1. [2 marks] (Multiple choice question). The limit $\lim_{x \to 0} \dfrac{x^2}{1 - \cos x}$ is equal to:

   A) -2 B) 1 C) 0 D) 2 E) None of these

   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. $\boxed{\text{D)}}$

2. [2 marks] (Multiple choice question). Let $G(x) = \int_x^1 \cos \sqrt{t} dt$. The derivative $G'(x)$ of the function G is equal to:

   A) $-\sin \sqrt{x}$ B) $-\dfrac{\sin \sqrt{x}}{2\sqrt{x}}$ C) $-\cos \sqrt{x}$ D) $\cos \sqrt{1}$

   E) None of these

   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. $\boxed{\text{C)}}$

3. [2 marks] (Multiple choice question). The integral $\int_{-1}^{1} |1 - 2x| dx$ is equal to

   A) $\dfrac{3}{2}$ B) 2 C) $\dfrac{5}{2}$ D) $-\dfrac{5}{2}$ E) None of these

   Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. $\boxed{\text{C)}}$

4. [9 marks] Evaluate the following integrals:

a) $\int_{\pi/3}^{\pi/2} (\theta - \sec\theta \tan\theta) d\theta$

b) $\int \frac{e^t + 1}{e^t + t} dt$

a) $\int_0^{\pi/3} (\theta - \sec\theta \tan\theta) d\theta = \left(\frac{\theta^2}{2} - \sec\theta\right)\Big|_0^{\pi/3}$

$= \frac{\pi^2}{18} - \frac{1}{\cos\frac{\pi}{3}} - \left(\frac{0^2}{2} - \frac{1}{\cos 0}\right) = \boxed{\frac{\pi^2}{18} - 1}$

b) $\int \frac{e^t+1}{e^t+t} dt$

$u = e^t + t$

$du = (e^t + 1) dt$

$= \int \frac{du}{u} = \ln|u|$

$= \boxed{\ln|e^t + t| + C}$

5. [5 marks] Consider the definite integral $\int_{-1}^{2}(3-5x)dx$.

a) Express the integral as a limit of Riemann sums

b) Find the value of the integral by evaluating the limit of the Rieman sum in part a)

Some useful facts: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

a) $\Delta x = \frac{2-(-1)}{n} = \frac{3}{n}$, $x_i = -1 + i\Delta x = -1 + \frac{3i}{n}$

$$\int_{-1}^{2}(3-5x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(3-5\left(-1+\frac{3i}{n}\right)\right)\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(8-\frac{15i}{n}\right)\cdot\frac{3}{n}$$

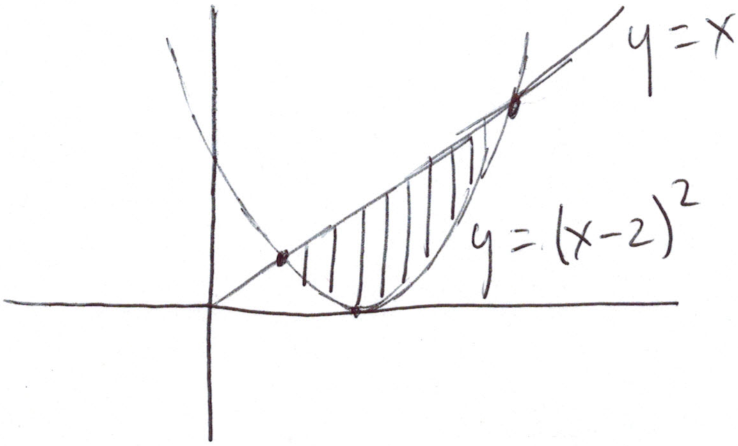
b) $$= \lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(\frac{24}{n}-\frac{45i}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\sum_{i=1}^{n}\frac{24}{n} - \sum_{i=1}^{n}\frac{45i}{n^2}\right]$$

$$= \lim_{n\to\infty}\left[24 - \frac{45}{n^2}\sum_{i=1}^{n} i\right] = 24 - \lim_{n\to\infty}\frac{45}{n^2}\cdot\frac{n(n+1)}{2}$$

$$= 24 - \frac{45}{2} = \boxed{\frac{3}{2}}$$

6. [6 marks] Find the area of the region enclosed by the curves $y = (x-2)^2$ and $y = x$.



