1. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ -1 & 3 & 1 \\ 4 & 2 & -5 \end{bmatrix}$ and let $B = \begin{bmatrix} -1 & 2 & 5 & -3 \\ 4 & -2 & 1 & 6 \\ -2 & 1 & 3 & 2 \end{bmatrix}$.
   What is the third column of AB.

   a) $\begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$ **b)** $\begin{bmatrix} -8 \\ 2 \\ 4 \end{bmatrix}$ **c)** $\begin{bmatrix} -2 \\ -8 \\ 4 \end{bmatrix}$ **d)** $\begin{bmatrix} 13 \\ -8 \\ -2 \\ 21 \end{bmatrix}$

2. Find all the values of k for which the system with the following augmented matrix is consistent:
   $\left[\begin{array}{cc|c} 1 & -5 & 4 \\ 0 & (k+3)(k-8) & (k+3)(k-5) \end{array}\right]$.

   a) $k \neq -3$ **b)** $k \neq 8$ **c)** $k = 8$ **d)** $k = -3, 8$

3. Consider the following augmented matrix of a system of linear equations:

   $\left[\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 3 \\ 1 & -2 & -2 & 2 & 3 \\ 1 & 0 & -2 & 2 & 3 \\ 1 & 1 & -2 & -2 & 3 \end{array}\right]$. Which of the following statements is TRUE?

   a) The system has a unique solution
   b) The system has infinitely many solutions with one parameter
   c) The system has infinitely many solutions with two parameters
   d) The system has no solutions

4. Consider the matrix equation $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 5 & 4 \\ 3 & 6 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} h \\ 7 \\ 9 \end{bmatrix}$.

   The above matrix equation has a UNIQUE solution if

   a) $k = 6$ and $h \neq 3$ **b)** For all k in R and $h \neq 3$.

   c) $k \neq 6$ and for all h in R. **d)** $k = 6$ and $h = 3$.

5. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 4 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$, $v_4 = \begin{bmatrix} 5 \\ 1 \\ -3 \\ h \end{bmatrix}$.

   For what value(s) of h the vector v_4 is in Span$\{v_1, v_2, v_3\}$?

   a) $h = -10$.
   b) $h = 10$.
   c)There is no value of h for which the vector v_4 is in Span$\{v_1, v_2, v_3\}$.
   d) For every real number h, the vector v_4 is in Span$\{v_1, v_2, v_3\}$.

6. What is the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$?

   a) $\begin{bmatrix} -2 & -1 & -1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **b)** $\begin{bmatrix} 2 & -1 & 1 \\ -2 & 2 & -1 \\ -1 & 1 & -1 \end{bmatrix}$ **c)** $\begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **d)** $\begin{bmatrix} -2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

7. Let A be a 5×4 matrix such that its reduced row echelon form has 4 pivot positions.

   Which of the following statements is TRUE?

   a) The linear transformation T defined by $T(x) = Ax$ is onto.
   b) $Ax = 0$ has a unique solution.
   c) Columns of A are linearly dependent.
   d) $Ax = b$ is consistent for every vector b in R^5.

8. If $A \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -1 & 3 \end{bmatrix}$, what is A?

   a) $\begin{bmatrix} -4 & -3 \\ 5 & 6 \end{bmatrix}$ **b)** $\begin{bmatrix} 4 & -3 \\ 5 & -6 \end{bmatrix}$ **c)** $\begin{bmatrix} -4 & 3 \\ -5 & 6 \end{bmatrix}$ **d)** $\begin{bmatrix} 4 & 3 \\ -5 & -6 \end{bmatrix}$

9. Let $T : R^2 \to R^2$ be the linear transformation which rotates vectors $\pi/3$ radians counterclockwise. Which of the following matrices is the standard matrix of T?

   a) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **b)** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **c)** $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **d)** $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

10. Let $A = \begin{bmatrix} 1 & 1 & 2 & 2 & 3 \\ 2 & 1 & 4 & 3 & 2 \\ 1 & 0 & 2 & 1 & -1 \\ 1 & 1 & 2 & 2 & 2 \end{bmatrix}$.

