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Faculty of Engineering

Civil engineering
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GNG 1105
ENGINEERING MECHANICS

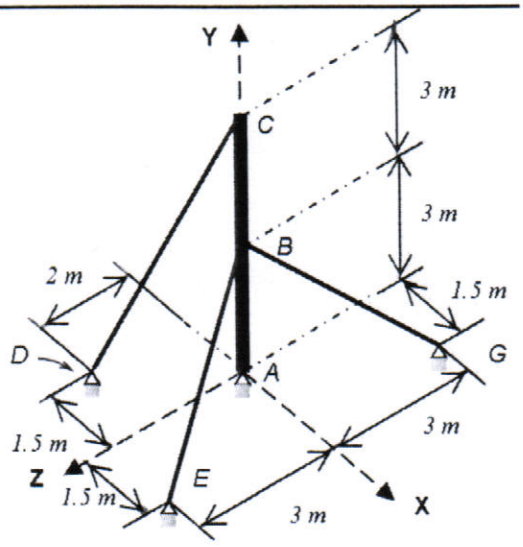
Final Examination
13 December 2010
Profs. Skaff, Van Blaeren and Flores-Vera

Time: 3 hrs

Closed Book Examination. Programmable calculators are not allowed.
Free-body diagrams must be drawn where appropriate.

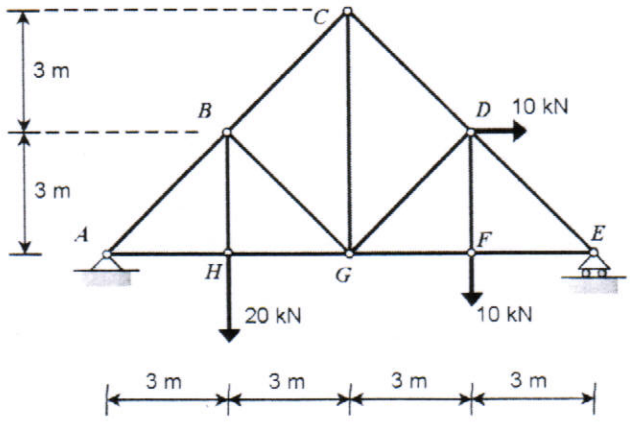
Problem 1 (16/60)
Mast ABC is being supported by a ball-and-socket joint at base A and by three cables BE, BG and CD as shown.

a) Draw the free-body diagram for mast ABC
b) Write the tensions in cables BE, BG and CD in vector form.
c) If the tension in cable CD is 500 N, determine the tensions in cables BE and BG.



Problem 2 (11/60)
For the truss shown and assuming that all joints are pinned,

a) Calculate the reactions at supports A & E.
b) Determine the forces in members CD, GD and GF using the method of sections. State whether each member is in tension or compression.

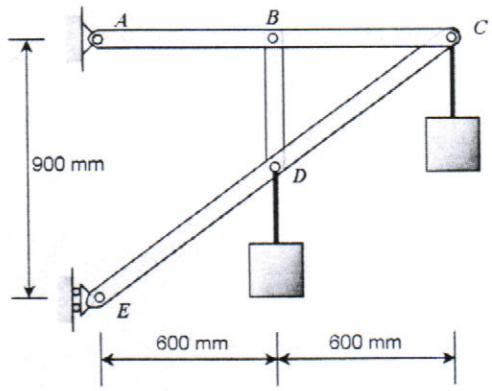


Problem 3 (11/60)

Two boxes are hanging from the frame shown to the right. The mass of each of these boxes is 30 Kg.

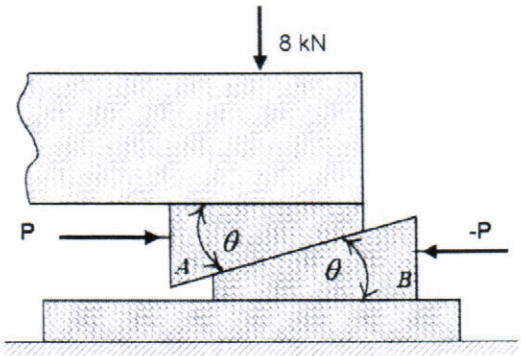
a) Determine the reactions at the supports A and E.

b) Determine all the forces acting on member ABC.



