COMM 225 WINTER 2012: SOLUTION TO REVIEW QUESTIONS

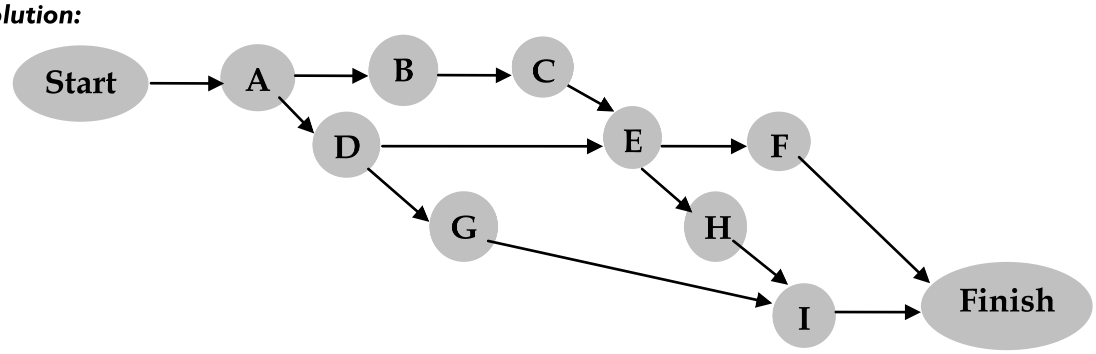
TOPIC: PROJECT MANAGEMENT

Q1.1: *An important client of RoboTech has asked for a redesign and redevelopment of one of RoboTech's industrial robots. The client wants to see a demonstration, a technical proposal and a cost proposal as soon as possible. The project team at RoboTech has identified the tasks involved as indicated in the table below. The activities and their predecessors are listed as follows:*

Activity	Description of the activity	Predecessor	Duration (days)
A	designing the robot	-	20
B	Building the prototype	A	10
C	testing the prototype	B	8
D	estimating the material costs	A	11
E	refining the design	C,D	7
F	demonstration	E	6
G	estimating labor	D	12
H	preparing the technical proposal	E	13
I	submission of the proposal	G,H	5

(a) *List all the paths and their durations.*
(b) *Identify the critical path(s) (by inspection).*
(c) *Determine the critical path(s) using early start, early finish, late start and late finish times. Also identify all the critical and non-critical activities.*

Solution:



(a) *The paths and their durations:*

Start- A –B – C – E –H- I – Finish, **Duration = 20+10+8+ 7+ 13+5 = 63 (Longest)**
Start- A –B – C – E –F – Finish, Duration = 20+10+8+ 7+ 6 =51
Start- A –D – G – I – Finish, Duration = 20+11+12+ 5 = 48
Start- A –D – E – H – I – Finish, Duration = 20+11+ 7+ 13+5 = 56
Start- A –D – E – F – Finish, Duration = 20+11+6+ 7 = 44

Critical path(s): The path with longest duration is Start- A –B – C – E –H- I – Finish with a duration of 20+10+8+ 7+ 13+5 = 63 days. Hence, this is the critical path.

(b) *Critical path(s) (using ES, EF, LS, and LF).*

Activity	*Predecessor*	*Duration (days)*	*Early Start*	*Early Finish*	*Late Start*	*Late Finish*	*Slack*
A	-	20	0	20	20-10 = 0	20	0
B	A	10	20	20+10=30	30-10= 20	30	0
C	B	8	30	30+8=38	38-8 = 30	38	0
D	A	11	20	20+11=31	38-11= 27	min(38, 46) =	7

						38	
E	C,D	7	max(38,31) = 38	38+7=45	45-7 = 38	min(57,45) =45	0
F	E	6	45	45+6=51	63-6 = 57	63	12
G	D	12	31	31+12=43	58-12=46	58	15
H	E	13	45	45+13=58	58-13=45	58	0
I	G,H	5	max(43,58) = 58	58+5=63	63-5= 58	63	0

For the table above, we know that activity A, B, C, E, H and I are critical activities (because the slack is zero). This implies that these activities must be completed in time so that the project is completed by 63rd day. And the path containing these activities is the critical path (i.e. Start- A –B – C – E –H- I – Finish). The other activities such as D, F, and G are noncritical as they have positive slacks.

Q1.2: *Refer to the following table which lists the activities of a project with completion time estimate for each activity.*

Activity	*Predecessor*	*Time Estimates (Weeks)*		
		a	*m*	*b*
1	*-*	*8*	*8*	*8*
2	*1*	*11*	*12*	*13*
3	*2*	*9*	*10*	*12*
4	*3*	*5*	*7*	*10*
5	*1*	*5*	*6*	*7*
6	*5*	*14*	*18*	*26*
7	*6*	*8*	*10*	*14*
8	*5*	*13*	*13*	*13*
9	*8*	*10.5*	*13*	*15.5*
10	*1*	*10*	*12*	*14*
11	*10*	*7*	*10*	*12*
12	*11*	*10*	*11*	*12*
13	*12*	*6*	*6*	*6*

(a) Calculate the expected duration of each activity. Also, calculate the expected duration of each path.

(b) Calculate the variance of each activity and variance of each path. Also, calculate the standard deviation of each activity and the standard deviation of each path.

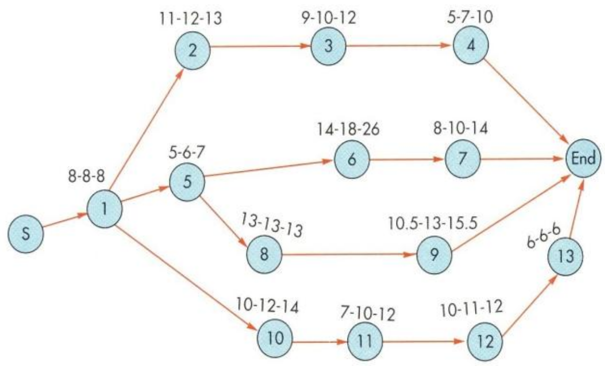
(c) Identify the critical path(s) (based on the expected durations).

(d) What is the probability that the project can be completed within 49 weeks? What is the probability that the project will NOT be completed within 49 weeks?

(e) What is the probability that the project will require 47 weeks or more? What is the probability that the project will NOT require more than 47 weeks?

(f) What is the probability that the project will be completed between 45 and 50 weeks?

Solution:



(a) and (b)

Activity	Mean (Expected Duration)	Variance	Standard Deviation
1	8	0	0
2	12	4/36	0.3334
5	6	4/36	0.3334
10	12	16/36	0.6667
3	10.17	9/36	0.5
6	18.67	144/36	2
8	13	0	0
11	9.83	25/36	0.8333
4	7.17	25/36	0.8333
7	10.33	36/36	1
9	13	25/36	0.8333
12	11	4/36	0.3334
13	6	0	0

Path	Mean (μ)	Var.	Std. dev. (σ)
1-2-3-4	37.34	38/36	1.027
1-5-6-7	43	184 /36	2.26
1-5-8-9	40	29/36	0.898
1-10-11-12-13	46.83	45/36	1.118

(c) *Identify the critical path(s) (based on the expected durations).*

Critical path is 1-10-11-12-13 with an expected duration/mean of 46.83 weeks.

(d) *What is the probability that the project <u>can be completed within 49 weeks</u>? What is the probability that the project <u>will not be completed within 49 weeks?</u>*

Caution: In order to calculate the probability of project-completion time, we will use the critical path(s) only. Note that this procedure on only approximate. Because there can be non-critical paths with high activity variances that might have the lower probability of completion time as compared to the critical path.

Path	Mean (μ)	Var.	Std. dev. (σ)	$z_{49} = \frac{49 - \mu}{\sigma}$	Probability
1-2-3-4	37.34	38/36	1.027	11.35	1
1-5-6-7	43	184 /36	2.26	2.65	0.9960
1-5-8-9	40	29/36	0.898	10.02	1
1-10-11-12-13	**46.83**	**45/36**	**1.118**	**1.94**	**0.9738**

- Probability that the project <u>can</u> be completed within 49 weeks =0.9738.
- Probability that the project <u>will not</u> be completed within 49 weeks = 1- 0.9738 = 0.0262

(e) *What is the probability that the project <u>will require 47 weeks or more</u>? What is the probability that the project <u>will not require more than 47 weeks?</u>*

Path	Mean (μ)	Var.	Std. dev. (σ)	$z_{47} = \frac{47 - \mu}{\sigma}$	Probability
1-2-3-4	37.34	38/36	1.027	9.406	1
1-5-6-7	43	184 /36	2.26	1.769	0.9608
1-5-8-9	40	29/36	0.898	7.795	1
1-10-11-12-13	**46.83**	**45/36**	**1.118**	**0.152**	**0.5596**

Probability that the project <u>will NOT require more than 47 weeks</u> = Probability that the project <u>can</u> be completed within 47 weeks=1- 0.5596 = 0.4404.

Probability that the project will require 47 weeks or more = Probability that the project will not be completed within 47 weeks = 1- 0.4404= 0.5596.

(f) *What is the probability that the project will be completed between 45 and 50 weeks?*

Path	Mean	Var.	Std. dev. (σ)	Z_{45}	P<=45	Z_{50}	P<=50
1-10-11-12-13	46.83	45/36	1.118	-1.64	1-0.9495= 0.0505	2.84	0.9977

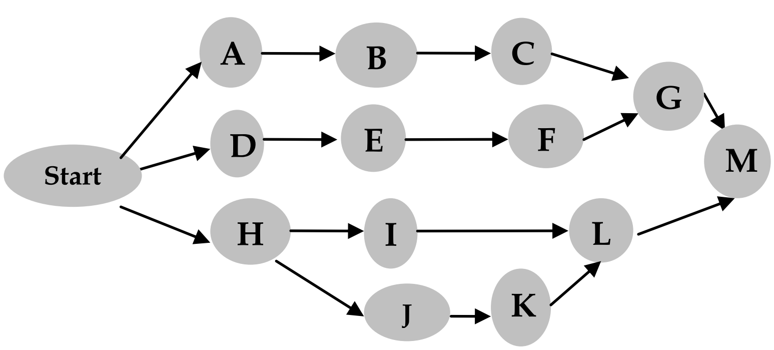
- Probability that the project can be completed within 45 weeks=0.0505
- Probability that the project can be completed within 50 weeks=0.9977
- Probability that the project will be completed between 45 and 50 weeks = Probability that the project can be completed within 50 weeks - Probability that the project can be completed within 45 weeks = 0.9977- 0.0505 = 0.9472.

