Chapter 5: The Time Value of Money

5.2 To find the FV of a lump sum, we use:

$FV = PV(1 + r)^t$

a. $FV = \$1,000(1.06)^{10}$ = \$1,790.85
b. $FV = \$1,000(1.09)^{10}$ = \$2,367.36
c. $FV = \$1,000(1.06)^{20}$ = \$3,207.14
d. Because interest compounds on the interest already earned, the interest earned in part *c* is more than twice the interest earned in part *a a, i.e. \$790.85 versus \$2,207.14.*. With compound interest, future values grow exponentially.

5.6 To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since each is the inverse of the other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for t, we get:

$t = \ln(FV / PV) / \ln(1 + r)$

The length of time to double your money is:

$FV = \$2 = \$1(1.09)^t$
$t = \ln 2 / \ln 1.09 = 8.04$ years

The length of time to quadruple your money is:

$FV = \$4 = \$1(1.09)^t$
$t = \ln 4 / \ln 1.09 = 16.09$ years

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the difference in these answers is due to rounding). This is an important concept of time value of money.

5.8 To answer this question, we can use either the FV or the PV formula. Both will give the same answer since each is the inverse of the other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for r, we get:

$r = (\text{FV} / \text{PV})^{1/t} - 1$
$r = (\$10{,}311{,}500 / \$12{,}377{,}500)^{1/4} - 1 = -0.0446$ or -4.46%

Notice that the interest rate is negative. This occurs when the FV is less than the PV.

5.19 Here, we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PV of an annuity equation:

$\text{PV} = C(\{1 - [1/(1 + r)]^t\} / r)$
$\$18{,}400 = \$600\{[1 - (1/1.009)^t] / 0.009\}$

Now, we solve for t:

$1/1.009^t = 1 - [(\$18{,}400)(0.009) / (\$600)]$
$1.009^t = 1/(0.724) = 1.381$
$t = \ln 1.381 / \ln 1.009 = 36.05$ months

5.26 This is a growing perpetuity. The present value of a growing perpetuity is:

$\text{PV} = \text{C} / (\text{r} - \text{g})$
$\text{PV} = \$215{,}000 / (0.10 - 0.04)$
$\text{PV} = \$3{,}583{,}333.33$

It is important to recognize that when dealing with annuities or perpetuities, the present value equation calculates the present value one period before the first payment. In this case, since the first payment is in two years, we have calculated the present value one year from now. To find the value today, we simply discount this value as a lump sum. Doing so, we find the value of the cash flow stream today is:

$\text{PV} = \text{FV} / (1 + \text{r})^{\text{t}}$
$\text{PV} = \$3{,}583{,}333.33 / (1 + 0.10)^1$
$\text{PV} = \$3{,}257{,}575.76$

5.31 Here, we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts. After the first six months, the balance will be:

$\text{FV} = \$6{,}000\,[1 + (0.024/12)]^6 = \$6{,}072.36$

This is the balance in six months. The FV in another six months will be:

$\text{FV} = \$6{,}072.36\,[1 + (0.18/12)]^6 = \$6{,}639.78$

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the FV. The interest accrued is:

Interest = $6,639.78 – 6,000 = $639.78

5.34 Since your salary grows at 4 percent per year, your salary next year will be:

Next year's salary = $60,000 (1 + 0.04)
Next year's salary = $62,400

This means your deposit next year will be:

Next year's deposit = $62,400(0.05)
Next year's deposit = $3,120

Since your salary grows at 4 percent, you deposit will also grow at 4 percent. We can use the present value of a growing perpetuity equation to find the value of your deposits today. Doing so, we find:

$PV = C\,\{[1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t\}$
$PV = \$3,120\{[1/(0.09 - 0.04)] - [1/(0.09 - 0.04)] \times [(1 + 0.04)/(1 + 0.09)]^{40}\}$
PV = $52,861.98

Now, we can find the future value of this lump sum in 40 years. We find:

$FV = PV(1 + r)^t$
$FV = \$52,861.98(1 + 0.09)^{40}$
FV = $1,660,364.12

This is the value of your savings in 40 years.