    What is the general solution of the matrix equation $Ax = 0$?

    a) $\begin{bmatrix} t \\ -2s-t \\ s \\ t \\ 0 \end{bmatrix}$ **b)** $\begin{bmatrix} 2s+t \\ -t \\ s \\ 0 \\ t \end{bmatrix}$ **c)** $\begin{bmatrix} -2s-t \\ t \\ -s \\ 0 \\ t \end{bmatrix}$ **d)** $\begin{bmatrix} -2s-t \\ -t \\ s \\ t \\ 0 \end{bmatrix}$ $(s,\ t \in R)$

11. Let $s \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$ be the general solution of a homogeneous system $Ax = 0$, and let $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ be a particular solution of a non-homogeneous system $Ax = b$ with the same coefficient matrix A. Which of the following is a solution of $Ax = b$?

    (I) $\begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$ (II) $\begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$

    a) Both (I) and (II) **b)** (I), but not (II) **c)** (II), but not (I) **d)** Neither (I) nor (II)

12. Let $T : R^2 \longrightarrow R^3$ be a linear transformation such that

    $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 2 \\ -5 \end{bmatrix}$. What is $T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right)$?

    a) $\begin{bmatrix} 3 \\ 2 \\ 11 \end{bmatrix}$ **b)** $\begin{bmatrix} -3 \\ -2 \\ 11 \end{bmatrix}$ **c)** $\begin{bmatrix} 3 \\ 2 \\ -11 \end{bmatrix}$ **d)** $\begin{bmatrix} 2 \\ -11 \\ 3 \end{bmatrix}$

13. Let $T : R^4 \longrightarrow R^3$ the linear transformations defined by $T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x - y + z + 2w \\ 2x + y + 5z - w \\ x + 2y + 3z + w \end{bmatrix}$.

    Which of the following statements about T is TRUE?

    a) T is onto but not one-to-one.
    b) T is one-to-one but not onto.
    c) T is one-to-one and onto.
    d) T neither one-to-one nor onto.

14. Let A be a 4×4 matrix such that $\det A = 0$.
    Exactly one of the following statements is TRUE. Which one?

    a) The linear transformation $T(x) = Ax$ is onto but not one-to-one.
    b) Each column of A is a linear combination of the other columns of A.
    c) Rank of A is 3.
    d) Rank$A \leq 3$.

15. Suppose that A is a 5×5 matrix, $\det A = 0$, and b is a vector in R^5.
    Exactly one of the following statements is TRUE. Which one?

    a) $Ax = b$ has exactly one solution.
    b) $Ax = b$ has no solution.
    c) $Ax = b$ has infinitely many solution.
    d) $Ax = b$ has either no solution or infinitely many solutions.

16. Let A be a 4×4 matrix such that $\det A = 9$. What is $\det(3A)$?

    a) 3^2 **b)** 3^4 **c)** 3^6 **d)** 3^8

17. Let A and B be two 4×4 matrices such that $\det A = 9$ and $\det B = 3$. What is $\det(3AB^{-1}A^T)$?

    a) 3^7 **b)** 3^6 **c)** 3^5 **d)** 3^4

18. Let $A = \begin{bmatrix} 1 & 2 & k \\ 2 & k & 8 \\ k & 0 & 0 \end{bmatrix}$. Find <u>all the values of k</u> such that $\det A = 0$.

    a) $k = -4, 0, 1$ **b)** $k = -4,\ 0,\ 4$ **c)** $k = -2,\ 1, 4$ **d)** $k = -2, 0, 2$

19. Let $A = \begin{bmatrix} k & 6 & 7 \\ 0 & k & 4 \\ 0 & 9 & k \end{bmatrix}$. Find <u>all the values of k</u> such that $\det A = 0$.

    a) $k = -6, 0$ **b)** $k = 6,\ 0$ **c)** $k = -6,\ 6$ **d)** $k = -6, 0, 6$

20. Let A and B be 3×3 matrices such that $\det A = 3$, and $\det B = -4$.
    What is $\det\left((2A)^{-1}B^2B^T\right)$?

    a) 8 **b)** 24 **c)** $-8/3$ **d)** $8/3$

21. Let $u_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 2 \end{bmatrix}$, $u_4 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix}$ and $H = \text{Span}\{u_1, u_2, u_3, u_4\}$.