Q1.3: *Fantasy products marketing manager has recently learned that its competitor is also in the process of developing a similar (new) product, which it intends to bring out to the market at exactly the same time and at lower price. The manager needs to decide whether to introduce the new product to the market 18 weeks from now. As a project management specialists, answer the following questions to help the marketing manager make a decision:*

(a) When would the project be completed using normal durations?
(b) Is it possible to complete the project in 18 weeks? What would the additional costs be? Which activities would need to the completed on a crash basis?
(c) Crash the project as much as possible. What is the optimum number of weeks to crash?
(d) Furthermore, market research has shown that if the early introduction is accomplished, it will bring a profit of $10,000 per week. Would you recommend some time frame shorter than 18 weeks? Justify your answer with appropriate calculation.

Activity	Predecessors	Normal Duration (Weeks)	Crash Duration (Weeks)	Normal Costs $	Crash Costs $
A	-	3	2	2,000	4,000
B	A	8	6	9,000	12,000
C	B	4	2	2,000	7,000
D	-	2	1	1,000	2,000
E	D	2	1	2,000	3,000
F	E	5	5	0	0
G	C, F	6	3	12,000	24,000
H	-	4	2	3,500	8,000
I	H	4	3	5,000	8,000
J	H	3	2	8,000	15,000
K	J	4	3	50,000	70,000
L	I, K	6	6	10,000	10,000
M	G, L	1	1	5,000	5,000

Solution:



Activity	Predecessors	Normal Duration (Weeks)	Crash Duration (Weeks)	Weeks can be Crashed	Normal Costs $	Crash Costs $	Crash Costs/ Weeks
A	-	3	2	1	2,000	4,000	2,000
B	A	8	6	2	9,000	12,000	1,500
C	B	4	2	2	2,000	7,000	2,500
D	-	2	1	1	1,000	2,000	1,000
E	D	2	1	1	2,000	3,000	1,000
F	E	5	5	-	0	0	-
G	C, F	6	3	3	12,000	24,000	4,000
H	-	4	2	2	3,500	8,000	2,250
I	H	4	3	1	5,000	8,000	3,000
J	H	3	2	1	8,000	15,000	7,000
K	J	4	3	1	50,000	70,000	20,000
L	I, K	6	6	-	10,000	10,000	-
M	G, L	1	1	-	5,000	5,000	-

(a) The normal duration of the project is **22 weeks** and the normal total cost is **$109,500.**

Path#	Path	Duration (Weeks)
1	Start-A-B-C-G-M-End	**3+8+4+6+1=22 weeks - Critical path**
2	Start-D-E-F-G-M-End	2+2+5+6+1=16
3	Start-H-I-L-M-End	4+4+6+1=15
4	Start-H-J-K-L-M-End	4+3+4+6+1=18

(b) Yes, it is possible to finish within 18 weeks. Some activities need to be crashed and additional crash cost will be incurred. The calculations are shown below:

Step 1:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	22	A	1	2,000
		B	2	1,500
		C	2	2,500
		G	3	4,000

- Crash activity B for 2 weeks at a crashing cost of $1500 x 2 =$3,000.
- Now, the duration of the A-B-C-G-M is 20 days.
- The resulting duration of the paths are as follows:

Path	Duration (Weeks)
Start-A-B-C-G-M-End	3+6+4+6+1=20*
Start-D-E-F-G-M-End	2+2+5+6+1=16
Start-H-I-L-M-End	4+4+6+1=15
Start-H-J-K-L-M-End	4+3+4+6+1=18

Step 2:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	20	A	1	2,000
		C	2	2,500
		G	3	4,000

- Crash activity A for one day at a crashing cost of $2,000.
- The resulting duration of the paths are as follows:

Path	Duration (Weeks)
Start-A-B-C-G-M-End	2+6+3+6+1=19*
Start-D-E-F-G-M-End	2+2+5+6+1=16
Start-H-I-L-M-End	4+4+6+1=15
Start-H-J-K-L-M-End	4+3+4+6+1=18

Step 3:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	20	C	2	2,500
		G	3	4,000

- Crash activity C for one week at a crashing cost of $2,500.
- Now, the duration of path A-B-C-G-M is 18 weeks. There are two critical paths:

Path	Duration (Weeks)
Start-A-B-C-G-M-End	2+6+3+6+1=18*
Start-D-E-F-G-M-End	2+2+5+6+1=16
Start-H-I-L-M-End	4+4+6+1=15
Start-H-J-K-L-M-End	4+3+4+6+1=18*

Project Length	Cumulative weeks shortened	Crashing costs per week	Cumulative crashing costs	Cumulative total costs
22	0	0	0	**$109,500**
21	1	$1,500	$1,500	**$111,000**
20	2	1,500	3,000	**$112,500**
19	3	2,000	5,000	**$114,500**
18	4	2,500	7,500	**$117,000**

(c)If we continue to further crash the project, we have:

Step 4:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	18	C	1	2,500
		G	3	4,000
H-J-K-L-M	18	H	2	2,250
		J	1	7,000
		K	1	20,000

- Crash activity C and H for one week at a crashing cost $2,500+$2,250 = $4,750
- When crashing H, path H-I-L-M is also crashed by one week to 14 weeks.

Path	Duration (Weeks)
Start-A-B-C-G-M-End	17*
Start-D-E-F-G-M-End	16
Start-H-I-L-M-End	14 (due to H)
Start-H-J-K-L-M-End	17*

Step 5:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	17	G	3	4,000
H-J-K-L-M	17	H	1	2,250
		J	1	7,000
		K	1	20,000

- Crash activity G and H for one week at a crashing cost of $4,000+$2,250 = $6,250
- When crashing G, path D-E-F-G-M is also crashed by one week to 15 weeks.
- When crashing H, path H-I-L-M is also crashed by one week to 13 weeks.

Path	Duration (Weeks)
Start-A-B-C-G-M-End	16*
Start-D-E-F-G-M-End	15 (due to G)
Start-H-I-L-M-End	13 (due to H)
Start-H-J-K-L-M-End	16*

Step 6:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	16	G	2	4,000
H-J-K-L-M	16	J	1	7,000
		K	1	20,000

- Crash activity G and J for one week at a crashing cost of $11,000.
- When crashing G, path D-E-F-G-M is also crashed by one week to 14 weeks.

Path	Duration (Weeks)
Start-A-B-C-G-M-End	15*
Start-D-E-F-G-M-End	14 (due to G)
Start-H-I-L-M-End	13
Start-H-J-K-L-M-End	15*

Step 7:

Critical Path	Duration	Activity	Crash Duration	Average Crashing Cost
A-B-C-G-M	15	G	1	4,000
H-J-K-L-M	15	K	1	20,000

- Crash activity G and K for one week at a crashing cost of 24,000 to reduce to 14 weeks.

Path	Duration (Weeks)
Start-A-B-C-G-M-End	14*
Start-D-E-F-G-M-End	13 (due to G)
Start-H-I-L-M-End	13
Start-H-J-K-L-M-End	14*

- Now, path A-B-C-G-M cannot be crashed any more.

Project Length	Cumulative weeks shortened	Crashing costs per week	Cumulative crashing costs	Cumulative total costs
22	0	0	0	**$109,500**
21	1	1,500	$1,500	**$111,000**
20	2	1,500	3,000	**$112,500**
19	3	2,000	5,000	**$114,500**
18	4	2,500	7,500	**$117,000**
17	5	4,750	12,250	**$121,750**
16	6	6,250	18,500	**$128,000**
15	7	**11,000**	29,500	**$139,000**
14	8	24,000	53, 500	**$163,000**

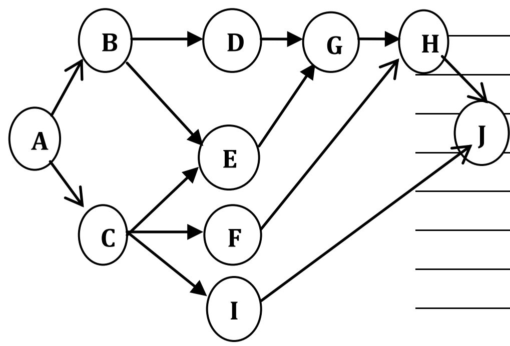
(d) Profit / week = $10,000. Because crash cost per week only jumps over $10,000 when the project is crashed from 16 down to 15 weeks, hence the project should be crashed to 16 weeks (see the table above).

Q 1.4: Given the following network and time & cost estimates, answer the following questions:

(a) What is the project completion time?

(b) What is the total cost required for completing this project on normal time?

(c) Crash the project by three weeks and calculate the new project completion cost.



