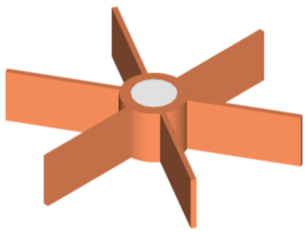
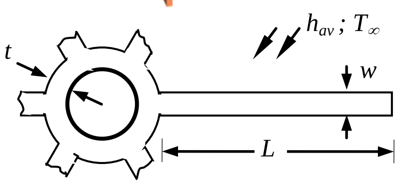
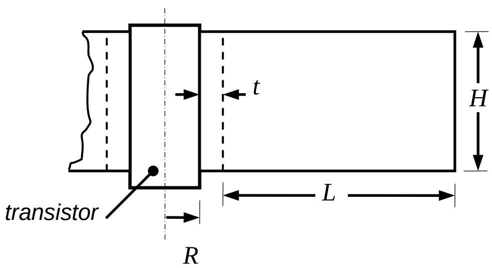
Problem 1:





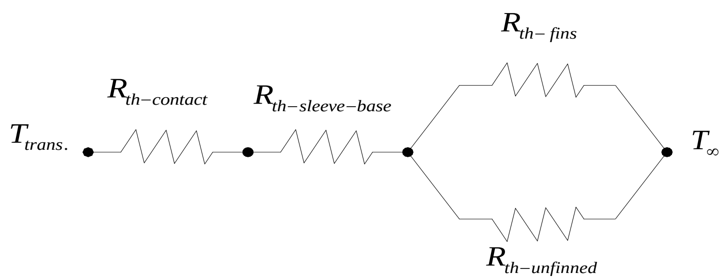
Top View



Front View

Assumptions
1) One-dimensional steady-state radial heat conduction;
2) Classical fin theory applies;
3) Constant properties;
4) Compensated length approach adequate;
5) Radiation heat transfer negligible.

Given
1) Fin dimensions : $H = 10$ mm ; $w = 1$ mm ; $L = 20$ mm
2) Sleeve dimensions : $R = 4$ mm ; $t = 2$ mm
3) $h_{\substack{contact \\ sleeve-trans}} = 10^3$ W/m^2-°C ; sleeve and the fin thermal cond.: $k_{cu} = 390$ W/m-°C
4) Air: T_∞ = 22°C; $h_{av} = 30$ W/m^2-°C
5) Transistor surface temperature: $T_{trans.}$ =80°C

a) 

b) $$q = \frac{T_{trans.} - T_\infty}{R_{eq}}\ ; \quad R_{eq} = R_{th-contact} + R_{th-sleeve-base} + \left(\frac{1}{R_{th-fins}} + \frac{1}{R_{th-unfinned}}\right)^{-1}$$

$$R_{th-contact} = 1/\left(h_{\substack{contact \\ sleeve-trans}} A_{contact}\right) = 1/\left(10^3 \times \underbrace{\pi \times (2\times 4\times 10^{-3}) \times 10\times 10^{-3}}_{A_{contact} = \pi 2RH}\right) = 3.98 \text{ K/W}$$

$$R_{th-sleeve-base} = \ln\left[\frac{R+t}{R}\right] / (2\pi k_{cu} H) = \ln\left[\frac{4+2}{4}\right] / (2\pi \times 390 \times 10\times 10^{-3}) = 0.0165 \text{ K/W}$$

$$R_{th-unfinned} = 1/(h_{av} A_{unfinned}) = 1/\left[30 \times \left[\underbrace{\underbrace{\pi \times (2\times(4+2)\times 10^{-3}) \times 10\times 10^{-3}}_{A_{total}} - \underbrace{6\times 1\times 10^{-3} \times 10\times 10^{-3}}_{A_{finned}}}_{A_{unfinned}}\right]\right] = 105.16 \text{ K/W}$$

