\ &= \$24,640.63 + .35 (\$12,320.18) \\ &= \$28,640.63 \end{aligned}$$

If D is 100% of \$24,640.63, then  $D = \$24,640$ , and

$$\begin{aligned} V_L &= \$24,375 + .35 (\$24,375) \\ &= \$32,906.25 \end{aligned}$$

#### Problem 5

Think of a balance sheet. Beta of assets (on the left side) = weighted average of Beta of equity and Beta of debt. Let's write Beta of debt as  $\beta_D$ . Thus

$$\beta_A = \beta_E \left( \frac{E}{V} \right) + \beta_D \left( \frac{D}{V} \right)$$

We are told in the problem that the debt is risk-free. Thus:

$$\beta_D = 0. \text{ So:}$$

$$\beta_A = \beta_E \left( \frac{E}{V} \right) \Rightarrow \beta_E = \beta_A \left( \frac{V}{E} \right)$$

We know  $V = D + E$ , so

$$\Rightarrow \beta_E = \beta_A \left( \frac{D + E}{E} \right)$$

$$\Rightarrow \beta_E = \beta_A \left( \frac{D}{E} + 1 \right)$$

$$\Rightarrow \beta_E = \beta_A \left( 1 + \frac{D}{E} \right)$$

### Problem 6

$$\frac{D}{E} = 0 \Rightarrow (i) \beta_E = 1(1 + 0) = 1 \quad (\text{we are given } \beta_A = 1)$$

$$\frac{D}{E} = 1 \Rightarrow (ii) \beta_E = 1(1 + 1) = 2$$

$$\frac{D}{E} = 5 \Rightarrow (iii) \beta_E = 1(1 + 5) = 6$$

$$\frac{D}{E} = 20 \Rightarrow (iv) \beta_E = 1(1 + 20) = 21$$

The message of the problem is:

The equity risk to the shareholders is composed of both business risk and financial risk from debt. Even if the assets of the firm are not very risky, the risk to the shareholders will be large if the financial leverage (reflected in high debt-equity ratio) is high. These higher levels of risk will be reflected in higher  $k_e^l$ .

### Problem 7

$$\begin{aligned} \text{If alive, PV} &= \frac{\$10M}{.18} \\ &= \$55.56M \end{aligned}$$

Since the PV if alive is less than the \$75M liquidation value, the firm should be liquidated. It is worth more “dead than alive”.

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