    What is the dimension of the subspace H?

    a) 1 **b)** 2 **c)** 3 **d)** 4

22. Which of the followings are a subspace of R^3?

(I) $\left\{ \begin{bmatrix} a+b \\ 2a-3c \\ b+5c \end{bmatrix} \middle| a,b,c \in R \right\}$ (II) $\left\{ \begin{bmatrix} a-b \\ 3 \\ b+3c \end{bmatrix} \middle| a,b,c \in R \right\}$ (III) $\left\{ \begin{bmatrix} a-2b \\ 5b+c \\ 0 \end{bmatrix} \middle| a,b,c \in R \right\}$

a) (III) only **b)** (II) and (III) only **c)** (I) and (III) only **d)** (I) and (II) only

23. What is the coordinate vector of $x = \begin{bmatrix} -7 \\ 4 \\ -3 \end{bmatrix}$ relative to basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$?

a) $\begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix}$ **b)** $\begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$ **c)** $\begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix}$ **d)** $\begin{bmatrix} 3 \\ -2 \\ -4 \end{bmatrix}$

24. Let $A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 4 \end{bmatrix}$. What is the characteristic equation of the matrix A?

a) $\lambda^3 - 5\lambda^2 - 6\lambda - 4 = 0$ **b)** $\lambda^3 + 5\lambda^2 - 5\lambda + 4 = 0$

c) $\lambda^3 - 6\lambda^2 + 6\lambda + 4 = 0$ **d)** $\lambda^3 + 7\lambda^2 - 5\lambda - 4 = 0$

25. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & k \\ 2 & 0 & 4 \end{bmatrix}$. For what value of k the rank of A is 2?

a) 2 **b)** 3 **c)** 4 **d)** 5

26. Let $A = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}$. What are the eigenvalues of A?

a) 1 and 6 **b)** $1 \pm 3i$ **c)** $1 \pm \sqrt{6}i$ **d)** 1 and -3

27. Let $A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$. What are the eigenvalues of A?

a) $-3 \pm 4i$ **b)** $4 \pm 3i$ **c)** $-4 \pm 3i$ **d)** $3 \pm 4i$

28. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 1 & 8 \end{bmatrix}$. What are the eigenvalues of A?

a) $1,\ 4,\ 8$ **b)** $1,\ 4, 6$ **c)** $1,\ 3,\ 9$ **d)** $2,\ 5,\ 8$

29. Let A be a 2×2 matrix with eigenvectors $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$ corresponding to eigenvalues $\lambda_1 = -3$ and $\lambda_2 = 3$, respectively. What is the matrix A?

a) $\begin{bmatrix} -15 & 6 \\ -36 & 15 \end{bmatrix}$ **b)** $\begin{bmatrix} -15 & -36 \\ 6 & 15 \end{bmatrix}$ **c)** $\begin{bmatrix} -45 & 18 \\ -108 & 45 \end{bmatrix}$ **d)** $\begin{bmatrix} -15 & -12 \\ 18 & 15 \end{bmatrix}$

30. Let $\lambda_1 = 0$, $\lambda_2 = 3$ and $\lambda_3 = -1$ be three distinct eigenvalues of a 3×3 matrix A. Exactly one of the following statements is FALSE. Which one?

a) Each eigenspace of the matrix A is one-dimensional.
b) $\det A = 0$
c) The matrix A is diagonalizable.
d) The matrix equation $Ax = 0$ has a unique solution.