Activity	Predecessor	Activity duration (weeks) Normal	Activity duration (weeks) Crash	Activity cost ($) Normal	Activity cost ($) Crash	Crash Cost / Week	# Days
A	-	6	4	10000	16000	3000	2
B	A	28	22	5000	9200	700	6
C	A	29	27	20000	20700	350	2
D	B	10	5	4000	6000	400	5
E	B,C	10	9	2500	3000	500	1
F	C	10	9	1000	7000	6000	1
G	D,E	15	14	1500	7500	6000	1
H	G,F	10	8	600	10600	5000	2
I	C	2	1	1000	2000	1000	1
J	H,I	10	8	900	8800	3950	2
TOTAL =				**46,500**			

Solution: The project has the following paths and their durations:

- A-B-D-G-H-J, Duration = 6 + 28 + 10 + 15 + 10 + 10 = 79
- A-B-E-G-H-J, Duration = 6 + 28 + 10 + 15 + 10 + 10 = 79
- **A-C-E-G-H-J, Duration = 6 + 29 + 10 + 15 + 10 + 10 = 80**
- A-C-F-H-J, Duration = 6 + 29 + 10 + 10 + 10 = 65
- A-C-I-J, Duration = 6 + 29 + 2 + 10 = 47

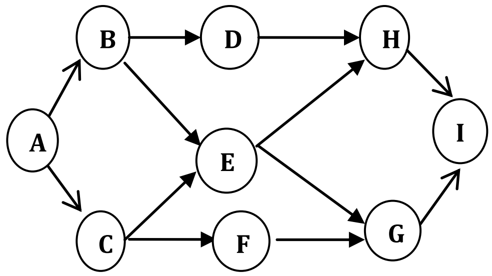
The project completion time is 80 weeks and the normal total cost (sum of all the activities) is **$46,500.**

Critical Path	Duration	It#1	It#2	It#3
A-B-D-G-H-J	79	79*	78*	77*
A-B-E-G-H-J	79	79*	78*	77*
A-C-E-G-H-J	80*	79*	78*	77*
A-C-F-H-J	65	64	64	63
A-C-I-J	47	46	46	45

It#	CP Before Crashing	Activity Crashed	CP After Crashing	Cumulative cost
1	A-C-E-G-H-J (80)	C by 1	A-B-D-G-H-J (79) A-B-E-G-H-J (79) A-C-E-G-H-J (79)	$ 350
2	A-B-D-G-H-J (79) A-B-E-G-H-J (79) A-C-E-G-H-J (79)	D by 1 E by 1	A-B-D-G-H-J (78) A-B-E-G-H-J (78) A-C-E-G-H-J (78)	$ 1250
3	A-B-D-G-H-J (78) A-B-E-G-H-J (78) A-C-E-G-H-J (78)	B by 1 C by 1	A-B-D-G-H-J (77) A-B-E-G-H-J (77) A-C-E-G-H-J (77)	$ 2300

After crashing by 3 weeks, the duration of the project is **77 weeks** and the total cost is **$46,500+ $2300 = $48,800.**

Q 1.5: The following table provides the necessary information for crashing a project. Project manager would like to crash the network by three weeks in the most economical way. Which activities should be crashed and by how many weeks?



Activity	Activity duration (weeks)		Activity cost ($)		Crash Cost/ Week
	Normal	Crash	Normal	Crash	
A	4	3	4000	6000	2000
B	3	2	5000	6000	1000
C	2	1	2000	2800	800
D	5	3	4000	6000	1000
E	6	5	2500	3000	500
F	3	2	1000	2000	1000
G	4	3	2000	2900	900
H	4	3	1500	2600	1100
I	6	5	5000	12000	7000

Solution:
The project has the following paths and their durations:

- A-B-D-H-I, Duration = 22
- **A-B-E-H-I, Duration = 23**
- **A-B-E-G-I, Duration = 23**
- A-C-E-H-I, Duration = 22
- A-C-E-G-I, Duration = 22
- A-C-F-G-I, Duration = 19

The normal duration of the project is 23 weeks and the normal total cost (sum of all the activities) is **$27,000.**

Critical Path	Duration	It#1	It#2	It#3
A-B-D-H-I	22	**22***	**21***	**20***
A-B-E-H-I	**23***	**22***	**21***	**20***
A-B-E-G-I	**23***	**22***	**21***	**20***
A-C-E-H-I	22	21	**21***	**20***
A-C-E-G-I	22	21	**21***	**20***
A-C-F-G-I	19	19	19	18

It#	CP Before Crashing	Activity Crashed	CP After Crashing	Cumulative cost
1	A-B-E-H-I (23) A-B-E-G-I (23)	E by 1	A-B-E-H-I (22) A-B-E-G-I (22) A-B-D-H-I (22)	$ 500
2	A-B-E-H-I (22) A-B-E-G-I (22) A-B-D-H-I (22)	B by 1	A-B-E-H-I (21) A-B-E-G-I (21) A-B-D-H-I (21)	$ 1500
3	A-B-E-H-I (21) A-B-E-G-I (21) A-B-D-H-I (21) A-C-E-H-I (21) A-C-E-G-I (21)	A by 1	A-B-E-H-I (20) A-B-E-G-I (20) A-B-D-H-I (20) A-C-E-H-I (20) A-C-E-G-I (20)	$ 3500

After crashing by 3 weeks, the duration of the project is **20 weeks** and the total cost is **$27,000+ $3500 = $30,500.**

Q 1.6: Kozar International, Inc. begun marketing a new instant-developing film project. The estimates of R&D activity time (weeks) for Kozar's project are given in the table below. The project has two paths: A-C-E-F and A-B-D-F. Assume the activity times are independent.

a) The company wants to be 95% confident that it can deliver the project without incurring any penalty, what time frame should it specify in the bid for project completion time?
b) If the time to complete the path A-B-D-F is normally distributed, what is the probability that this path will take at least 38 weeks to be completed?

Activity	Time(weeks)				Mean	Variance
	Predecessors	**Optimistic time**	**Probable time**	**Pessimistic time**		
A	-	9	9	9	9	0.00
B	A	8	10	12	10	0.44
C	A	9	12	18	12.5	2.25
D	B	5	8	11	8	1.00
E	C	5	7	10	7.166	0.69
F	D, E	10	12	14	12	0.44

Solution:

The project has two paths:

A-C-E-F:

- Expected Duration = 9 + 12.5 + 7.166 + 12 = 40.66 weeks (Critical Path).
- Variance $\sigma^2 = 0 + 2.25 + 0.69 + 0.44 = 3.38$, Standard Deviation = $\sigma = 1.838$

A-B-D-F

- Expected Duration = 9+ 10 + 8 + 12 = 39 weeks.
- Variance = 0+0.44+1.00+0.44 = 1.88, Standard Deviation = 1.371

Assumption: The probability of completing the project within a given timeframe depends on the critical path only. Note that this will provide us with approximate probability because there can be non-critical paths with high activity variances that might have the lower probability of completion time as compared to the critical path.

- The critical path A-C-E-F has an expected duration of 40.66 weeks and standard deviation of 1.838.
- The z value that corresponds to 95% is 1.645.
- This implies $z = 1.645 = \frac{\text{T−Expected Duration}}{\sigma} = \frac{T-40.66}{1.838}$ or T = 43.68.
- Hence, if the company wants to be 95% confident that it can deliver the project without incurring any penalty, then the time frame it should specify in the bid for project completion is **<u>43.68 weeks.</u>**

If the time to complete the path A-B-D-F is normally distributed, what is the probability that this path will take at least 38 weeks to be completed?

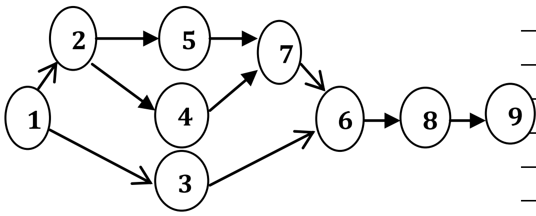
- The non-critical path A-B-D-F has an expected duration of 39 weeks and standard deviation of 1.371.
- This implies $z = \frac{\text{T−Expected Duration}}{\sigma} = \frac{38-39}{1.371}$ =-0.73.
- This z value corresponds to a probability of 0.2327.
- Probability that this path will take <u>less than 38 weeks</u> to be completed is 23.27%.
- Hence, the probability that this path will <u>at least 38 weeks</u> to be completed is 76.73%.

Q 1.7: Given the following network and probabilistic time estimates, answer the following questions:

(a) Indicate the critical path and determine the expected project duration.

(b) The company wants to be 95% confident that it can deliver the project without incurring any penalty, what time frame should it specify in the bid for project completion time?

(c) What is the probability that the project will take at least 60 weeks to finish?



Activity	Estimated time (weeks)			Mean	Variance
	Optimistic	Most likely	Pessimistic		
1	10	10	10	10	0.00
2	5	5	5	5	0.00
3	10	18	20	17	2.78
4	4	4	4	4	0.00
5	3	3	3	3	0.00
6	6	7	8	7	0.11
7	8	9	10	9	0.11
8	6	10	14	10	1.78
9	10	14	18	14	1.78

Solution:

The project has three paths:

1-2-5-7-6-8-9:

- Expected Duration = 58 weeks
- Variance = 3.78, Standard Deviation = 1.94

1-2-4-7-6-8-9

- Expected Duration = 59 weeks **(Critical Path).**
- Variance = 3.78, Standard Deviation = 1.94

1-3-6-8-9

- Expected Duration = 58 weeks
- Variance = 6.44, Standard Deviation = 2.54

Assumption: The probability of completing the project within a given timeframe depends on the critical path only. *Note that this will provide us with approximate probability because there can be non-critical paths with high activity variances that might have the lower probability of completion time as compared to the critical path.*

- The critical path 1-2-4-7-6-8-9 has an expected duration of 59 weeks and standard deviation of 1.94.
- The z value that corresponds to 95% is 1.645.
- This implies $z = 1.645 = \frac{T - Expected\ Duration}{\sigma} = \frac{T-59}{1.94}$ or T = 62.19 weeks.
- Hence, if the company wants to be 95% confident that it can deliver the project without incurring any penalty, then the time frame it should specify in the bid for project completion is **62.19 weeks.**

What is the probability that the project will take at least 60 weeks to be completed?

- The z statistics is $z = \frac{T - Expected\ Duration}{\sigma} = \frac{60-59}{1.94}$ = +0.515.
- This z value corresponds to a probability of 0.6968.
- Probability that this path will take less than 60 weeks to be completed is 69.68%.
- Hence, the probability that this path will at least 60 weeks to be completed is 30.32%.