Intersection Points:

$(x-2)^2 = x$

$x^2 - 4x + 4 = x$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

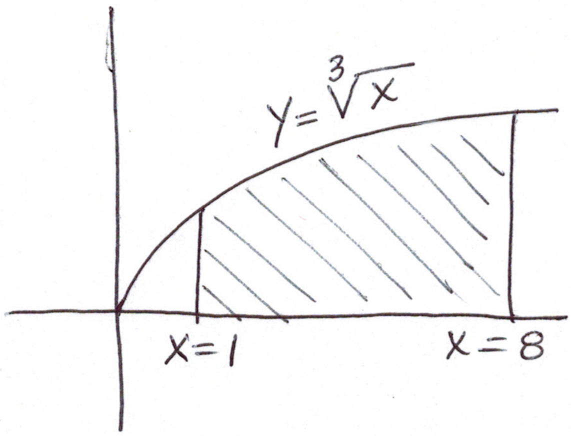
$\boxed{x=1}, \boxed{x=4}$

$$A = \int_1^4 \left(x - (x-2)^2\right) dx$$

$$= \int_1^4 \left(-x^2 + 5x - 4\right) dx = \left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right)\Big|_1^4$$

$$= \left(-\frac{64}{3} + 40 - 16\right) - \left(-\frac{1}{3} + \frac{5}{2} - 4\right) = \boxed{\frac{9}{2}}$$

7. [6 marks] Find the volume of the solid generated by rotating the region bounded by $y = \sqrt[3]{x}$, $y = 0$, $x = 1$ and $x = 8$ about the y-axis.



Method: Cylindrical Shells

$$V=\int_1^8 2\pi x\cdot f(x)\,dx=\int_1^8 2\pi x\sqrt[3]{x}\,dx=\int_1^8 2\pi x^{4/3}dx$$

$$=2\pi\,\frac{x^{7/3}}{7/3}\Big|_1^8=\frac{6\pi}{7}\left(8^{7/3}-1^{7/3}\right)=\frac{6\pi}{7}(128-1)$$

$$=\boxed{\frac{762\pi}{7}}$$

8. [10 marks] Consider the function $f(x) = \dfrac{2x-7}{(x-1)^2}$.

a)[2 marks] Use f' to find the intervals on which f is increasing or decreasing

b)[2 marks] Find the local maximum and minimum values of f

c)[2 marks] Find the intervals on which f is concave upward or concave downward.

d)[2 marks] Find the vertical and horizontal asymptotes of f

e)[2 marks] Use the information from parts a)-d) to sketch the graph of f

a) $$f'(x) = \frac{2(x-1)^2 - (2x-7)\cdot 2(x-1)}{(x-1)^4} = \frac{2(x-1) - 2(2x-7)}{(x-1)^3}$$

$$= \frac{-2x+12}{(x-1)^3} = \frac{-2(x-6)}{(x-1)^3}$$

- If $x<1$ then $f'(x)<0 \Rightarrow f$ decreasing
- If $1<x<6$ then $f'(x)>0 \Rightarrow f$ increasing
- If $x>6$ then $f'(x)<0 \Rightarrow f$ decreasing

b) At $x=6$ the derivative changes sign (from + to −) so at $x=6$ there is a local maximum.

c) $$f''(x) = \frac{-2(x-1)^3 + 2(x-6)\cdot 3(x-1)^2}{(x-1)^6}$$

$$= \frac{-2(x-1)+6(x-6)}{(x-1)^4} = \frac{4x-34}{(x-1)^4} = \frac{4\left(x-\frac{17}{2}\right)}{(x-1)^4}$$

c) Cont.

- If $x<1$ then $f''(x)<0$: Concave downward
- If $1<x<\frac{17}{2}$ then $f''(x)<0$: Concave downward
- If $x>\frac{17}{2}$ then $f''(x)>0$: Concave upward.

d)