5.38 The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the $1,200 monthly payments is:

$PV = \$1,200[(1 - \{1 / [1 + (0.068/12)]\}^{360}) / (0.068/12)] = \$184,070.20$

The monthly payments of $1,200 will amount to a principal payment of $184,070.20. The amount of principal you will still owe is:

$250,000 – 184,070.20 = $65,929.80

This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal will be:

$\text{Balloon payment} = \$65,929.80[1 + (0.068/12)]^{360} = \$504,129.05$

5.41 Here, we are finding interest rate for an annuity cash flow. We are given the PV of the annuity, number of periods, and the amount of the annuity. We need to solve for the number of payments. We should also note that the PV of the annuity is not the amount borrowed since we are making a down payment on the warehouse. The amount borrowed is:

Amount borrowed = 0.80($2,600,000) = $2,080,000

Using the PV of an annuity equation:

$PV = \$2{,}080{,}000 = \$14{,}000[\{1 - [1 / (1 + r)]^{360}\}/ r]$

Unfortunately, this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PV of an annuity and decreasing the interest rate increases the PV of an annuity. Using a spreadsheet, we find:

$r = 0.00593$ or 0.593%

The APR is the monthly interest rate times the number of months in the year, so:

APR = 12(0.593%) = 7.12%

And the EAR is:

$EAR = (1 + .00593)^{12} - 1 = .0735$ or 7.35%

5.45 Here, we are trying to find the dollar amount invested today that will equal the FV of the annuity with a known interest rate, and payments. First, we need to determine how much we would have in the annuity account. Finding the FV of the annuity, we get:

$FV = \$1{,}200\ [\{[\ 1 + (0.098/12)]^{180} - 1\}\ /\ (0.098/12)] = \$488{,}328.61$

Now, we need to find the PV of a lump sum that will give us the same FV. So, using the FV of a lump sum with continuous compounding, we get:

$FV = \$488{,}328.61 = PVe^{0.09(15)}$
$PV = \$488{,}328.61e^{-1.35} = \$126{,}594.44$

5.51 The monthly interest rate is the annual interest rate divided by 12, or:

Monthly interest rate = 0.104 / 12
Monthly interest rate = 0.00867

Now we can set the present value of the lease payments equal to the cost of the equipment, or $3,500. The lease payments are in the form of an annuity due, so:

PV $\text{Annuity}_{\text{due}} = (1 + r)\ C(\{1 - [1/(1 + r)]^t\ \} / r)$
$\$3{,}500 = (1 + 0.00867)\ C(\{1 - [1/(1 + 0.00867)]^{24}\ \} / 0.00867)$
$C = \$160.76$

5.53 The salary is a growing annuity, so using the equation for the present value of a growing annuity. The salary growth rate is 3.5 percent and the discount rate is 12 percent, so the value of the salary offer today is:

$\text{PV} = C\ \{[1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t\}$
$\text{PV} = \$45{,}000\{[1/(0.12 - 0.035)] - [1/(0.12 - 0.035)] \times [(1 + 0.035)/(1 + 0.12)]^{25}\}$
$\text{PV} = \$455{,}816.18$

The yearly bonuses are 10 percent of the annual salary. This means that next year's bonus will be:

Next year's bonus = 0.10($45,000)
Next year's bonus = $4,500

Since the salary grows at 3.5 percent, the bonus will grow at 3.5 percent as well. Using the growing annuity equation, with a 3.5 percent growth rate and a 12 percent discount rate, the present value of the annual bonuses is:

$\text{PV} = C\ \{[1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t\}$
$\text{PV} = \$4{,}500\{[1/(0.12 - 0.035)] - [1/(0.12 - 0.035)] \times [(1 + 0.035)/(1 + 0.12)]^{25}\}$
$\text{PV} = \$45{,}581.62$

Notice the present value of the bonus is 10 percent of the present value of the salary. The present value of the bonus will always be the same percentage of the present value of the salary as the bonus percentage. So, the total value of the offer is:

PV = PV(Salary) + PV(Bonus) + Bonus paid today
PV = $455,816.18 + 45,581.62 + 10,000
PV = $511,397.80

5.55 We need to find the first payment into the retirement account. The present value of the desired amount at retirement is:

$\text{PV} = \text{FV}/(1 + r)^t$
$\text{PV} = \$1{,}500{,}000/(1 + 0.10)^{30}$
$\text{PV} = \$85{,}962.83$

This is the value today. Since the savings are in the form of a growing annuity, we can use the growing annuity equation and solve for the payment. Doing so, we get:

$\text{PV} = C\ \{[1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t\}$
$\$85{,}962.83 = C\{[1/(0.10 - 0.03)] - [1/(0.10 - 0.03)] \times [(1 + 0.03)/(1 + 0.10)]^{30}\}$

C = \$6,989.68

This is the amount you need to save next year. So, the percentage of your salary is:

Percentage of salary = \$6,989.68/\$70,000
Percentage of salary = 0.0999 or 9.99%

Note that this is the percentage of your salary you must save each year. Since your salary is increasing at 3 percent, and the savings are increasing at 3 percent, the percentage of salary will remain constant.

5.58 To answer this question, we should find the PV of both options, and compare them. Since we are purchasing the car, the lowest PV is the best option. The PV of the leasing is simply the PV of the lease payments, plus the \$1. The interest rate we would use for the leasing option is the same as the interest rate of the loan. The PV of leasing is:

$PV = \$1 + \$520\{1 – [1 / (1 + 0.08/12)^{12(3)}]\} / (0.08/12) = \$16,595.14$

The PV of purchasing the car is the current price of the car minus the PV of the resale price. The PV of the resale price is:

$PV = \$26,000 / [1 + (0.08/12)]^{12(3)} = \$20,468.62$

The PV of the decision to purchase is:

\$38,000 – 20,468.62 = \$17,531.38

In this case, it is cheaper to lease the car than buy it since the PV of the leasing cash flows is lower. To find the breakeven resale price, we need to find the resale price that makes the PV of the two options the same. In other words, the PV of the decision to buy should be:

\$38,000 – PV of resale price = \$16,595.14
PV of resale price = \$21,404.86

The resale price that would make the PVs of the two alternatives the same is the FV of this value, so:

$\text{Breakeven resale price} = \$21,404.86[1 + (0.08/12)]^{12(3)} = \$27,189.25$

5.61 Here, we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate, so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:
$APR = 12[(1.09)^{1/12} – 1] = 0.0865$ or 8.65%

To find the value today of the back pay from two years ago, we will find the FV of the annuity (salary), and then find the FV of the lump sum value of the salary. Doing so gives us:

$FV = (\$42,000/12) [\{[1 + (0.0865/12)]^{12} – 1\} / (0.0865/12)] (1 + 0.09) = \$47,639.05$

Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way.

Now, we need to find the value today of last year's back pay:

$FV = (\$45,000/12) [\{[1 + (0.0865/12)]^{12} – 1\} / (0.0865/12)] = \$46,827.37$

Next, we find the value today of the five year's future salary:

$PV = (\$49,000/12)\{[\{1 – \{1 / [1 + (0.0865/12)]^{12(5)}\}] / (0.0865/12)\}= \$198,332.55$

The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

Award = \$47,639.05 + 46,827.37 + 198,332.55 + 150,000 + 25,000
Award = \$467,798.97

As the plaintiff, you would prefer a lower interest rate. In this problem, we are calculating both the PV and FV of annuities. A lower interest rate will decrease the FV of an annuity, but increase the PV of an annuity. So, by a lower interest rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time, this is the more important cash flow to the plaintiff.

5.65 Here, we are solving a two-step time value of money problem. Each question asks for a different possible cash flow to fund the same retirement plan. Each savings possibility has the same FV, that is, the PV of the retirement spending when your friend is ready to retire. The amount needed when your friend is ready to retire is:

$PV = \$110,000\{[1 – (1/1.09)^{25}] / 0.09\} = \$1,080,483.76$

This amount is the same for all three parts of this question.

a. If your friend makes equal annual deposits into the account, this is an annuity with the FV equal to the amount needed in retirement. The required savings each year will be:

$FV = \$1,080,483.76 = C[(1.09^{30} – 1) / 0.09]$
$C = \$7,926.81$

b. Here we need to find a lump sum savings amount. Using the FV for a lump sum equation, we get:

FV = \$1,080,483.76 = PV$(1.09)^{30}$
PV = \$81,437.29

c. In this problem, we have a lump sum savings in addition to an annual deposit. Since we already
know the value needed at retirement, we can subtract the value of the lump sum savings at retirement to find out how much your friend is short. Doing so gives us:

FV of trust fund deposit = \$50,000$(1.09)^{10}$ = \$118,368.18

So, the amount your friend still needs at retirement is:

FV = \$1,080,483.76 – 118,368.18 = \$962,115.58

Using the FV equation, and solving for the payment, we get:

\$962,115.58 = $C[(1.09^{30} – 1) / 0.09]$

C = \$7,058.42

This is the total annual contribution, but your friend's employer will contribute \$1,500 per year, so your friend must contribute:

Friend's contribution = \$7,058.42 – 1,500 = \$5,558.42

5.66 We will calculate the number of periods necessary to repay the balance with no fee first. We simply need to use the PV of an annuity equation and solve for the number of payments.

Without fee and annual rate = 18.6%:

PV = \$9,000 = \$200{[1 – $(1/1.0155)^t$] / .0155 } where .0155 = .186/12

Solving for t, we get:

t = ln{1 / [1 – (\$9,000/\$200)(0.0155)]} / ln(1.0155)
t = ln 3.3058 / ln 1.0155
t = 77.74 months

Without fee and annual rate = 8.2%:

PV = \$9,000 = \$200{[1 – $(1/1.006833)^t$] / 0.006833 } where 0.006833 = 0.082/12

Solving for t, we get:

t = ln{1 / [1 – ($9,000/$200)(.006833)]} / ln(1.006833)
t = ln 1.4440 / ln 1.006833
t = 53.96 months

Note that we do not need to calculate the time necessary to repay your current credit card with a fee since no fee will be incurred. The time to repay the new card with a transfer fee is:

With fee and annual rate = 8.20%:

PV = $9,180 = $200{ [1 – (1/1.006833)^t] / 0.006833 } where 0.006833 = 0.092/12

Solving for t, we get:

t = ln{1 / [1 – ($9,180/$200)(0.006833)]} / ln(1.006833)
t = ln 1.45698 / ln 1.006833
t = 55.27 months

5.75 Since it is only an approximation, we know the Rule of 72 is exact for only one interest rate. Using the basic future value equation for an amount that doubles in value and solving for t, we find:

$FV = PV(1 + r)^t$
$\$2 = \$1(1 + r)^t$
$\ln(2) = t \ln(1 + r)$
$t = \ln(2) / \ln(1 + r)$

We also know the Rule of 72 approximation is:

$t = 72 / r$

We can set these two equations equal to each other and solve for r. We also need to remember that the exact future value equation uses decimals, so the equation becomes:

$.72 / r = \ln(2) / \ln(1 + r)$
$0 = (0.72 / r) / [\ln(2) / \ln(1 + r)]$

It is not possible to solve this equation directly for R, but using Solver, we find the interest rate for which the Rule of 72 is exact is 7.846894 percent.

Chapter 6: How to Value Bonds and Stocks

6.5 The price of any bond is the PV of the interest payment, plus the PV of the par value. The fact that the bond is denominated in euros is irrelevant. Notice this problem assumes an annual coupon. The price of the bond will be:

PV = €84 $A^{15}_{7.6\%}$ + €1,000/$(1 + 0.076)^{15}$
PV = €1,070.18

6.7 Here we are finding the price of semiannual coupon bonds for various maturity lengths. The bond price equation is:

PV = $C\,A^{t}_{r\%}$ + \$1,000/$(1+ r)^{t}$

Miller Corporation bond:

PV_0 = \$45 $A^{26}_{3.5\%}$ + \$1,000/$(1+0.035)^{26}$ = \$1,168.90
PV_1 = \$45 $A^{24}_{3.5\%}$ + \$1,000/$(1+0.035)^{24}$ = \$1,160.58
PV_3 = \$45 $A^{20}_{3.5\%}$ + \$1,000/$(1+0.035)^{20}$ = \$1,142.12
PV_8 = \$45 $A^{10}_{3.5\%}$ + \$1,000/$(1+0.035)^{10}$ = \$1,083.17
PV_{12} = \$45 $A^{2}_{3.5\%}$ + \$1,000/$(1+0.035)^{2}$ = \$1,019.00
PV_{13} = \$1,000