TOPIC: FORECASTING

***Q2.1:** Monthly sales for National Mixer, Inc. for a seven-month period were as follows:*

Month (T)	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*Jul.*	*Aug.*
Sales (1000 UNITS)	*19*	*18*	*15*	*20*	*18*	*22*	*20*

Forecast the sales volume for September using each of the following method:

a) 5-month moving average;
b) Weighted average, where the weights are - 0.60 (August), 0.30 (July), 0.10 (June)
c) Exponential smoothing with a smoothing constant equal to 0.20
*d) Linear trend equation, Y = 16.86+0.5*T;*

Solution:

a) 5-month moving average;

Using the 5-month moving average, the forecast for September $= \frac{15+20+18+22+20}{5} = 19$ (in thousands of units)

b) Weighted average, where the weights are - 0.60 (August), 0.30 (July), 0.10 (June)
Using weighted moving average, for the month of September, the forecast for September $0.1*18+0.3*22+0.6*20$ = 20.4 (in thousands of units)

c) Exponential smoothing with a smoothing constant equal to 0.20
To use this method, set the forecast for period 1 equal to the actual observation for the period 1. Also while calculating the error, do not include this data point.

MONTH	*SALES (1000 UNITS)*	*Forecast (alpha=0.2)*	*Error = (Sales-Forecast)*
Feb.	*19*	*19*	
Mar.	*18*	*19*	-1
Apr.	*15*	*18.8*	-3.8
May	*20*	*18.04*	1.96
June	*18*	*18.432*	-0.432
Jul.	*22*	*18.3456*	3.6544
Aug.	*20*	*19.07648*	0.92352
September		*19.26118*	

The forecast for September = 19.2612 (in thousands of units)

*d) Linear trend equation, Y = 16.86+0.5*T;*
For the month of September, T = 8, as the data starts from Feb (T=1), and hence the forecast for September = 16.86+0.5*8 = 20.86 (in thousands of units).

***Q2.2:** The following past data is available on the weekly demand for a product:*

Week (T)	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Demand	*150*	*157*	*162*	*166*	*177*	*185*	*192*	*197*	*209*	*214*

a) Forecast the demand for weeks 4-11 using 3-period moving average method.
b) Forecast the demand for weeks 1-11 using exponential smoothing with a = 0.2 (assume the first week's forecast to be equal to the first week's demand).
*c) Forecast the demand for weeks 1-11 using linear trend, Y = 140.87+7.28*T.*

Solution:

a) *Forecast the demand for weeks 4-11 using 3-period moving average method.*

Week	Demand	3 –period MA
1	150	
2	157	
3	162	
4	166	156.33
5	177	161.67
6	185	168.33
7	192	176.00
8	197	184.67
9	209	191.33
10	214	199.33
11		**206.67**

b) *Forecast the demand for weeks 1-11 using single exponential smoothing with a = 0.2 (assume the first week's forecast to be equal to the first week's demand).*

c) *Forecast the demand for weeks 1-11 using linear trend, Y = 140.87+7.28*T.*

Week	*Demand*	*Forecast (alpha = 0.2)*	*Error(D-F)*	Forecast Using Y = 140.87+7.28*T.
1	150	150.00		148.15
2	157	150.00	7.00	155.43
3	162	151.40	10.60	162.71
4	166	153.52	12.48	169.99
5	177	156.02	20.98	177.27
6	185	160.21	24.79	184.55
7	192	165.17	26.83	191.83
8	197	170.54	26.46	199.11
9	209	175.83	33.17	206.39
10	214	182.46	31.54	213.67
11		188.77	193.85	220.95

Q2.3: *Two independent methods of forecasting based on the managers experience have been prepared each month for the past 10 months. The forecasts and actual sales are as follows. Which forecast seem superior? Justify your answer with appropriate calculations.*

MONTH	*FORECAST 1*	*FORECAST 2*	*ACTUAL SALES*
1	*771*	*769*	*770*
2	*785*	*787*	*789*
3	*790*	*792*	*794*
4	*784*	*798*	*780*
5	*770*	*774*	*768*
6	*768*	*770*	*772*
7	*761*	*759*	*760*
8	*771*	*775*	*775*
9	*784*	*788*	*786*
10	*788*	*788*	*790*

Solution:

Month	Actual Sales	F1	Error (A-F1)	Abs Error	Error^2
1	770	771	-1	1	1
2	789	785	4	4	16
3	794	790	4	4	16
4	780	784	-4	4	16
5	768	770	-2	2	4
6	772	768	4	4	16
7	760	761	-1	1	1
8	775	771	4	4	16
9	786	784	2	2	4
10	790	788	2	2	4
SUM	**7784**		**12**	**28**	**94**

Month	Actual Sales	F2	Error (A-F2)	Abs Error	Error^2
1	770	769	1	1	1
2	789	787	2	2	4
3	794	792	2	2	4
4	780	798	-18	18	324
5	768	774	-6	6	36
6	772	770	2	2	4
7	760	759	1	1	1
8	775	775	0	0	0
9	786	788	-2	2	4
10	790	788	2	2	4
SUM	**7784**		**-16**	**36**	**382**

Note that the demand and forecast values for period 1 are not equal, hence we will use n=10 for the calculation of MAD and MSE.

Forecast 1:

$$MAD = \frac{\sum |error|}{n} = 2.8, \quad MSE = \frac{\sum error^2}{n} = 9.4,$$

$$MAPD = \frac{\sum \text{Absolute Error}}{\sum D_t} = \frac{28}{7784} = 0.00035 = 0.3587\%$$

Forecast 2:

$$MAD = \frac{\sum |error|}{10} = 3.6, \quad MSE = \frac{\sum error^2}{10} = 38.2$$

$$MAPD = \frac{\sum \text{Absolute Error}}{\sum D_t} = \frac{382}{7784} = 0.0004622 = 0.4622\%$$

All the three measures, MAD, MSE and MAPE are **smaller** for forecasting 1 and so that is **superior.**

Q2.4: *Sales of CD-ROM sales over the past 12 weeks are shown in the following table. The manager has decided to use single exponential smoothing to forecast sales using a smoothing constant of either 0.10 or 0.40. Using the data from weeks 1 through 7, determine which smoothing constant would produce the smaller errors.*

Week	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Sales	*40*	*44*	*46*	*43*	*45*	*44*	*40*	*43*	*44*	*42*	*46*	*45*

Solution:

MONTH	DEMAND	Forecast (alpha=0.1)	Error (D-F)	Abs(Error)	Forecast (alpha=0.4)	Error (D-F)	Abs(Error)
1	40	40.00			40.00		
2	44	40.00	4.00	4.00	40.00	4.00	4.00
3	46	40.40	5.60	5.60	41.60	4.40	4.40
4	43	40.96	2.04	2.04	43.36	-0.36	0.36
5	45	41.16	3.84	3.84	43.22	1.78	1.78

6	44	41.55	2.45	2.45	43.93	0.07	0.07
7	40	41.79	-1.79	1.79	43.96	-3.96	3.96
SUM				**19.72**			**14.57**

$$MAD_{\alpha=0.1} = \frac{\sum|error|}{6} = \frac{19.72}{6} \quad \text{and} \quad MAD_{\alpha=0.4} = \frac{\sum|error|}{6} = \frac{14.57}{6}$$

Using data from weeks 1 through 7 (assuming 1st period forecast = 1st period demand), based on MAD, alpha = 0.4 is better. Remember when considering periods 1 – 7, just take the sum of the absolute deviations for those periods and divide by 6.

MONTH	Error (D-F)	Error^2	Forecast (alpha=0.4)	Error (D-F)	Error^2
1			40.00		
2	4.00	16.00	40.00	4.00	16.00
3	5.60	31.36	41.60	4.40	19.36
4	2.04	4.16	43.36	-0.36	0.13
5	3.84	14.75	43.22	1.78	3.17
6	2.45	6.00	43.93	0.07	0.00
7	-1.79	3.20	43.96	-3.96	15.68
SUM		**75.47**			**54.34**

$$MSE_{\alpha=0.1} = \frac{\sum error^2}{6} = \frac{75.47}{6} \quad \text{and} \quad MAD_{\alpha=0.4} = \frac{\sum error^2}{6} = \frac{54.34}{6}$$

Looking at the MAD and MSE calculations, alpha = 0.4 performs better forecast.

Q2.5: *Consider the demand for a chemical in a pulp and paper industry in the first 10 months of 1998.*

Month:	*Jan*	*Feb*	*March*	*April*	*May*	*June*	*July*	*August*	*Sept.*	*October*
Demand	*10*	*10*	*66*	*32*	*34*	*18*	*24*	*9*	*14*	*48*

Forecast November sales volume using each of the following:
(1) 3-month moving average;
(2) 5-month moving average;
(3) Exponential smoothing with a smoothing constant equal to 0.10;
*(4) Linear trend (Regression), Y = 25+0.2727*T; where T refers to the month 1, 2, 3,...*
(5) Which forecast seem superior? <u>Justify your answer with appropriate calculations.</u>

Solution:
(1) 3-month moving average;
(2) 5-month moving average;

Month	Demand	MA3	MA5
Jan	10		
Feb	10		
Mar	66		
Apr	32	28.67	
May	34	36.00	
June	18	44.00	30.4
July	24	28.00	32
Aug	9	25.33	34.8
Sept	14	17.00	23.4

Oct	48	15.67	19.8
NOV		23.67	22.6

(3) Exponential smoothing with a smoothing constant equal to 0.10;

Month	**Demand**	Forecast (alpha = 0.1)	Error(D-F)
Jan	10	10	0
Feb	10	10	0
Mar	66	10.00	56.00
Apr	32	15.60	16.40
May	34	17.24	16.76
June	18	18.92	-0.92
July	24	18.82	5.18
Aug	9	19.34	-10.34
Sept	14	18.31	-4.31
Oct	48	17.88	30.12
NOV		20.89	

(4) Linear trend, Y = 25+0.2727*T; where T refers to the month 1, 2, 3,...

For T = 11, the demand forecast is Y = 27.999

(5) Which forecast seem superior? Justify your answer with appropriate calculations.