Problem 4 (11/60)

To level a wood deck, wood wedges A&B are placed under a corner of the deck. Wedge B rests on a wood board as shown, and a bar clamp is used to apply equal an opposite forces to the wedges. The load transmitted from the deck to the wedges is 8 kN. Knowing that $\theta=18°$ and that the coefficient of static friction between all wood surfaces is 0.35 and between the board and the ground is 0.6, determine the magnitude of **P** of the clamping forces for which upward motion of the deck is impending.

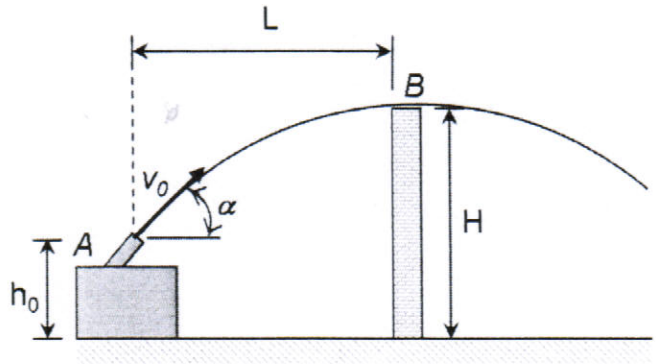


Problem 5 (11/60)

A projectile is fired from point A, located at $h_0 = 2$m above the ground, with an initial velocity v_0 and at an angle $\alpha=50°$.

(a) What should be the minimum value of v_0 for the projectile to clear the wall if its height is H=5 m?

(b) For the value of v_0 obtained in part (a), determine the distance L that separates point A from the wall.



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SOLUTIONS

Dec. 13, 2010

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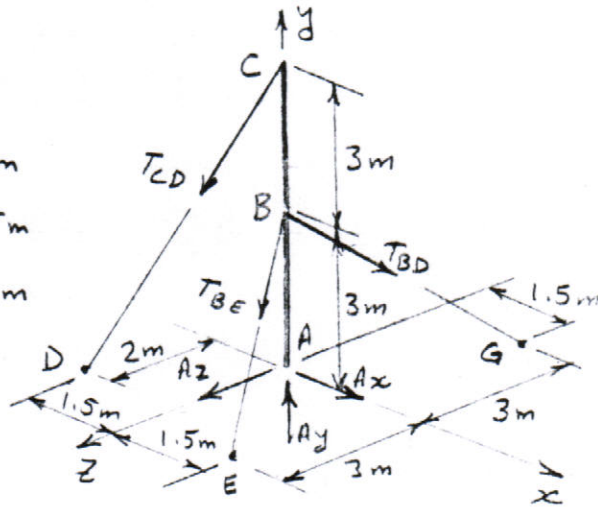
1. a) See FBD to the right.



b) $\bar{BE} = 1.5\bar{i} - 3.0\bar{j} + 3.0\bar{k}$; $BE = 4.5m$

$\bar{BG} = 1.5\bar{i} - 3.0\bar{j} - 3.0\bar{k}$; $BG = 4.5m$

$\bar{CD} = -1.5\bar{i} - 6.0\bar{j} + 2.0\bar{k}$; $CD = 6.5m$

$$\bar{T}_{BE} = T_{BE}\bar{\lambda}_{BE} = T_{BE}\frac{\bar{BE}}{BE} = \frac{T_{BE}}{4.5}(1.5\bar{i} - 3.0\bar{j} + 3.0\bar{k})$$

$$\bar{T}_{BG} = T_{BG}\bar{\lambda}_{BG} = T_{BG}\frac{\bar{BG}}{BG} = \frac{T_{BG}}{4.5}(1.5\bar{i} - 3.0\bar{j} - 3.0\bar{k})$$

$$\bar{T}_{CD} = T_{CD}\bar{\lambda}_{CD} = T_{CD}\frac{\bar{CD}}{CD} = \frac{T_{CD}}{6.5}(-1.5\bar{i} - 6.0\bar{j} + 2.0\bar{k})$$