Modigliani Company bond:

PV_0 = \$35 $A^{26}_{4.5\%}$ + \$1,000/$(1+0.045)^{26}$ = \$848.53
PV_1 = \$35 $A^{24}_{4.5\%}$ + \$1,000/$(1+0.045)^{24}$ = \$855.05
PV_3 = \$35 $A^{20}_{5\%}$ + \$1,000/$(1+0.045)^{20}$ = \$869.92
PV_8 = \$35 $A^{10}_{4.5\%}$ + \$1,000 /$(1+0.045)^{10}$= \$920.87
PV_{12} = \$35 $A^{2}_{4.5\%}$ + \$1,000/$(1+0.045)^{2}$ = \$981.27
PV_{13} = \$1,000

All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called "pull to par." In both cases, the largest percentage price changes occur at the shortest maturity lengths.
Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

6.9 Initially, at a YTM of 10 percent, the prices of the two bonds are:

$PV_{Faulk} = \$30\ A_{5\%}^{16} + \$1{,}000/(1+0.05)^{16} = \$783.24$

$PV_{Gonas} = \$70\ A_{5\%}^{16} + \$1{,}000/(1+0.05)^{16} = \$1{,}216.76$

If the YTM rises from 10 percent to 12 percent:

$PV_{Faulk} = \$30\ A_{6\%}^{16} + \$1{,}000/(1+0.06)^{16} = \$696.82$

$PV_{Gonas} = \$70\ A_{6\%}^{16} + \$1{,}000/(1+0.06)^{16} = \$1{,}101.06$

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$\Delta P_{Faulk}\% = (\$696.82 - 783.24)/\$783.24 = -0.1103$ or -11.03%
$\Delta P_{Gonas}\% = (\$1{,}101.06 - 1{,}216.76)/\$1{,}216.76 = -0.0951$ or -9.51%

If the YTM declines from 10 percent to 8 percent:

$PV_{Faulk} = \$30\ A_{4\%}^{16} + \$1{,}000/(1+0.04)^{16} = \$883.48$

$PV_{Gonas} = \$70\ A_{4\%}^{16} + \$1{,}000/(1+0.04)^{16} = \$1{,}349.57$

$\Delta PV_{Faulk}\% = (\$883.48 - 783.24)/\$783.24 = +0.1280$ or 12.80%

$\Delta PV_{Gonas}\% = (\$1{,}349.57 - 1{,}216.76)/\$1{,}216.76 = +0.1092$ or 10.92%

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

6.10 The bond price equation for this bond is:

$PV = \$960 = \$37\ A_{r}^{18} + \$1{,}000/(1+r)^{18}$

Using a spreadsheet, financial calculator, or trial and error we find:

$r = 4.016\%$

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 4.016% = 8.03%

The current yield is:

Current yield = Annual coupon payment / Price = \$74 / \$960 = 0.0771 or 7.71%

The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

Effective annual yield = $(1+0.04016)^2 - 1$ = 0.0819 or 8.19%

6.11 The company should set the coupon rate on its new bonds equal to the required return. The required return can be observed in the market by finding the YTM on outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:

$PV = \$1{,}063 = \$50\,A_r^{40} + \$1{,}000/(1+r)^{40}$

Using a spreadsheet, financial calculator, or trial and error we find:

$r = 4.650\%$

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 4.650% = 9.30%

6.13 The bond has 10 years to maturity, so the bond price equation is:

$PV = \$871.55 = \$41.25\,A_r^{20} + \$1{,}000/(1+r)^{10}$

Using a spreadsheet, financial calculator, or trial and error we find:

$r = 5.171\%$

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 5.171% = 10.34%

The current yield is the annual coupon payment divided by the bond price, so:

Current yield = \$82.50/\$871.55 = 0.0947 or 9.47%

6.16 *a*. The rate of return you expect to earn if you purchase a bond and hold it until maturity is the YTM. The bond price equation for this bond is:

$PV = \$1{,}140 = \$90\,A_r^{10} + \$1{,}000/(1+r)^{10}$

Using a spreadsheet, financial calculator, or trial and error we find:

r = YTM = 7.01%

b. To find our HPY, we need to find the price of the bond in two years. The price of the bond in two years, at the new interest rate, will be:

$$PV = \$90\,A^{8}_{6.01\%} + \$1{,}000/(1+0.0601)^{8} = \$1{,}185.87$$

To calculate the HPY, we need to find the interest rate that equates the price we paid for the bond with the cash flows we received. The cash flows we received were $90 each year for two years, and the price of the bond when we sold it. The equation to find our HPY is:

$$PV = \$1{,}140 = \$90\,A^{2}_{r\%} + \$1{,}185.87/(1+r)^{5}$$

Solving for r, we get:

r = HPY = 9.81%

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

6.17 The price of any bond (or financial instrument) is the PV of the future cash flows. Even though Bond M makes different coupons payments, to find the price of the bond, we just find the PV of the cash flows. The PV of the cash flows for Bond M is:

$$PV_M = \$800\,A^{16}_{4\%}\ (1/(1+0.04)^{12}) + \$1{,}000\,A^{12}_{4\%}\ (1/(1+0.04)^{28}) + \$20{,}000/(1+0.04)^{40}$$
$$PV_M = \$13{,}117.88$$

Notice that for the coupon payments of $800, we found the PV of the annuity for the coupon payments, and then discounted the lump sum back to today.

Bond N is a zero coupon bond with a $20,000 par value; therefore, the price of the bond is the PV of the par, or:

$$PV_N = \$20{,}000/(1+0.04)^{40} = \$4{,}165.78$$

6.26 The growth rate of earnings is the return on equity times the retention ratio, so:

$g = \text{ROE} \times b$
$g = 0.15(0.70)$
$g = 0.1050$ or 10.50%

To find next year's earnings, we simply multiply the current earnings times one plus the growth rate, so:

Next year's earnings = Current earnings(1 + g)
Next year's earnings = \$28,000,000(1+0.1050)
Next year's earnings = \$30,940,000

6.30 With differential dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the differential growth period. The stock begins constant growth in Year 5, so we can find the price of the stock in Year 4, one year before the constant dividend growth begins, as:

$P_4 = D_4(1 + g)/(r - g) = \$2.50(1.05)/(0.13 - 0.05) = \$32.81$

The price of the stock today is the PV of the first four dividends, plus the PV of the Year 4 stock price. So, the price of the stock today will be:

$P_0 = \$9/1.13 + \$7/1.13^2 + \$5/1.13^3 + (\$2.50+32.81)/1.13^4 = \$38.57$

6.33 The constant growth model can be applied even if the dividends are declining by a constant percentage, just make sure to recognize the negative growth. So, the price of the stock today will be:

$P_0 = D_0(1 + g)/(r - g) = \$12(1 - 0.06)/[(0.11 - (- 0.06)] = \$66.35$

6.37 Here we have a stock paying a constant dividend for a fixed period, and an increasing dividend thereafter. We need to find the present value of the two different cash flows using the appropriate quarterly interest rate. The constant dividend is an annuity, so the present value of these dividends is:

$PV_{Annuity} = C\ A^{t}_{r\%}$
$PV_{Annuity} = \$0.75\ A^{12}_{2.5\%}$
$PV_{Annuity} = \$7.69$

Now we can find the present value of the dividends beyond the constant dividend phase. Using the present value of a growing annuity equation, we find:

$P_{12} = D_{13} /(r - g)$
$P_{12} = \$0.75(1 + 0.01)/(0.025 - 0.01)$
$P_{12} = \$50.50$

This is the price of the stock immediately after it has paid the last constant dividend. So, the present value of the future price is:

$PV = \$50.50/(1 + 0.025)^{12}$
$PV = \$37.55$

The price today is the sum of the present value of the two cash flows, so:

$P_0 = \$7.69+37.55$
$P_0 = \$45.24$

6.40 First, we need to find the annual dividend growth rate over the past four years. To do this, we can use the future value of a lump sum equation, and solve for the interest rate. Doing so, we find the dividend growth rate over the past four years was:

$FV = PV(1 + r)^t$
$\$1.93 = \$1.20(1 + r)^4$
$r = (\$1.93 / \$1.20)^{1/4} – 1$
$r = 0.1261$ or 12.61%