	Month	**Demand**	**MA3**	**MA5**	**ES**	**Linear R.**	**E(MA3)**	**E(MA5)**	**E(ES)**	**E(LR)**
1	Jan	10			10	25.27				
2	Feb	10			10	25.55				
3	Mar	66			10	25.82				
4	Apr	32	28.67		15.6	26.09				
5	May	34	36.00		17.24	26.36				
6	June	18	44.00	30.40	18.92	26.64	-26.00	-12.40	-0.92	-8.64
7	July	24	28.00	32.00	18.82	26.91	-4.00	-8.00	5.18	-2.91
8	Aug	9	25.33	34.80	19.34	27.18	-16.33	-25.80	-10.34	-18.18
9	Sept	14	17.00	23.40	18.31	27.45	-3.00	-9.40	-4.31	-13.45
10	Oct	48	15.67	19.80	17.88	27.73	32.33	28.20	30.12	20.27

Note that we are using the period June-October only to compare the quality of forecasts because these are the only periods for which we have the forecasts using all the four methods.

Err(MA3)	Abs(Err(MA3))	Err(MA3)^2
-26.00	26	676
-4.00	4	16
-16.33	16.33	266.78
-3.00	3	9
32.33	32.33	1045.44
	81.67	**2013.22**

Err(MA5)	Abs(Err(MA5))	Err(MA5)^2
-12.4	12.4	153.76
-8.0	8	64
-25.8	25.8	665.64
-9.4	9.4	88.36
28.2	28.2	795.24
	83.8	**1767**

Err(ES)	Abs(ES)	Err(ES)^2
-0.9	0.92	0.85

Err(LR)	Abs(LR)	Err(LR)^2
-8.6	8.64	74.58

5.2	5.18	26.83	-2.9	2.91	8.46
-10.3	10.34	106.92	-18.2	18.18	330.57
-4.3	4.31	18.58	-13.5	13.45	181.02
30.1	30.12	907.21	20.3	20.27	410.99
	50.87	**1060.38**		**63.45**	**1005.63**

- MAD for forecast 1 = 81.67/5 = 16.33, MSE for forecast 1 = 2013.22/5 = 402.64
- MAD for forecast 2 = 83.8/5 = 16.76, MSE for forecast 2 = 1767/5 = 353.4
- MAD for forecast 3 = 50.87/5 = 10.173, MSE for forecast 3 = 1060.38/5 = 212.067
- MAD for forecast 4 = 63.45/5 = 12.69, MSE for forecast 4 = 1005.63/5 = 201.126
- MAD indicates that forecast 3 is better; MSE indicates that forecast 4 is better. Any of these two forecasts can be selected.

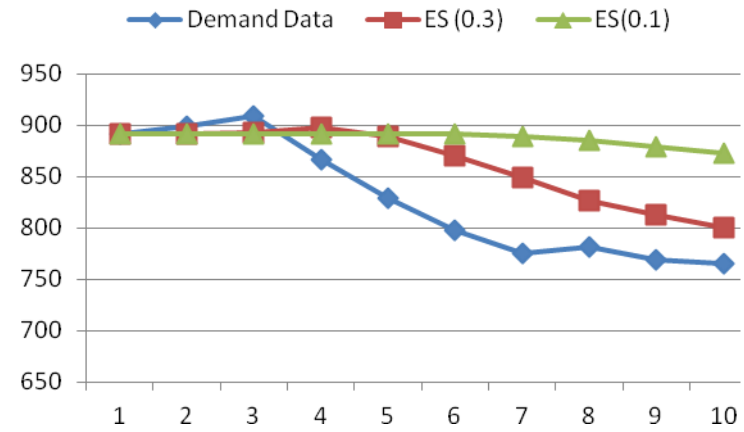
Q 2.6: *Mr. Harvey wishes to compare forecasts using different forecasting approaches. Demand for a particular product had shifted downward last April because of a price increase. The following numbers of units were demanded in the past 10 months:*

MONTH	*UNITS*	*MONTH*	*UNITS*
January	*891*	*June*	*798*
February	*899*	*July*	*775*
March	*909*	*August*	*782*
April	*867*	*September*	*769*
May	*829*	*October*	*765*

(a) *Using a smoothing constant of 0.3, prepare an exponential smoothing forecast of the data. Assume an initial value of 891.*
(b) *Using a smoothing constant of 0.1, prepare an exponential smoothing forecast of the data. Assume an initial value of 891.*
(c) *Compare the smoothed forecasts in parts (a) and (b). Does a decrease in the smoothing constant make the forecasting model less sensitive or more sensitive to a shift in demand?*
(d) *If the random movements in monthly demand are relatively large, would it be better to set the smoothing constant relatively high or relatively low? Explain.*
(e) *Forecast demand for November.*

Solution: (a) and (b)

MONTH	Demand	ES(0.3)	Error(D-F)	ES(0.1)	Error(D-F)
January	891	891		891	
February	899	891.0	8.0	891.0	8.0
March	909	893.4	15.6	891.8	17.2
April	867	898.1	-31.1	893.5	-26.5
May	829	888.8	-59.8	890.9	-61.9
June	798	870.8	-72.8	884.7	-86.7
July	775	849.0	-74.0	876.0	-101.0
August	782	826.8	-44.8	865.9	-83.9
September	769	813.4	-44.4	857.5	-88.5
October	765	800.0	-35.0	848.7	-83.7
November		789.5		840.3	



c) As can be seen from the figure, the higher the value of alpha, the greater is the reaction to the most recent demand. Hence, alpha=0.3 produces a forecast that is more sensitive to the changes in the recent demand, and smoothing will be less. The forecast with alpha= 0.1 produces a forecast with dampening or smoothing effect.

d) If the random movements in monthly demand are relatively large, then it would be better to set the smoothing constant relatively high. This will produce forecast that is more representative of the actual demand. Relatively low value of alpha will dampen or smoothen the effect.

e) The forecast for November using exponential smoothing with alpha = 0.3 and alpha= 0.1 is 789.5 and 840.3.

Q 2.7: *The following past data is available on the monthly demand for a product:*

Period	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***
Demand	*215*	*250*	*275*	*285*	*350*	*285*	*305*	*300*	*300*	*380*	*350*	*325*

(a) *Using adjusted exponential smoothing with $\alpha = 0.4$ and $\beta = 0.1$, determine the forecast for period 13.*

(b) *Suppose that the actual demand for period 13 turned out to be 395. What would be your forecast for period 14 at the end of period 13? What would be your forecast for period 15 at the end of period 13?*

Solution: The forecast is shown in table below:

Period	Demand	ES Forecast	Trend	Adj. Trend ES Forecast
1	215	215.00		
2	250	215.00	0	
3	275	229.00	1.40	230.40
4	285	247.40	3.10	250.50
5	350	262.44	4.29	266.73
6	285	297.46	7.37	304.83
7	305	292.48	6.13	298.61
8	300	297.49	6.02	303.51
9	300	298.49	5.52	304.01
10	380	299.10	5.03	304.12
11	350	331.46	7.76	339.22
12	325	338.87	7.73	346.60
13	395	333.32	6.40	**339.72**
14		357.99	8.23	**366.22**

The forecast for period 13 is 339.72 and the forecast for period 14 is 366.22. In order to determine the forecast for period 15, we would need the actual demand for 14th period.

***Q 2.8:** The following past data is available on the weekly demand for a product:*

Week (t)	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
Demand	*150*	*157*	*162*	*166*	*177*	*185*	*192*	*197*	*209*	*214*

(a) *Forecast the demand for weeks 1-10 using linear trend, weeks 1-10 using exponential smoothing with a = 0.2 (assume the first week's forecast to be equal to the first week's demand) and weeks 4-10 using 3-period moving average method.*

(b) *Plot the forecast and the actual demand data. Comment about using averaging techniques when actual demand has a trend.*

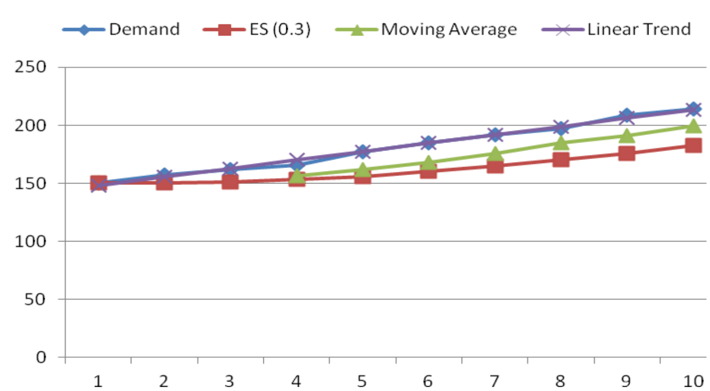
Solution:

(a) The forecast is shown in the table below:

				Moving Average		Linear Trend				
Week	Demand	ES(0.2)	Error (D-F)	MA(3)	Error (D-F)	**Week (x)**	**Demand (y)**	**x. y**	**x^2**	**Forecast**
1	150	150				1	150	150	1	148.1
2	157	150.0	7.0			2	157	314	4	155.4
3	162	151.4	10.6			3	162	486	9	162.7
4	166	153.5	12.5	156.3	9.7	4	166	664	16	170.0
5	177	156.0	21.0	161.7	15.3	5	177	885	25	177.3
6	185	160.2	24.8	168.3	16.7	6	185	1110	36	184.5
7	192	165.2	26.8	176.0	16.0	7	192	1344	49	191.8
8	197	170.5	26.5	184.7	12.3	8	197	1576	64	199.1
9	209	175.8	33.2	191.3	17.7	9	209	1881	81	206.4
10	214	182.5	31.5	199.3	14.7	10	214	2140	100	213.7
Mean						5.5	180.9	1055	38.5	
SUM						55	1809	10550	385	

Linear Trend:

- Linear Trend Line: $y = a + bx$, Where $b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$, and $a = \bar{y} - b\bar{x}$
- $b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{10550 - 10*5.5*180.9}{385 - 10*5.5*5.5} = 7.28$
- $a = \bar{y} - b\bar{x} = 180.9 - 7.28 * 5.5 = 140.86$
- $y = a + bx = 140.86 + 7.28.\text{week}$



(b) As can be seen from the above figure, the linear trend performs great when the data has a trend. The moving average method also performs well, however it lags behind the actual demand. The lag is ever higher in the case of exponential smoothing.