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c) $\Sigma \bar{M}_A = 0$

$\Sigma \bar{M}_A = \bar{r}_{B/A}\bar{T}_{BE} + \bar{r}_{B/A}\bar{T}_{BG} + \bar{r}_{C/A}\bar{T}_{CD}$; where $\bar{r}_{B/A} = 3.0\bar{j}$ and $\bar{r}_{C/A} = 6.0\bar{j}$

$$\therefore \Sigma \bar{M}_A = 3.0\bar{j} \times \frac{T_{BE}}{4.5}(1.5\bar{i} - 3.0\bar{j} + 3.0\bar{k}) + 3.0\bar{j} \times \frac{T_{BG}}{4.5}(1.5\bar{i} - 3.0\bar{j} - 3.0\bar{k}$$
$$+ 6.0\bar{j} \times \frac{T_{CD}}{6.5}(-1.5\bar{i} - 6.0\bar{j} + 2.0\bar{k}) = 0$$

$$\Sigma \bar{M}_A = -T_{BE}\bar{k} + 2T_{BE}\bar{i} - T_{BG}\bar{k} - 2T_{BG}\bar{i} + 1.38T_{CD}\bar{k} + 1.85T_{CD}\bar{i} = 0$$

Equate coefficients of $\bar{i}$ & $\bar{k}$ to zero.

$(\bar{i})$: $2T_{BE} - 2T_{BG} + 1.85T_{CD} = 0$ ——— (1)

$(\bar{k})$: $-T_{BE} - T_{BG} + 1.38T_{CD} = 0$ ——— (2)

1. Cont'd.

Substitute $T_{CD} = 500N$ in eqs. (1) & (2):

$$2T_{BE} - 2T_{BG} + 1.85 \times 500N = 0$$

$$2T_{BE} - 2T_{BG} + 925 = 0 \quad \text{——— (1)}$$

and $$-T_{BE} - T_{BG} + 1.38 \times 500N = 0$$

$$-T_{BE} - T_{BG} + 690 = 0 \quad \text{——— (2)}$$

Solving (1) & (2). Multiply (2) by 2 and add to (1)

$$-4T_{BG} + 2305 = 0$$

$$\therefore T_{BG} = \frac{2305}{4} = \underline{\underline{576.25N}} \quad \text{ANS.}$$

6/ From (2): $-T_{BE} - 576.25 + 690 = 0$

$$\therefore T_{BE} = 690 - 576.25 = \underline{\underline{113.75N}} \quad \text{ANS.}$$

Another Method

$$\Sigma \bar{M}_A = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 3 & 0 \\ 1.5 & -3 & 3 \end{vmatrix} \times \frac{T_{BE}}{4.5} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 3 & 0 \\ 1.5 & -3 & -3 \end{vmatrix} \times \frac{T_{BG}}{4.5}$$

$$+ \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 6 & 0 \\ -1.5 & -6 & +2 \end{vmatrix} \times \frac{T_{CD}}{6.5} = 0$$

$$\Sigma \bar{M}_A = \frac{9T_{BE}}{4.5}\bar{i} - \frac{4.5T_{BE}}{4.5}\bar{k} - \frac{9T_{BG}}{4.5}\bar{i} - \frac{4.5T_{BG}}{4.5}\bar{k}$$

$$+ \frac{12T_{CD}}{6.5}\bar{i} + \frac{9T_{CD}}{6.5}\bar{k} = 0$$

i.e. $2T_{BE}\bar{i} - T_{BE}\bar{k} - 2T_{BG}\bar{i} - T_{BG}\bar{k} + 1.85T_{CD}\bar{i} + 1.38T_{CD}\bar{k} = 0$