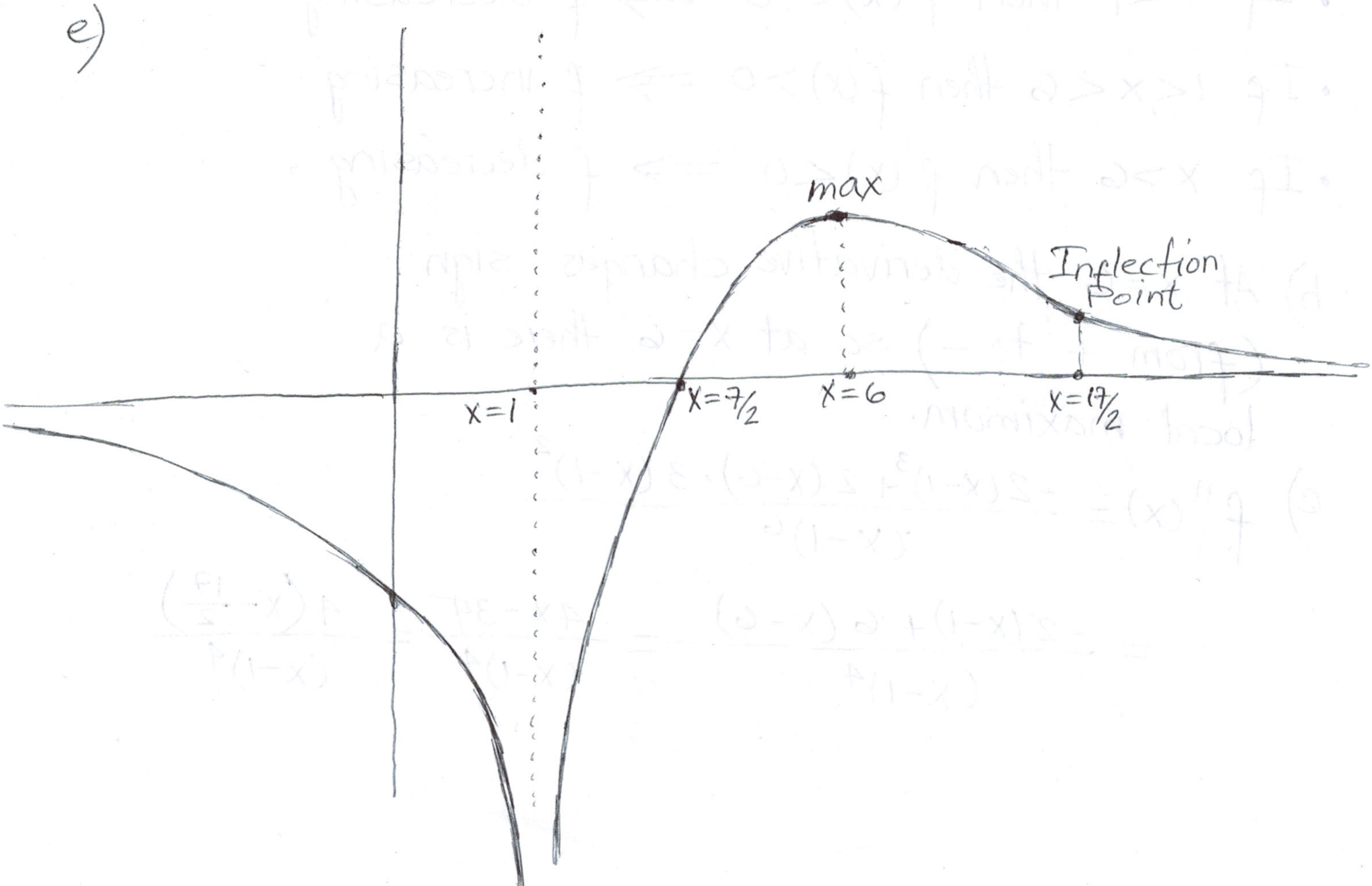
- $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = -\infty$

So $x=1$ is a vertical asymptote.

- $\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(x) = 0$

So $y=0$ is a horizontal asymptote.

e)



9. [8 marks] A cylindrical can without a top is made to contain $8\pi cm^3$ of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

$$V = \pi r^2 h = 8\pi \Rightarrow r^2 h = 8 \Rightarrow h = \frac{8}{r^2}$$

$$A = 2\pi r h + \pi r^2$$

$$= 2\pi r \cdot \frac{8}{r^2} + \pi r^2 = \frac{16\pi}{r} + \pi r^2$$

$$A'(r) = -\frac{16\pi}{r^2} + 2\pi r = \frac{2\pi(-8 + r^3)}{r^2}$$

So $r=2$ is a critical number.

- If $0 < r < 2$ then $A'(r) < 0$
- If $r > 2$ then $A'(r) > 0$

$\Big\}$ Then at $r=2$ there is an absolute minimum.

Dimensions: $\boxed{r=2}$ and $\boxed{h=2}$