TOPIC: INVENTORY MANAGEMENT

Q3.1: The Zertex Manufacturing Company produces fertilizer to sell to wholesalers. One raw material, calcium nitrite, is purchased from a supplier located near Zertex's plant. 5,750,000 tons of calcium nitrite is forecast to be required next year to support production. If calcium nitrite costs $22.50 per ton, carrying cost is 40% of acquisition cost, and ordering cost is $595 per order:

a) In what quantities should Zertex buy calcium nitrite?
b) What annual stocking costs will be incurred if calcium nitrite is ordered at EOQ?
c) How many orders per year must take place for calcium nitrite?
d) How much time will elapse between two orders?

Given: (BASIC EOQ MODEL)

- Annual Demand (D) = 5,750,000 tons/year
- Holding cost (C_h) = 40% of $22.50 = $ 9.0
- Ordering/ Setup Cost (C_o)= $ 595/order

(a) Optimal Ordering Quantity (EOQ) =

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*595*5{,}750{,}000}{9.00}} = 27{,}573.135 \text{ tons}$$

(b) Total Annual Cost = Annual Holding Cost + Annual Ordering Cost

Annual Holding Cost = $\frac{Q}{2}C_h = \frac{27{,}573.135*9}{2} = \$124{,}079.10$

Annual Ordering Cost = $\frac{D}{Q}C_o = \frac{5{,}750{,}000*595}{27{,}573.135} = \$124{,}079.10$

Total Annual Cost = Annual Holding Cost + Annual Ordering Cost = $ 248,158.20

c) Number of orders placed per year = D/Q* = 5750, 000/27,573.135 = 208.53 orders/year
d) Time between the placement of orders (Cycle Time) =Q*/D = 27,573.135/5750,000=0.00479 year

Q3.2: *A large automobile repair shop installs about 1250 mufflers per year, 18% of which are for imported cars. All the imported car mufflers are purchased from a single local supplier at a cost of $18.50 cents each. The shop uses a holding cost based on a 25% annual interest rate. The setup cost for placing an order is estimated to be $28.*

(a) *Determine the optimal number of imported-car mufflers the shop should purchase each time an order is placed, and the time between the placements of orders.*
(b) *If the replenishment lead time is six weeks, what is the reorder point based on the levels of on-hand inventory?*
(c) *The current reorder policy is to buy imported-car mufflers only once a year. What are the additional holding and setup costs incurred by the policy?*

Solution: Basic EOQ Model (with Positive Lead Times)
Given: Demand (D) = 18% of 1250 units/year = 225 units/year

- Holding cost (Ch) = 25% of $18.50 = $ 4 .625
- Ordering/ Setup Cost (Co)= $ 28/order

(a) Optimal Ordering Quantity (EOQ) =

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*28*225}{4.625}} = 52.20 \approx 52 \text{ units}$$

Time between the placement of orders (Cycle Time) =Q*/D = 52/225 =0.2311 years

(b) Note that demand and lead times are constant.
Weekly Demand = 225/52 = 4.33 units, Lead Time = 6 weeks
Reorder Points = Demand during lead time = 4.33*6 = 25.98 = 26 units.
An order should be placed when the inventory on hand reaches 26 units.

(c) Let us first annual holding and setup cost for the optimal policy (Q* = 52 units/order).

- $\textit{Holding Cost @52 units/order} = \frac{Q}{2}C_h = \frac{52*4.625}{2} = 120.25$
- $\textit{Setup Cost @52 units/order} = \frac{D}{Q}C_o = \frac{225*28}{52} = \$121.15$

For the optimal policy, annual Holding costs+ Setup costs = 120.25 + 121.15 = $ 241.40.

If we place only one order per year (then Q = total demand = 225 units/year), which is their current policy, then

- $\textit{Holding Cost @225 units/order} = \frac{Q}{2}C_h = \frac{225*4.625}{2} = 520.31$
- $\textit{Setup Cost @225 units/order} = \frac{D}{Q}C_o = \frac{225*28}{225} = \$28$

Annual Holding costs+ Setup costs = 520.31 + 28 = $ 548.31

Additional Costs = $ 548.31 - $ 241.40 = $ 306.91

Q3.3: *A consumer electronics store that sells a flash drive knows that the annual demand for this drive is fairly constant at 5000 units/year at the selling price of $30/unit. The manufacturer of this drive charges a delivery fee of $200 for each order, regardless of the number delivered. In addition, in-house costs associated with each order (e.g., unloading, order processing) total $50. The inventory holding costs have been estimated at 20 cents/dollar/year. The manufacturer charges $20/unit for 0-1000 units ordered, but is willing to give a discount of 3% per unit for all units on order sizes between 1001-2000 units and 5% per unit for all units on order sizes of 2001 units or more. What order size will maximize the profit for the store from sales of this flash drive, and what will be the profit at that order size?*

Solution: (EOQ Model with Quantity Discounts)
Given:

- Demand (D) (Constant) = 5000 units/year
- Selling Price = $30/unit
- Ordering/Setup Cost (Co) = $200+ $50 = $250/order
- Holding Cost (Ch) = $ 0.20 * Unit cost of the product
- Unit price = $20 for 0-1000 units, $19.4 for 1001-2000 units, $19 for 2001-more units

If Q is priced at $ 20/unit then,

$$D = 5000 \text{ units/year}, \; C_o = 200 + 50 = \$250/\text{order}, \; C_h = 0.2*20 = \$4/\text{unit/year}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*5000*250}{4}} = 790 \text{ units (It is feasible).}$$

If Q is priced at $ 19.4/unit then,

$D = 5000$ units/year, $C_o = 200 + 50 = \$250/\text{order}$, $C_h = 0.2 * 20 * (1 - 0.03) = \$3.88/\text{unit/year}$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*5000*250}{3.88}} = 802 \text{ units (not feasible, hence bump up to 1001 units)} \approx 1001 \text{ units}$$

If Q is priced at $ 19/unit then,

$D = 5000$ units/year, $C_o = 200 + 50 = \$250/\text{order}$, $C_h = 0.2 * 20 * (1 - 0.05) = \$3.8/\text{unit/year}$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*5000*250}{3.8}} = 811 \text{ units (not feasible, hence bump up to 2001 units)} \approx 2001 \text{ units}$$

(1) If Q* = 790 units,

$$TC = D*C + (Q/2)*C_h + (D/Q)*C_o = 5000*20 + 790/2*4 + (5000/790)*250 = \$103{,}163$$

(2) If Q*= 1001 units,

$$TC = 5000*19.4 + 1001/2*3.88 + 5000/1001*250 = \$100{,}189$$

(3) If Q* = 2001 units,

$$TC = 5000*19 + 2001/2*3.8 + 5000/2001*250 = \$99{,}425 \text{ (this is minimum) - select}$$

Therefore, the best order quantity is 2001 units, and the profit is 5000*30 – 99425 = $ 50575/year.

Q3.4: *A mail ordering company uses 800 boxes a year. The boxes can be purchased from either the supplier A or supplier B. Holding cost is 25% of unit cost and the ordering cost is $ 40 per order. The following quantity discounts are available.*

Supplier A		*Supplier B*	
Quantity	*Unit Price*	*Quantity*	*Unit Price*
1-199	*$14.00*	*1-149*	*$14.10*
200-499	*$13.80*	*150-349*	*$13.90*
500+	*$13.60*	*350+*	*$13.70*

Which supplier should be used and what is (a) the optimal order quantity and (b) the number of orders per year if the intent is to minimize the total annual cost.

Solution: (EOQ Model with Quantity Discounts)

Given:

Annual Demand (D) = 800 units /year
Ordering Costs Co = $40/order
Holding Costs Ch = (25%) x Unit Price of product

For Supplier A:

Range	*C*	*Ch*	*Co*	*FEASIBLE*
1-199	*14*	*3.5*	*40*	*YES*
200-499	*13.8*	*3.45*	*40*	
500+	*13.6*	*3.4*	*40*	

The optimal ordering quantity for the first price range is

$$Q_1^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*800*40}{3.5}} = 135.22 \text{ (This is realizable)}$$

Total annual cost of this policy is:

$$TC_1 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 800*14 + \frac{135.22}{2}*3.5 + \frac{800}{135.22}*40 = \$11{,}673.29$$

The optimal ordering quantity for the second price range is

$$Q_2^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*800*40}{3.45}} = 136.20 \text{ (This is not realizable, increase it up 200 units)}$$

Total annual cost of this policy is:

$$TC_2 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 800*13.8 + \frac{200}{2}*3.45 + \frac{800}{200}*40 = \$11{,}545 \text{ (Lowest)}$$

The optimal ordering quantity for the third price range is

$$Q_3^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*800*40}{3.40}} = 137.20 \text{ (This is not realizable, increase it up 500 units)}$$

Total annual cost of this policy is:

$$TC_3 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 800*13.6 + \frac{500}{2}*3.40 + \frac{800}{500}*40 = \$11{,}794$$

Supplier B:

Range	C	H	S	FEASIBLE
1-149	14.1	3.525	40	YES
150-349	13.9	3.475	40	NO
350+	13.7	3.425	40	

The optimal ordering quantity for the first price range is

$$Q_1^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*800*40}{3.525}} = 134.74 \text{ (This is realizable)}$$

Total annual cost of this policy is:

$$TC_1 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 800*14.1 + \frac{134.74}{2}*3.525 + \frac{800}{134.74}*40 = \$11{,}754.96$$

The optimal ordering quantity for the second price range is

$$Q_2^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*800*40}{3.475}} = 135.71 \text{ (This is not realizable, increase it up 150 units)}$$

Total annual cost of this policy is:

$$TC_2 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 800*13.9 + \frac{150}{2}*3.475 + \frac{800}{150}*40 = \$11{,}593.96 \text{ (Lowest)}$$

The optimal ordering quantity for the third price range is

$$Q_3^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*800*40}{3.425}} = 136.70 \text{ (This is not realizable, increase it up 350 units)}$$

Total annual cost of this policy is:

$$TC_3 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 800*13.7 + \frac{3500}{2}*3.425 + \frac{800}{350}*40 = \$11{,}650.80$$

Conclusion: Since supplier A's cost of 11, 545 is less than Supplier B's 11,593.96, we should use supplier A. The optimal order quantity is 200 units. The number of order per year to minimize cost is (800/200=) 4.

