

1. Let  $A = \begin{bmatrix} a & -b & c \\ b & c & -a \\ -c & a & b \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ .

What is the second column of  $A^T B$ ?

a)  $\begin{bmatrix} b+c \\ a-c \\ a-b \end{bmatrix}$     b)  $\begin{bmatrix} a-c \\ a-b \\ c+b \end{bmatrix}$     c)  $\begin{bmatrix} a+b \\ -b+c \\ -a+c \end{bmatrix}$     d)  $\begin{bmatrix} b-c \\ a+c \\ -a+b \end{bmatrix}$

2. Let  $A = \begin{bmatrix} k & 5 & 7 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ . For which value of  $k$  is the matrix  $A$  not invertible?

- a) 14    b) 10    c) 7    d) -10

3. What is the standard form of the complex number  $\frac{3-2i}{1+3i}$ ?

a)  $\frac{3}{10} - \frac{11}{10}i$     b)  $-\frac{3}{10} - \frac{11}{10}i$     c)  $\frac{3}{10} + \frac{11}{10}i$     d)  $-\frac{3}{10} + \frac{11}{10}i$

4. Let  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$  and  $B = \begin{bmatrix} 2a & 2b & 2c \\ g & h & i \\ d+3g & e+3h & f+3i \end{bmatrix}$ .

If  $\det A = 4$ , what is  $\det B$ ?

- a) -8    b) 8    c) -12    d) 12

5. Let  $T: R^3 \rightarrow R^4$  be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 - x_3 \\ 2x_1 + x_2 - x_3 \\ x_1 - x_3 \\ x_2 - x_3 \end{bmatrix}.$$

Which one of the following statements is TRUE?

- a)  $T$  is one-to-one and onto.  
 b)  $T$  is one-to-one but not onto.  
 c)  $T$  is onto but not one-to-one.  
 d)  $T$  is neither one-to-one nor onto.

6. If  $z = \sqrt{3} + i$ , what is the standard form of  $z^{30}$ ?

- a)  $2^{30}$     b)  $-i2^{30}$     c)  $-2^{30}$     d)  $i2^{30}$

7. Let  $\lambda = 5$  be an eigenvalue of a  $3 \times 3$  matrix  $A$ .

Exactly one of the following statement is TRUE. Which one?

- a)  $\det(A - 5I) = 0$   
 b)  $\det(A - 5I) \neq 0$   
 c)  $Ax = 5x$  for all  $x$  in  $R^3$ .  
 d) The equation  $(A - 5I)x = 0$  has a unique solution.

8. Consider

$$\begin{aligned} x_1 - 5x_2 + 2x_3 &= 0 \\ x_1 + x_2 + 8x_3 &= 0 \\ -x_1 + 3x_2 - 4x_3 &= 0. \end{aligned}$$

What is the general solution of the above system of linear equations?

a)  $\begin{bmatrix} t \\ 7t \\ t \end{bmatrix}, t \in R$     b)  $\begin{bmatrix} t \\ -7t \\ -t \end{bmatrix}, t \in R$     c)  $\begin{bmatrix} t \\ -t \\ 7t \end{bmatrix}, t \in R$     d)  $\begin{bmatrix} -7t \\ -t \\ t \end{bmatrix}, t \in R$

9. Let  $T: R^3 \rightarrow R^3$  be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 8 \\ 8 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}.$$

What is  $T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = ?$

a)  $\begin{bmatrix} 10 \\ 10 \\ 14 \end{bmatrix}$     b)  $\begin{bmatrix} 14 \\ 14 \\ 10 \end{bmatrix}$     c)  $\begin{bmatrix} 14 \\ 10 \\ 10 \end{bmatrix}$     d)  $\begin{bmatrix} 14 \\ 10 \\ 14 \end{bmatrix}$

10. Let  $A$  be a  $3 \times 3$  matrix with the characteristic polynomial  $p(\lambda) = \lambda(\lambda + 4)(\lambda - 5)$ . Exactly one of the following statement is FALSE. Which one?

- a) Each eigenspace of  $A$  is one-dimensional.  
 b)  $A$  is not invertible.  
 c)  $A$  is diagonalizable.  
 d) The equation  $Ax = b$  is consistent for every vector  $b$  in  $R^3$ .

11. Exactly one of the following is NOT an elementary matrix. Which one?

a)  $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$     b)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$     c)  $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 7 & 0 & 0 \end{bmatrix}$     d)  $\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

12. What are the eigenvalues of the matrix  $\begin{bmatrix} 4 & -5 \\ 1 & 0 \end{bmatrix}$ ?

- a)  $\lambda_1 = 1 + 2i, \lambda_2 = 1 - 2i$     b)  $\lambda_1 = 2 + i, \lambda_2 = 2 - i$   
 c)  $\lambda_1 = 2i, \lambda_2 = -2i$     d)  $\lambda_1 = 3 + 2i, \lambda_2 = 3 - 2i$

13. Let  $H = \left\{ \begin{bmatrix} a+b \\ b+c \\ c+d \\ c+d \end{bmatrix} \mid a, b, c, d \in R \right\}$ . What is the dimension of the subspace  $H$ ?

- a) 1    b) 2    c) 3    d) 4

14. If  $\det A = 2, \det B = -3$  and  $\det C = -6$  then, what is  $\det(A^{-1}B^2C^T)$ ?

- a) 18    b) -18    c) 27    d) -27

15. If  $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$ , what is  $A^{1107}$ ?

a)  $\begin{bmatrix} -3 & 2 \\ -4 & 3 \end{bmatrix}$     b)  $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$     c)  $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$     d)  $\begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$

16. What is the angle between the vectors  $x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$  and  $y = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ ?

- a)  $\pi/2$     b)  $\pi/3$     c)  $\pi/4$     d)  $\pi/6$

Solutions:

- |      |       |       |       |       |       |       |       |
|------|-------|-------|-------|-------|-------|-------|-------|
| 1. d | 2. a  | 3. b  | 4. a  | 5. b  | 6. c  | 7. a  | 8. d  |
| 9. b | 10. d | 11. c | 12. b | 13. c | 14. d | 15. a | 16. c |