MCAOA1 – OAC CALCULUS: KEY FORMULAS TO SUCCESS!

Unit One: Limits and the Derivative of a Function

Factoring: Sums & Differences of Cubes

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

Finding Derivative (Slope of a Tangent) using First Principles

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Unit Two: Differentiation Rules

Power Rule

$$f(x) = x^n \qquad f'(x) = nx^{n-1}$$

Product Rule

$$F(x) = f(x) \cdot g(x) \qquad F'(x) = f(x)g'(x) + g(x)f'(x)$$

Quotient Rule

$$f(x) = \frac{g(x)}{h(x)} \qquad f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

Chain Rule

$$\frac{d}{dx}u^n = nu^{n-1} \cdot \frac{du}{dx}$$

Chain Rule Using Composition of Functions

$$\frac{d}{dx} f[g(x)] = f'[g(x)]g'(x)$$

Implicit Differentiation

e.g. $xy = y + 5$ (treat y as an unspecified but differentiable function of x --> multiply by $\frac{dy}{dx}$)

Unit Three: Curve Sketching

Vertical Asymptotes

occur when *f (x)* is undefined

Horizontal Asymptotes

$y = \lim_{x \to +\infty} f(x) \qquad y = \lim_{x \to -\infty} f(x)$

Oblique (Slant) Asymptotes

occur when the degree of the numerator is greater than the degree of the denominator

$$y = \frac{1 + x - x^2}{x - 1}$$

$$\begin{array}{r|l} & -x + 0 + 0 \\ \hline x - 1 & -x^2 + x + 1 \\ & \underline{-x^2 + x \quad} \\ & 0 + 1 \\ & \underline{0 + 0} \\ & 1 \end{array}$$

$$y = (-x) + \frac{1}{x - 1}$$

As x approaches infinity, the function approaches $y = -x + 0$, or simply $y = -x$

Unit Four: Extreme Value Applications

Accurately sketch diagram, define variables, find equation of constraint, and write down the main equation to take the derivative!

Unit Five: Related Rates

Average Rate of Change

$$\text{avg rate of change} = \frac{\Delta y}{\Delta x} \quad \text{(slope of the secant)}$$

Instantaneous Rate of Change

$$\text{instnt. rate of change} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \quad \text{(slope of the tangent)}$$

Displacement, Velocity, and Acceleration

$s(t) = \text{displacement}$

$s'(t) = v(t) = \text{velocity}$

$v'(t) = a(t) = \text{acceleration}$

Unit Six: Trigonometry

Pythagorean Identity Formula – Very Important!

$$\sin^2\theta + \cos^2\theta = 1$$

Trigonometric Identities

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Angle Addition and Subtraction Formulas

these are found on the last page of the course pack (information also provided on examination)

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{\tan 2\theta}{1 - \tan^2\theta}$$

Trigonometric Limits

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin v = \cos v \frac{dv}{dx}$$

$$\frac{d}{dx}\cos v = -\sin v \frac{dv}{dx}$$

More trigonometric derivatives are shown at the back of course pack.

$$\frac{d}{dx}\tan v = \sec^2 v \frac{dv}{dx}$$

*Note: More trigonometric derivatives are shown at the back of course pack (information will be available in the examination).

Unit Seven: Logarithm and Exponential Functions

Remember: A logarithm (log) is an exponent.

Inverse Logarithmic Identities

$$a^{\log_a x} = x$$
$$\log_a \left(a^y\right) = y$$
$$\log_a a = 1$$

Change of Base Identity

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Base e

$$y = e^x \qquad D = \{x \mid x \in \mathrm{R}\} \qquad \mathrm{R} = \{y \mid \mathrm{y} > 0, \mathrm{y} \in \mathrm{R}\}$$
$$y = \ln x \qquad D = \{x \mid \mathrm{x} > 0,\ x \in \mathrm{R}\} \qquad \mathrm{R} = \{y \mid \mathrm{y} \in \mathrm{R}\}$$

$$\log_e x = \ln x$$
$$\ln e = 1 \qquad \ln 1 = 0$$

Derivatives of Logarithms

$$\frac{d}{dx}\log_a v = \frac{1}{v}\log_a e \cdot \frac{dv}{dx}$$
$$\frac{d}{dx}\ln v = \frac{1}{v} \cdot \frac{dv}{dx}$$

Derivatives of Exponential Functions

$$\frac{d}{dx}a^v = a^v \ln a \cdot \frac{dv}{dx}$$
$$\frac{d}{dx}e^v = e^v \cdot \frac{dv}{dx}$$

Logarithmic Differentiation

used when both base and power are not constants
take ln of both sides and then take derivative

Unit Eight: Integrals and Applications

Integration Formulas

$$\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$$

$$\int a \cdot f(x)dx = a\int f(x)dx \qquad \text{* Note : "a" is a constant}$$

$$\int \cos v dv = \sin v + c$$

$$\int \sin v dv = -\cos v + c$$

$$\int \sec^2 v dv = \tan v + c$$

$$\int e^v dv = e^v + c$$

$$\int \frac{1}{v} dv = \ln|\mathrm{v}| + \mathrm{c}$$

$$\int v^n dv = \frac{v^{n+1}}{n+1} + c \qquad \mathrm{n} \neq \text{-1}$$

Separable Differential Equations

To solve: $\frac{dy}{dx} = f(x,y)$

Put in form: $\int h(y)dy = \int g(x)dx$

Unit Nine: Area

Area of a Trapezoid

$$A = \frac{1}{2}h(a+b)$$

Area Between 2 Curves

$$A = \int_a^b f(x) - g(x)dx$$

b = upper limit, a = lower limit, *f(x)* = top curve, *g(x)* = bottom curve