Q3.5: *An electronic retailer stocks a popular model of alarm clock in his warehouse. The lead time has been so stable in the past year that it is safely assumed to be a constant 9 days. Past data also indicates that the probability distribution of the daily demand is approximately normal with a mean of 20 and a variance of 16.*

a) *What are the reordering point and the safety stock that provides a 96% service level?*
b) *The ordering point is arbitrarily set to 189 clocks by the manager. What is the corresponding service level?*
c) *Determine the safety stock that is needed to attain a 1% risk of stockout during lead-time.*

Solution:

Given:

- Lead Time = 9 days (Constant)
- Demand (D) is normally distributed with mean of 20 units/day,
- Standard Deviation of demand is 4 units/day

Hence, use EOQ model with Stochastic Demand formulas.

From this, we will calculate the expected demand during lead time (μ_L) and the standard deviation of demand during lead time (σ_L) as follows:

- Expected demand during lead time $\mu_L = \mu_t * L = 20*9 = 180$ units
- Standard deviation of demand during lead time $\sigma_L = \sigma_t \sqrt{L} = 4*\sqrt{9} = 12$ units.

(a) From the normal distribution table, a 96% service level implies that 4% upper tail area. This corresponds to a standard normal z-value of 1.75.

- That is the reorder point is $r = \mu_L + z\sigma_L = 180 + 1.75*12 = 201$ units.
- The amount of safety tock to be carried is $z\sigma_L = 1.75*12 = 21$ units.

(b) If the reordering point is set to 189 units, this implies that the safety stock is 9 units as the expected demand during the lead time is 180 units.

Safety stock of 9 units implies that the service level corresponds to $z = \frac{9 \text{ units}}{\sigma_L} = \frac{9}{12} = 0.75$.

This corresponds to a service level of 77.34% (from the normal distribution table).

(c) Risk of stock out of 1% implies that the service level is 99%. For a 99% service level, the z-score is 2.33.

- Safety Stock = $z\sigma_L = 2.33*12 = 27.96$ units ≈ 28 units

Q3.6: A product with an annual demand of 1,000 SKUs has C_0 = \$30 and C_c = \$8. The demand exhibits some variability such that the lead-time demand follows a normal distribution, with a mean of 25 and a standard deviation of 5.

(a) What is the recommended order quantity?
(b) What are the reorder point and safety-stock level if the firm desires at most a 2-percent probability of a stockout on any given order cycle?
(c) If the manager sets the reorder point at 30, what is the probability of a stockout on any given order cycle? How many times would you expect to stockout during the year if this reorder point were used?

Solution:

Given:

Annual Demand (D) = 1000 units /year, Ordering Costs Co = \$30/order
Holding Costs Ch = \$8/unit/year

(a) The optimal ordering quantity is

$$Q_1^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*1000*30}{8}} = 86.6\text{units}$$

(b) Using a normal distribution table, a *2-percent probability of a stockout on any given order cycle* implies 98% service level. This implies that 2% upper tail area. This corresponds to a standard normal z-value of 2.05.

- That is the reorder point is $r = \mu_L + z\sigma_L = 25 + 2.05*5 = 35.3 \approx 35$ units.
- The amount of safety tock to be carried is $z\sigma_L = 2.05*5 = 10.3 \approx 10$ units.
- Annual safety stock cost = *10(\$8) = \$80/year*

(c) If R = 30, the standardized z-value is (30 – 25)/5 = 1, giving a tail area under the normal distribution of 0.1587. This is the probability of a stock-out each cycle.

The number of stockouts per year is therefore 0.1587 times the number of orders per year = 0.1587(D/Q) = 2.

***Q 3.7:** The Sum and Yang (SY) produces copper contacts that is uses in switches and relays. SY needs to determine the order quantity Q to meet the annual demand at the lowest cost. SY consumes 5000 kg per month. The holding cost is 28% per year and ordering cost is $30 per order. The price of copper depends on the quantity ordered. The following are the price-break data and other relevant data for the problem:*

Order Quantity (Kg)	*Price per Kg*
0 ≤Q≤ 3999	*$1.00*
4000 ≤Q≤ 5249	*0.81*
5250 ≤Q≤ 6999	*0.80*
7000 ≤Q≤ 19,999	*0.79*
Q ≥ 20,000	*0.78*

Solution: (EOQ Model with Quantity Discounts)

Given: Annual Demand (D) = 5000 units/months *12 months/year= 60,000 units/year

Ordering Costs Co = $30/order

Holding Costs Ch = (28%) x Unit Price of product

Range	*C*	*Ch*	*Co*	*Q**
0 ≤Q≤ 3999	$1.00	$0.2800	30	3585.686
4000 ≤Q≤ 5249	0.81	$0.2268	30	~~3984.095~~→4000
5250 ≤Q≤ 6999	0.80	$0.2240	30	~~4008.919~~→5250
7000 ≤Q≤ 19,999	0.79	$0.2212	30	~~4034.212~~→7000
Q ≥ 20,000	0.78	$0.2184	30	~~4059.99~~→20,000

The optimal ordering quantity for the first price range is

$$Q_1^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*60,000*30}{0.28}} = 3585.69 \text{ (This is realizable)}$$

Total annual cost of this policy is:

$$TC_1 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 60,000*1 + \frac{3585.69}{2}*0.28 + \frac{60,000}{3585.69}*30 = \$61,004$$

The optimal ordering quantity for the second price range is

$$Q_2^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*60,000*30}{0.2268}} = 3984.095 \text{ (This is unrealizable) Hence, change it to 4000 units}$$

Total annual cost of this policy is:

$$TC_2 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 60,000*0.81 + \frac{4000}{2}*0.2268 + \frac{60,000}{4000}*30 = \$49098.6$$

The optimal ordering quantity for the three price range is

$$Q_3^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*60,000*30}{0.2240}} = 4008.92 \text{ (This is unrealizable) Change it to 5250 units}$$

Total annual cost of this policy is:

$$TC_3 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 60,000*0.80 + \frac{5250}{2}*0.2240 + \frac{60,000}{5250}*30 = \$48,930.86$$

The optimal ordering quantity for the forth price range is

$$Q_4^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*60,000*30}{0.2212}} = 4034.20 \text{ (This is unrealizable) Change it to 7000 units}$$

Total annual cost of this policy is:

$$TC_4 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 60,000*0.79 + \frac{7,000}{2}*0.2212 + \frac{60,000}{7000}*30 = \$48,431.34 \text{ (LOWEST)}$$

The optimal ordering quantity for the fifth price range is

$$Q_5^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*60,000*30}{0.2184}} = 4060 \text{ (This is realizable) Change it to 20,000 units}$$

Total annual cost of this policy is:

$$TC_5 = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o = 60,000*0.78 + \frac{20,000}{2}*0.2184 + \frac{60,000}{20,000}*30 = \$49,074$$

Hence, the optimal policy is to order 7000 units. The annual cost of this policy is \$48,431.34 .

Q 3.8: The Friendly Sausage Factory (FSF) can produce European Wieners at a rate of 2000 kg per week. FSF supplies wieners to local stores and restaurants at a steady rate of 150 kg per day. The cost to prepare the equipment for producing wieners is \$50. Annual holding cost is \$ 5 per kg of wiener. The factory operates throughout the year.

a) Determine the optimal run quantity, total annual inventory cost, the number of production runs per year.
b) If the factory has only enough storage space to hold a maximum of 300 kg in inventory, how will that effect the total inventory costs? (show with complete calculation)

Solution:

- Production Rate = 2000 kg/ week = 104,000 kg/year (@52 weeks per year)
- Demand = 150 kg per day = 1050 kg/week = 54,600 kg/year (@7 days/week & 52 weeks/year)
- Holding Cost (Cc) = \$5/kg/year
- Setup Cost (Co) = \$50/order
- $\frac{d}{p} = \frac{150}{285.7} \approx 0.525$

We will is the production quantity model:

- Optimal Run Quantity: $Q_{Opt} = \sqrt{\frac{2C_O D}{C_c(1-\frac{d}{p})}} = \sqrt{\frac{2C_O D}{C_c(1-\frac{d}{p})}} = \sqrt{\frac{2*50*54,600}{5(1-0.525)}} =$ **1516.22 Kg**
- Total Annual Cost: $TC = \frac{C_O D}{Q} + \frac{C_c Q}{2}\left(1 - \frac{d}{p}\right) = \frac{50\times54600}{1516.22} + \frac{5*1516.22}{2}(1 - 0.525) = \$\ \mathbf{3601.03}$
- Number of runs/year = $\frac{D}{Q} = \frac{54,600}{1516.22} \approx \mathbf{36}$

<u>With a maximum storage space of 300 Kg, we have</u>

- Maximum inventory Level: $I_{max} = Q\left(1 - \frac{d}{p}\right) \Rightarrow 300 = Q_{new}\left(1 - \frac{150}{285.71}\right) \Rightarrow \boldsymbol{Q_{new}} = \mathbf{631.57}$ Kg
- Total Cost: $TC = \frac{C_O D}{Q} + \frac{C_c Q}{2}\left(1 - \frac{d}{p}\right) = \frac{50\times54,600}{631.57} + \frac{5\times631.573}{2}(1 - 0.525) = \$\mathbf{5072.5}$
- **Total annual cost increases by \$5072.5 - \$ 3601.03 = \$1471.52.**

Q 3.9: Allen's Shoe store carries a basic black dress shoes for men that sell at an approximate constant rate of 1000 pairs of shoes every three months. Allen's current buying policy is to order 500 pairs each time an order is placed. It costs $500 to place an order, an inventory- carrying costs at an annual rate of 20 percent is $8 per pair. Lead time is one month.

a) What are the estimated annual inventory-holding and –ordering costs associated with this product?
b) What is the cycle time for this product?
c) What is the re-order point for this policy?
d) Allen's supplier offered to reduce the price by 15% if the order size is doubled. What is your recommendation for Allen?

