MATH 3705* B Test 1 January 2009

LAST NAME: ______________________ **ID#:**____________

Questions 1-6 are multiple choice. Circle the correct answer. Only the answer will be marked.

1. [**2**] $\mathcal{L}\{e^{-2t}\cos(3t)\} =$

$(a)\ \dfrac{s+2}{(s+2)^2+9}$ $(b)\ \dfrac{s}{s^2+9}$ $(c)\ \dfrac{s-2}{(s-2)^2+9}$ (d) None of the above

2. [**2**] $\mathcal{L}\{t\sin(3t)\} =$

$(a)\ \dfrac{6s}{(s^2+9)^2}$ $(b)\ \dfrac{s^2-9}{(s^2+9)^2}$ $(c)\ \dfrac{9-s^2}{(s^2+9)^2}$ (d) None of the above

3. [**2**] $\mathcal{L}\{u(t-1)\,e^{2t}\} =$

$(a)\ \dfrac{e^{-s}}{s-2}$ $(b)\ \dfrac{e^{1-s}}{s+2}$ $(c)\ \dfrac{e^{2-s}}{s-2}$ (d) None of the above

4. [**3**] $\mathcal{L}^{-1}\left\{\dfrac{2s}{s^2-25}\right\} =$

$(a)\ e^{5t}-e^{-5t}$ $(b)\ e^{5t}+e^{-5t}$ $(c)\ -\cos(5t)$ (d) None of the above

5. [**3**] $\mathcal{L}^{-1}\left\{\dfrac{se^{-3s}}{s^2+4}\right\} =$

$(a)\ u(t-3)\,\cos(2t)$ $(b)\ u(t+3)\,\cos[2(t+3)]$

$(c)\ u(t-3)\,\cos[2(t-3)]$ (d) None of the above

6. [**3**] $\mathcal{L}\left\{\displaystyle\int_0^t e^x\sin(t-x)\,dx\right\} =$

$(a)\ \dfrac{e^{-s}}{s^2+1}$ $(b)\ \dfrac{1}{(s-1)(s^2+1)}$ $(c)\ \dfrac{1}{(s-1)^2+1}$ (d) None of the above

Answers: 1.(a), 2.(a), 3.(c), 4.(b), 5.(c), 6.(b).

7. [**5 marks**] Let $f(t) = e^t$ for $0 < t < 1$ and $f(t+1) = f(t)$ for all $t \geq 0$. Find $\mathcal{L}\{f(t)\}$.

Solution:

Since f is periodic with the period $\omega = 1$, then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-s}} \int_0^1 t\,e^{-st}\,dt = \frac{1}{1-e^{-s}} \left\{ -\frac{t}{s}e^{-st} - \frac{1}{s^2}e^{-st} \right\}_0^1 = \frac{1}{1-e^{-s}} \cdot \left(-\frac{1}{s}\right) \left\{ te^{-st} + \frac{1}{s}e^{-st} \right\}_0^1 =$$

$$\frac{1}{s(e^{-s}-1)} \left(e^{-s} + \frac{1}{s}e^{-s} - \frac{1}{s} \right).$$

8. [**10 marks**] Employ the Laplace transform to solve the initial-value problem

$$y'' + 4y' + 8y = 0, \quad y(0) = 2, \; y'(0) = 4.$$

Solution:

$$[s^2Y(s) - sy(0) - y'(0)] + 4[sY(s) - y(0)] + 8Y(s) = 0$$

$$\Rightarrow (s^2 + 4s + 8)Y(s) - 2s - 12 = 0 \Rightarrow Y(s) = 2\frac{s+6}{s^2+4s+8} = 2\frac{(s+2)+4}{(s+2)^2+4} =$$

$$2\frac{s+2}{(s+2)^2+4} + 4\frac{2}{(s+2)^2+4} \quad \Rightarrow \quad y(t) = 2e^{-2t}\cos(2t) + 4e^{-2t}\sin(2t).$$