$$R_{th-finned} = R_{th-one-fin}/N = \left[1/(\eta_{fin} h_{av} A_{surface-of-fin})\right]/6; \quad \eta_{fin} = \tanh(mL_c)/mL_c;$$

$$m = \sqrt{\frac{h_{av}P_{cs}}{k_{cu}A_{cs}}}; \quad P_{cs} = 2(H+w) = 22\times 10^{-3} \text{ m}; \quad A_{cs} = (H\times w) = 10\times 10^{-6} \text{ m}^2 \Rightarrow m = \sqrt{\frac{30\times 22\times 10^{-3}}{390\times 10\times 10^{-6}}} = 13.0 \text{ m}^{-1}$$

$$L_c = L + \frac{A_{cs}}{P_{cs}} = 20\times 10^{-3} + \frac{10\times 10^{-6}}{22\times 10^{-3}} = 0.02045 \text{ m} \Rightarrow \quad \eta_{fin} = \tanh(13\times 0.02045)/(13\times 0.02045) = 0.977$$

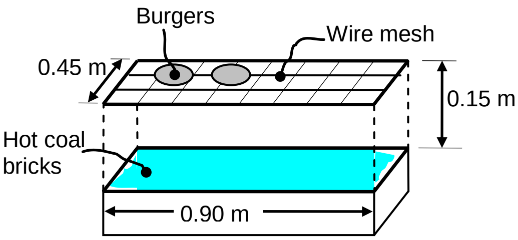
$$A_{surface-of-fin} = P_{cs} \times L_c = 22\times 10^{-3} \times 0.02045 = 0.0004499 \text{ m}^2$$

$$R_{th-finned} = \left[1/(0.977\times 30\times 0.0004499)\right]/6 = 12.63 \text{ K/W}$$

$$R_{eq} = 3.98 + 0.0165 + \left(\frac{1}{12.63} + \frac{1}{105.16}\right)^{-1} = 15.27 \text{ K/W} \Rightarrow q = \frac{80-22}{15.27} = 3.8 \text{ W}$$

Note: with no sleeve the resistance is $R_{th-unfinned} = 1/(h_{av}A_{transistor}) = 1/(30\times\pi\times(2\times 4\times 10^{-3})\times 10\times 10^{-3}) = 132.6$ K/W!!

Problem 2:



Assumptions
1) Initial rate of heat transfers: assume steady-state;
2) Constant surface temperatures
3) Constant properties;
4) Convection heat transfer negligible.
5) The burger and the hot brick surfaces are behaving as black

Given
1) $T_{c-brick} = 650°\mathrm{C}$; $T_{burger} = 5°\mathrm{C}$
2) $\varepsilon_1 = \varepsilon_2 = 1$ for brick (surface 1) and burger (surface 2)
3) $\varepsilon_3 = \varepsilon_{al} = 0$ for aluminum foil

a) Assume the bricks is surface 1 and the burgers are surface 2; thus the *initial* net rate of radiation heat transfer exchange between the coal bricks and the burgers bottom faces

$$q = A_1F_{12}\left(e_{b1} - e_{b2}\right);$$

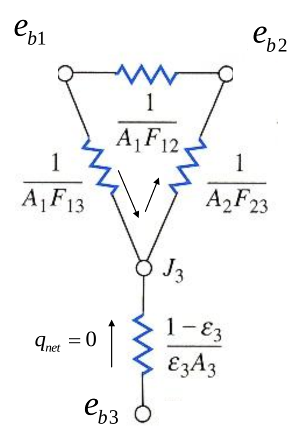
$$e_{b1} = \sigma T_1^4 = 5.669\times10^{-8}\times\left(650+273.15\right)^4 = 41171.4 \quad \mathrm{W/m^2}$$

$$e_{b2} = \sigma T_2^4 = 5.669\times10^{-8}\times\left(5+273.15\right)^4 = 339.3 \quad \mathrm{W/m^2}$$

$$\xrightarrow[\substack{X/D=3\\ Y/D=6}]{charts-cribsheets} F_{12} \approx 0.6; \quad A_1 = 0.90\times0.45 = 0.405 \quad m^2$$

$$\Rightarrow q = A_1F_{12}\left(e_{b1} - e_{b2}\right) = 0.405\times0.6\times\left(41171.4 - 339.3\right) = 9922.22 \text{ W}$$

b) As the open area is covered with the aluminum that practically has no emissivity its surface resistance is infinity and there is no net rate of radiation for this surface. Thus the aluminum sheet acts as a ***reradiating*** surface.