We know the dividend will grow at this rate for five years before slowing to a constant rate indefinitely. So, the dividend amount in seven years will be:

$D_7 = D_0(1 + g_1)^5(1 + g_2)^2$
$D_7 = \$1.93(1 + 0.1261)^5(1 + 0.07)^2$
$D_7 = \$4.00$

6.41 *a.* We can find the price of all the outstanding company stock by using the dividends the same way we would value an individual share. Since earnings are equal to dividends, and there is no growth, the value of the company's stock today is the present value of a perpetuity, so:

$P = D / R$
$P = \$750,000/0.14$
$P = \$5,357,142.86$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings ratio of each company with no growth is:

P/E = Price / Earnings
P/E = \$5,357,142.86/\$750,000
P/E = 7.14 times

b. Since the earnings have increased, the price of the stock will increase. The new price of the all the outstanding company stock is:

$P = D / r$
$P = (\$750,000+100,000)/0.14$
$P = \$6,071,428.57$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings with the increased earnings is:

P/E = Price / Earnings
P/E = $6,071,428.57/$750,000
P/E = 8.10 times

c. Since the earnings have increased, the price of the stock will increase. The new price of the all the outstanding company stock is:

P = D / *r*
P = ($750,000+200,000)/0.14
P = $6,785,714.29

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings with the increased earnings is:

P/E = Price / Earnings
P/E = $6,785,714.29/$750,000
P/E = 9.05 times

6.43*a.* The price of the stock is the present value of the dividends. Since earnings are equal to dividends, we can find the present value of the earnings to calculate the stock price. Also, since we are excluding taxes, the earnings will be the revenues minus the costs. We simply need to find the present value of all future earnings to find the price of the stock. The present value of the revenues is:

$PV_{Revenue} = C_1 / (r - g)$
$PV_{Revenue} = \$6,000,000(1+0.05)/(0.15 - 0.05)$
$PV_{Revenue} = \$63,000,000$

And the present value of the costs will be:

$PV_{Costs} = C_1 / (r - g)$
$PV_{Costs} = \$3,100,000(1+0.05)/(0.1–0.05)$
$PV_{Costs} = \$32,550,000$

Since there are no taxes, the present value of the company's earnings and dividends will be:

$PV_{Dividends} = \$63,000,000 - 32,550,000$
$PV_{Dividends} = \$30,450,000$

Note that since revenues and costs increase at the same rate, we could have found the present value of future dividends as the present value of current dividends. Doing so, we find:

D_0 = Revenue$_0$ – Costs$_0$
D_0 = \$6,000,000 – 3,100,000
D_0 = \$2,900,000

Now, applying the growing perpetuity equation, we find:

$PV_{Dividends} = C_1 / (r - g)$
$PV_{Dividends}$ = \$2,900,000(1+0.05)/(0.15–0.05)
$PV_{Dividends}$ = \$30,450,000

This is the same answer we found previously. The price per share of stock is the total value of the company's stock divided by the shares outstanding, or:

P = Value of all stock / Shares outstanding
P = \$30,450,000/1,000,000
P = \$30.45

b. The value of a share of stock in a company is the present value of its current operations, plus the present value of growth opportunities. To find the present value of the growth opportunities, we need to discount the cash outlay in Year 1 back to the present, and find the value today of the increase in earnings. The increase in earnings is a perpetuity, which we must discount back to today. So, the value of the growth opportunity is:

NPVGO = $C_0 + C_1 / (1 + r) + (C_2 / r) / (1 + r)$
NPVGO = –\$22,000,000–\$8,000,000/(1+0.15)+(\$7,000,000/0.15)/(1+0.15)
NPVGO = \$11,623,188.41

To find the value of the growth opportunity on a per share basis, we must divide this amount by the number of shares outstanding, which gives us:

$NPVGO_{Per\ share}$ = \$11,623,188.41/\$1,000,000
$NPVGO_{Per\ share}$ = \$11.62

The stock price will increase by \$11.62 per share. The new stock price will be:

New stock price = \$30.45 + 11.62
New stock price = \$42.07

6.47 *a.* If the company does not make any new investments, the stock price will be the present value of the constant perpetual dividends. In this case, all earnings are paid as dividends, so, applying the perpetuity equation, we get:

P = Dividend / r
P = \$6.25/0.13
P = \$48.08

b. The investment occurs every year in the growth opportunity, so the opportunity is a growing perpetuity. So, we first need to find the growth rate. The growth rate is:

g = Retention Ratio × Return on Retained Earnings
g = 0.20 × 0.11
g = 0.022 or 2.20%

Next, we need to calculate the NPV of the investment. During year 3, 20 percent of the earnings will be reinvested. Therefore, $1.25 is invested ($6.25 × 0.20). One year later, the shareholders receive an 11 percent return on the investment, or $0.138 ($1.25 × 0.11), in perpetuity. The perpetuity formula values that stream as of year 3. Since the investment opportunity will continue indefinitely and grows at 2.2 percent, apply the growing perpetuity formula to calculate the NPV of the investment as of year 2. Discount that value back two years to today.

NPVGO = $[(\text{Investment} + \text{Return} / r) / (r - g)] / (1 + r)^2$
NPVGO = $[(-\$1.25+\$0.138/0.13)/(0.13-0.022)]/(1.13)^2$
NPVGO = –$1.39

The value of the stock is the PV of the firm without making the investment plus the NPV of the investment, or:

P = PV(EPS) + NPVGO
P = $48.08 – 1.39
P = $46.68

c. Zero percent! There is no retention ratio which would make the project profitable for the company. If the company retains more earnings, the growth rate of the earnings on the investment will increase, but the project will still not be profitable. Since the return of the project is less than the required return on the company stock, the project is never worthwhile. In fact, the more the company retains and invests in the project, the less valuable the stock becomes.

6.48 Here we have a stock with differential growth, but the dividend growth changes every year for the first four years. We can find the price of the stock in Year 3 since the dividend growth rate is constant after the third dividend. The price of the stock in Year 3 will be the dividend in Year 4, divided by the required return minus the constant dividend growth rate. So, the price in Year 3 will be:

$P_3 = \$4.20(1.20)(1.15)(1.10)(1.05)/(0.12-0.05) = \$95.63$

The price of the stock today will be the PV of the first three dividends, plus the PV of the stock price in Year 3, so:

$P_0 = \$4.20(1.20)/(1.12) + \$4.20(1.20)(1.15)/1.12^2 + \$4.20(1.20)(1.15)(1.10)/1.12^3 + \$95.63/1.12^3$

$P_0 = \$81.73$

6.50 In this problem, growth is occurring from two different sources: The learning curve and the new project. We need to separately compute the value from the two different sources. First, we will compute the value from the learning curve, which will increase at 5 percent. All earnings are paid out as dividends, so we find the earnings per share are:

EPS = Earnings/total number of outstanding shares
EPS = ($15,000,000×1.05)/10,000,000
EPS = $1.58

From the NPVGO mode:

$P = EPS/(r - g) + NPVGO$
P = $1.58/(0.10–0.05) + NPVGO
P = $31.50 + NPVGO

Now we can compute the NPVGO of the new project to be launched two years from now. The earnings per share two years from now will be:

$EPS_2 = \$1.58(1+ 0.05)^2$
$EPS_2 = \$1.6538$

Therefore, the initial investment in the new project will be:

Initial investment = 0.30($1.6538)
Initial investment = $0.50

The earnings per share of the new project is a perpetuity, with an annual cash flow of:

Increased EPS from project = $6,500,000/10,000,000 shares
Increased EPS from project = $0.65

So, the value of all future earnings in year 2, one year before the company realizes the earnings, is:

PV = $0.65/0.10
PV = $6.50

Now, we can find the NPVGO per share of the investment opportunity in year 2, which will be:

$NPVGO_2 = -\$0.50 + 6.50$
$NPVGO_2 = \$6.00$

The value of the NPVGO today will be:

$$NPVGO = \$6.00/(1+0.10)^2$$
$$NPVGO = \$4.96$$

Plugging in the NPVGO model we get;

$$P = \$31.50 + 4.96$$
$$P = \$36.46$$

Note that you could also value the company and the project with the values given, and then divide the final answer by the shares outstanding. The final answer would be the same.