Solution:

- Demand = 1000 pairs every 3 months = 4000 pairs/year (Constant)
- Carrying Cost (Cc) = $8/pair/year
- Ordering Cost (Co) = $500/order
- Lead Time = 1 month = 1/12 year
- 20% of Unit Price = $8, Hence, unit price of product = $40
- Current Ordering Quantity = $Q_{Current} = 500\ pairs$

(a) Annual Inventory Holding Cost = $\frac{C_c Q}{2} = \frac{8*500}{2} = \$\ 2000$

Annual Inventory Ordering Cost = $\frac{C_O D}{Q} = \frac{500*4000}{500} = \$\ 4000$

Since, the annual holding cost is not equal to annual ordering costs, the current policy if 500 pairs is not optimal.

(b) Cycle time for this product = $\frac{Q}{D} = \frac{500}{4000} = 0.125\ years = 1.5$ months

(c) Re-order point for this policy = Demand during the lead time = 1000/3 = 333.33 pairs.

(d) For the current policy, we have

- Annual Inventory Holding Cost = $\frac{C_c Q}{2} = \frac{8*500}{2} = \$\ 2000$
- Annual Inventory Ordering Cost = $\frac{C_O D}{Q} = \frac{500*4000}{500} = \$\ 4000$
- Since, carrying cost is 20% of purchase cost = $8.00, unit purchase cost = $40.
- Annual Purchase Cost = 40000*$40 = $ 160,000.
- Total Annual Cost of this Policy = $2000+4000+160,000 = **$166, 000.**

If Allen's supplier offers to reduce the price by 15% if the order size is doubled, then the cost of the policy is:

- Ordering Quantity = $Q_{new} = 1000\ pairs$
- Price of the product = 85% of $40 (due to 15% discount) = $34/pair
- Carrying Cost = 20% of the cost of product = 20% of 34 = $6.8/pair/year
- Annual Inventory Holding Cost = $\frac{C_c Q}{2}$ = $3400/pair
- Annual Inventory Ordering Cost = $\frac{C_O D}{Q} = \frac{500*4000}{1000} = \$\ 2000$
- Annual Purchase Cost = 40000*$34 = $ 136,000.
- Total Annual Cost of this Policy = $3400+2000+136,000 = **$141, 400.**

If Allen's supplier offers to reduce the price by 15% if the order size is doubled, then the cost of the policy is $141,400 which is less than the current policy, hence Allen should accept the offer.

Q 3.10: The Old Town Microbrewery makes Towside beer, which it bottles and sells in its adjoining restaurant. It costs $1700 to setup, brew, and bottle a batch of the beer. The annual cost to store the beer in inventory is

$1.25 per bottle. The annual demand for beer is 21,000 bottles and the brewery has the capacity to produce 30,000 bottles annually.

(a) Determine the optimal ordering quantity, total annual inventory costs, and the number of production runs per year.

(b) If the microbrewery has only enough storage space to hold a maximum of 2500 bottles of beer in inventory, how will that effect the total inventory costs?

Solution:

- Demand = 21, 000 bottles/year (Constant)
- Production Capacity = 30,000/year
- Carrying Cost (Cc) = $1.25/bottle/year
- Setup Cost (Co) = $1700/order
- Assuming 360 days/year, d =21,000/360 = 58.33 bottles/year
- Assuming 360 days/year, p= 30,000/360 = 83.33 bottles/year

Production Quantity Model:

- Optimal Quantity: $Q_{Opt} = \sqrt{\frac{2C_O D}{C_c(1-\frac{d}{p})}} = \sqrt{\frac{2\times1700\times21000}{1.25(1-\frac{58.33}{83.33})}} = 13{,}798.55 \approx 13{,}799$ bottles
- Total Cost: $TC = \frac{C_O D}{Q} + \frac{C_c Q}{2}\left(1-\frac{d}{p}\right) = \frac{1700\times21000}{13799} + \frac{1.25\times13{,}799}{2}\left(1-\frac{58.33}{83.33}\right) = \$5174.64$
- Number of runs/year $= \frac{D}{Q} = \frac{21{,}000}{13{,}799} \approx 1.52$

With a maximum storage space of 2500 bottles,

- Maximum inventory Level: $Q\left(1-\frac{d}{p}\right) \Rightarrow 2500 = Q_{new}\left(1-\frac{58.33}{83.33}\right) \Rightarrow Q_{new} = 8333.33$ bottles
- Total Cost: $TC = \frac{C_O D}{Q} + \frac{C_c Q}{2}\left(1-\frac{d}{p}\right) = \frac{1700\times21000}{8333.33} + \frac{1.25\times8333.33}{2}\left(1-\frac{58.33}{83.33}\right) = \$5846.5$

- **Total annual cost increases by $ 5846.5- $ 5174.64 = $671.9.**

Q 3.4: Bell computers purchases integrated chips at $350 per chip. The holding cost is 10% per year, the ordering cost is $120 per order, and sales are steady, at 400 per month. The company's supplier, Rich Blue Chip Manufacturing, Inc., decides to offer price concessions in order to attract larger orders. The price structure is given in the table below. What is the optimal order quantity and the minimum cost for Bell Computers to order, purchase, and hold these integrated chips?

Quantity Purchased	Price per unit
1-99 units	$350
100-199 units	$325
200 or more	$300

Solution:

- Demand (D) (Constant) = 400 units/month = 4800 units/year
- Ordering/Setup Cost (Co) = $120/order
- Holding Cost (Cc) = 0.10 * Unit price of the product

If Q is priced at $ 350/unit then,

$D = 4800$ units/year, $C_o = \$120/\text{order}$, $C_c = 0.1*350 = \$35/\text{unit/year}$

$$Q^* = \sqrt{\frac{2DC_o}{C_c}} = \sqrt{\frac{2*4800*120}{35}} = 181.42 \text{ units (It is not feasible).}$$

If Q is priced at $ 325/unit then,

$$D = 4800 \text{ units/year},\ C_o = \$120/\text{order},\ C_c = 0.1*325 = \$32.5/\text{unit/year}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_c}} = \sqrt{\frac{2*4800*120}{32.5}} = 188.27 \text{ units}$$

If Q is priced at $ 300/unit then,

$$D = 4800 \text{ units/year},\ C_o = \$120/\text{order},\ C_c = 0.1*300 = \$30/\text{unit/year}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_c}} = \sqrt{\frac{2*4800*120}{30}} = 195.95 \text{units (not feasible, hence bump up to 200 units)} \approx 200 \text{ units}$$

(4) If Q* = 181.42 units, the range is not feasible.

(5) If Q*= 188.27 units,

$$TC = 4800*325 + 188.27/2*32.5 + 4800/188.27*120 = \$1{,}566{,}118.87$$

(6) If Q* = 200 units,

$$TC = 4800*300 + 200/2*30 + 4800/200*120 = \$1{,}445{,}880 \text{ (this is minimum) - select}$$

Therefore, the best order quantity is 200 units, and the annual cost is $1445, 880.

Q 3.5: Brauch's Pharmacy has an expected annual demand for a leading pain reliever of 800 boxes, which sell for $6.50 each. Each order costs $6.00, and the inventory carrying charge is 20 ~~cents~~ percent. The expected demand during the lead time is normal, with a mean of 25 and a standard deviation of 3. Assuming 52 weeks per year, what reorder point provides a 95-percent service level? How much safety stock will be carried? If the carrying charge were 25 ~~cents~~ percent instead, what would be the total annual inventory-related cost?

Solution:

- Demand (D) = 800 boxes/year (Normally distributed)
- Selling Price = $6.50/unit
- Ordering Cost (Co) = $6/order
- Holding Cost (Cc) = 20% of the price of the product
- Demand during Lead Time = 25 units, Std. Dev = 3 units

If we apply EOQ using the average demand, we will find that the optimal ordering quantity is

Optimal Ordering Quantity (EOQ) = $Q^* = \sqrt{\frac{2DC_o}{C_c}} = \sqrt{\frac{2*800*6}{0.20*6.50}} = 85.93 \approx 86 \text{ units}$

- Average Demand during Lead Time = μ_L = 25 boxes
- Standard Deviation of Demand during Lead Time = σ_L = 3 boxes

From the normal distribution table, a 5% upper tail area corresponds to a standard normal z-value of 1.645.

That is the reorder point is $r = \mu_L + z\sigma_L = 25 + 1.645*3 = 29.935 \text{ boxes} \approx 30 \text{ boxes}$.

The amount of safety tock to be carried is $z\sigma_L = 1.645*3 = 4.935 \text{ boxes} \approx 5 \text{ boxes}$.

This suggests that a policy of ordering 86 boxes whenever the inventory position reaches a reorder point of 30 will minimize inventory costs and risks at most a 5% probability of stock out during the lead time period.

The total annual cost of this policy is the sum of the following costs:

- $\text{Ordering Costs} = \frac{D}{Q}C_o = \frac{800}{86}*6 = 55.81$
- $\text{Holding Cost of Cycle Inventory} = \frac{Q}{2}C_c = \frac{86}{2}*1.3 = \$55.90$
- $\text{Holding Cost of Safety Stock} = (z\sigma_L)C_c = 1.645*3*1.3 = \$6.42$

- Hence, the total cost is $118.13

If the carrying charge were 25 % instead, what would be the total annual inventory-related cost?

If we apply EOQ using the average demand, we will find that the optimal ordering quantity is

- Optimal Ordering Quantity (EOQ) = $Q^* = \sqrt{\frac{2DC_o}{C_c}} = \sqrt{\frac{2*800*6}{0.25*6.50}} = 76.86 \approx 77 \text{ boxes}$
- $\text{Ordering Costs} = \frac{D}{Q}C_o = \frac{800}{77}*6 = 62.34$
- $\text{Holding Cost of Cycle Inventory} = \frac{Q}{2}C_c = \frac{77}{2}*(0.25*6.5) = \$62.56$

  $\text{Holding Cost of Safety Stock} = (z\sigma_L)C_c = (1.645*3)*(0.25*6.5) = \$8.02$
- The total annual cost of this policy is the sum of the above costs = $132.92.