$$q = \frac{e_{b1} - e_{b2}}{R_{eq}}; \quad R_{eq} = \left[\left(\frac{1}{A_1F_{12}}\right)^{-1} + \left[\frac{1}{A_1F_{13}} + \frac{1}{A_2F_{23}}\right]^{-1}\right]^{-1}$$

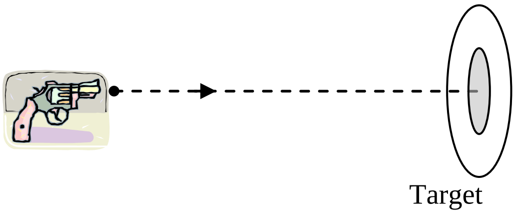
$$F_{12} = 0.6 \xrightarrow[F_{11}=0]{} F_{13} = 0.4; \quad symmetry \Rightarrow A_1F_{13} = A_2F_{23}$$

$$\Rightarrow R_{eq} = \left(A_1F_{12} + A_1F_{13}/2\right)^{-1} = \left(0.405\times0.6 + 0.405\times0.4/2\right)^{-1} = 3.0864 \quad \mathrm{m^2}$$

$$q = \frac{e_{b1} - e_{b2}}{R_{eq}} = \frac{41171.4 - 339.3}{3.0864} = 13229.7 \text{ W}$$

$$\%\text{increase} = \frac{13229.7 - 9922.2}{9922.2} = 33.3\%$$

Problem 3:



Assumptions
1) Transient heat conduction problem;
2) Sphere in cross flow
2) h_{av} and U_∞ and T_∞ remain constant
3) Constant properties;
4) Radiation heat transfer negligible

Given
6) D = 4.0 mm; $T_i = 200°C$; $T_\infty = 27°C$; $U_\infty = 250$ m/s ; $t_f = 0.56$ s
7) The thermo-physical properties of lead:
ρ = 11000 kg/m^3; c_p = 2000 J/kg-°C; k = 35 W/m-°C.

a) h_{av} =?

$$Nu_{av} = \frac{h_{av}D}{k_{fluid}} = 2 + [0.4Re_D^{1/2} + 0.06Re_D^{2/3}]Pr^{0.4}\left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}$$

The thermo-physical properties of air at T_∞ (300K):

ρ = 1.1614 kg/m^3; c_p = 1007 J/kg-°C; k = 0.026 W/m-°C; Pr=0.707; $\mu = 184.6\times10^{-7}$ N.s/m^2;

Guess wall temperature average of $\overline{T}_w = T_i = 200°C=473K$ thus $\mu_w = 260.4\times10^{-7}$ N.s/m^2;

$$Re_D = \frac{\rho U_\infty D}{\mu} = \frac{1.1614\times250\times4\times10^{-3}}{184.6\times10^{-7}} = 62914.4$$

$$Nu_{av} = \frac{h_{av}D}{k_{fluid}} = 2 + [0.4\times(62914.4)^{1/2} + 0.06(62914.4)^{2/3}](0.707)^{0.4}\left(\frac{184.6}{260.4}\right)^{1/4} = 155.95$$

$$\frac{h_{av}D}{k_{fluid}} = 155.25 \Rightarrow h_{av} = \frac{155.25\times0.026}{4\times10^{-3}} = 1013.1 \text{ W/m}^2\text{-K}$$