31. Let $z = 2 - 2i$. What is z^{20} ?

a) -2^{15} **b)** 2^{30} **c)** $2^{15}\ i$ **d)** -2^{30}

32. Let $A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Which of the following sets is a basis for the null space of A?

a) $\left\{ \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$ **b)** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ **c)** $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$ **d)** $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

33. $\lambda = 3$ is an eigenvalue for the matrix $A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 5 & -10 \\ 1 & 0 & 3 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$.

What is the dimension of the eigenspace for $\lambda = 3$?

a) 1 **b)** 2 **c)** 3 **d)** 4

34. $\lambda = 2$ is an eigenvalue for the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$.

Which one of the following is the eigenspace of A corresponding to $\lambda = 2$?

a) Span$\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} \right\}$ **b)** Span$\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \right\}$ **c)** Span$\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \right\}$ **d)** Span$\left\{ \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

35. You are given that $\lambda = 3$ is an eigenvalue of the matrix $A = \begin{bmatrix} 5 & 2 & -4 \\ 2 & 5 & -4 \\ 2 & 2 & -1 \end{bmatrix}$.

Which one of the following sets is a basis for E_3?

a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ **b)** $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ **c)** $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ **d)** $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

36. Exactly one of the following statement is TRUE. Which one?

a) Every invertible matrix is diagonalizable.
b) Every diagonalixable matrix is invertible.
c) If an $n \times n$ matrix A is diagonalizable, then it must have n distinct eigenvalues.
d) If A is an $n \times n$ diagonalizable matrix, then it has n linearly independent eigenvectors.

37. Let $u_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, $x = \begin{bmatrix} 3 \\ 0 \\ -9 \\ 6 \end{bmatrix}$ and $W = \text{Span}\{u_1, u_2\}$.

    What is the orthogonal projection of x on W?

    a) $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ **b)** $\begin{bmatrix} 2 \\ 3 \\ -5 \\ 1 \end{bmatrix}$ **c)** $\begin{bmatrix} 3 \\ 2 \\ 1 \\ 5 \end{bmatrix}$ **d)** $\begin{bmatrix} 1 \\ -5 \\ -2 \\ 3 \end{bmatrix}$

38. Let $u_1 = \begin{bmatrix} 0 \\ 1 \\ -4 \\ -1 \end{bmatrix}$ $u_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -4 \end{bmatrix}$, $u_4 = \begin{bmatrix} 5 \\ -3 \\ -1 \\ 1 \end{bmatrix}$ and $x = \begin{bmatrix} 10 \\ -8 \\ 2 \\ 0 \end{bmatrix}$.

    You are given that $B = \{u_1,\ u_2,\ u_3,\ u_4\}$ is an orthogonal basis for R^4. What is the value of c_2 in the equation $x = c_1u_1 + c_2u_2 + c_3u_3 + c_4u_4$?

    a) $-8/9$ **b)** $-2/9$ **c)** $2/3$ **d)** 2

39. Let $W = \text{Span}\left\{\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}\right\}$ and $x = \begin{bmatrix} 6 \\ -2 \\ 4 \end{bmatrix}$. What is the distance from x to W?

    a) 2 **b)** 36 **c)** $\sqrt{6}$ **d)** 6

40. Let $y = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$ and $u = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

    What is the distance from y to the line through u and the origin?

    a) 20 **b)** $\sqrt{40}$ **c)** $\sqrt{116}$ **d)** 4

41. Let $u = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$. What is the angle between the vectors u and v?

    a) 0 **b)** $\pi/6$ **c)** $\pi/4$ **d)** $\pi/3$

Answer Key

1.a, 2.b, 3.b, 4.c, 5.b, 6.b, 7.b, 8.b, 9.c, 10.d, 11.c, 12.c, 13.a, 14.d, 15.d, 16.c, 17.a, 18.b, 19.d, 20.c, 21.b, 22.c, 23.d, 24.c, 25.c, 26.c, 27.d, 28.c, 29.a, 30.d, 31.d, 32.c, 33.b, 34.c, 35.b, 36.d, 37.d, 38.b, 39.d, 40.b, 41.c.

If there are any typos/errors in the questions/answers, please let me know.