b) The final temperature of the bullet right before the impact

$$Bi = \frac{h_{av}R/3}{k_{solid}} = \frac{1013.1\times(2\times10^{-3})/3}{35} = 0.01929 < 0.1 \Rightarrow LPA \text{ valid}$$

$$\frac{T-T_\infty}{T_i-T_\infty} = \exp\left(-\frac{Ah_{av}}{V\rho c_p}t\right) \rightarrow \frac{T_f-27}{200-27} = \exp\left(-\frac{3\times1013.1}{(2\times10^{-3})\times11000\times2000}0.56\right)$$

$$T_f = 27 + 173\times0.962 = 193.4°C$$

c) During the travel time what is the amount of heat loss (in J) for the bullet?

$$Q = V\rho c_p\left[T_f - T_i\right] = \frac{4}{3}\pi(2\times10^{-3})^3\times11000\times2000\times(193.4-200) = -4.86 \text{ J}$$

Problem 4:

Assumptions
1) Steady-state heat transfer;
2) Heat loss to ambient negligible
3) Constant properties;
Note: Approximate water properties by saturated liquid data.

Given
$T_{hi} = 100°C; T_{ci} = 15°C; \dot{m}_c = 0.6$ kg/s ;
$C_{oil} = 3350$ W/K; U =500 W/m^2-°C; A = 10 m^2;
$h_{oil} / h_{water} = 0.8$

a) Determine the *exit temperatures* and the *rate of heat transfer* in

$C_{oil} = \dot{m}_h c_h = 3350$ W/K

Guess water temperature rises about 20°C

Thus, $\bar{T}_w = 15°C \xrightarrow[table]{} c_c = 4184$ J/kg-K

$C_c = \dot{m}_c c_c = 0.6 \times 4184 = 2510.4$ W/K

thus $\dot{m}_c c_c < \dot{m}_h c_h$

$$R_{\min} = \frac{(\dot{m}c)_{\min}}{(\dot{m}c)_{\max}} = \frac{C_{\min}}{C_{\max}} = \frac{2510.4}{3350} = 0.749$$

$$\text{NTU} = \frac{UA}{(\dot{m}c)_{\min}} = \frac{500 \times 10}{2510.4} = 1.99$$

$\varepsilon = (T_{c,o} - T_{c,i}) / (T_{h,i} - T_{c,i}) \qquad \dot{m}_c c_c < \dot{m}_h c_h$

I) Counter flow arrangement

$\xrightarrow[chart-10-13]{} \varepsilon_{cf} = 0.72 = (T_{c,o} - T_{c,i}) / (T_{h,i} - T_{c,i})$

$$0.72 = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})} = \frac{(T_{c,o} - 10)}{(100 - 10)} \Rightarrow T_{c,o} = 74.8°C$$

$$q = (\dot{m}c)_c (T_{c,o} - T_{c,i}) = (\dot{m}c)_h (T_{h,i} - T_{h,o}) \Rightarrow T_{h,o} = T_{h,i} - \frac{(\dot{m}c)_c}{(\dot{m}c)_h}(T_{c,o} - T_{c,i}) = 100 - 0.749 \;\; 74.8 - 10 \;\; \rightarrow T_{h,o} = 51.5°C$$

$$q = (\dot{m}c)_c (T_{c,o} - T_{c,i}) = 2510.4 \left(74.8 - 10\right) = 162673.9 \text{ W}$$

b) If the ratio of ratio of convection heat transfer coefficients of oil to water is 0.8; and assuming thin wall heat exchangers, calculate the wall temperature at the inlet of the counter flow heat exchanger.

$$q'' = U \;\; T_{hi} - T_{co} \;\; = h_{oil} \;\; T_{hi} - T_w \;\; = h_{water} \;\; T_w - T_{co}$$

$$h_{oil} / h_{water} = 0.8 \Rightarrow 0.8T_w + T_w = 0.8T_{hi} + T_{co} \Rightarrow T_w = \frac{0.8 \times 100 + 74.8}{1.8} = 86°C$$

Good Luck