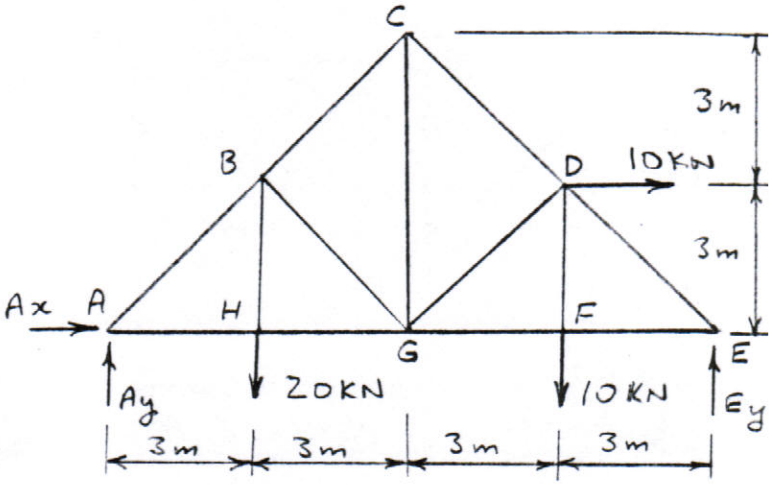
$(\bar{i})$: $2T_{BE} - 2T_{BG} + 1.85T_{CD} = 0 \quad \text{——— (1)}$

$(\bar{k})$: $-T_{BE} - T_{BG} + 1.38T_{CD} = 0 \quad \text{——— (2)}$

Substitute $T_{CD} = 500N$ and solve both equations:

$\therefore T_{BG} = \underline{\underline{576.25N}}$ and $T_{BE} = \underline{\underline{113.75N}}$ (check) ANS.

2. a)



$\pm \Sigma F_x = 0$

$A_x + 10\,KN = 0$

$\therefore A_x = -10\,KN = 10\,KN \leftarrow$

$+\circlearrowleft \Sigma M_E = 0$

$10\,KN \times 3m + 20\,KN \times 9m - 10\,KN \times 3m - A_y \times 12m = 0$

$\therefore 12A_y = 180$

Hence, $A_y = \frac{180}{12} = 15\,KN \uparrow$ AN

$+\uparrow \Sigma F_y = 0$

$E_y - 10\,KN - 20\,KN + 15\,KN = 0$

$\therefore E_y = 15\,KN \uparrow$ ANS

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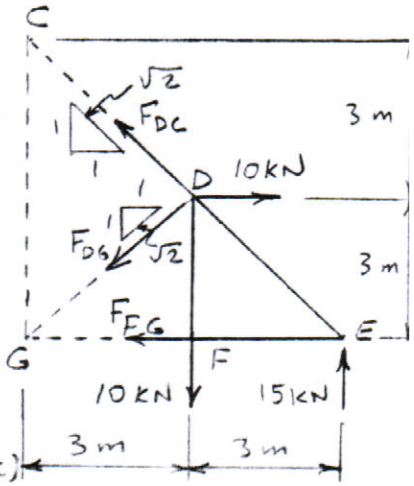
b)



$+\circlearrowleft \Sigma M_D = 0$

$15\,KN \times 3m - F_{FG} \times 3m = 0$

$\therefore F_{FG} = 15\,KN\,(T)$, i.e. $F_{GF} = 15\,KN\,(T)$ ANS.

$+\circlearrowleft \Sigma M_G = 0$

$15\,KN \times 6m - 10KN \times 3m - 10\,KN \times 3m + F_{DC} \times \frac{1}{\sqrt{2}} \times 3m + F_{DC} \times \frac{1}{\sqrt{2}} \times 3m = 0$

$90 - 30 - 30 + 4.24\,F_{DC} = 0$

$\therefore F_{DC} = \frac{-30}{4.24} = -7.07\,KN = 7.07\,KN\,(C)$

i.e. $F_{CD} = 7.07\,KN\,(C)$ A

$+\circlearrowleft \Sigma M_E = 0$

$10\,KN \times 3m - 10KN \times 3m + F_{DG} \times \frac{1}{\sqrt{2}} \times 3m + F_{DG} \times \frac{1}{\sqrt{2}} \times 3m = 0$

$30 - 30 + 2F_{DG} \times \frac{1}{\sqrt{2}} \times 3m = 0$

$\therefore 2F_{DG} = 0$ and $F_{DG} = F_{GD} = 0$ A

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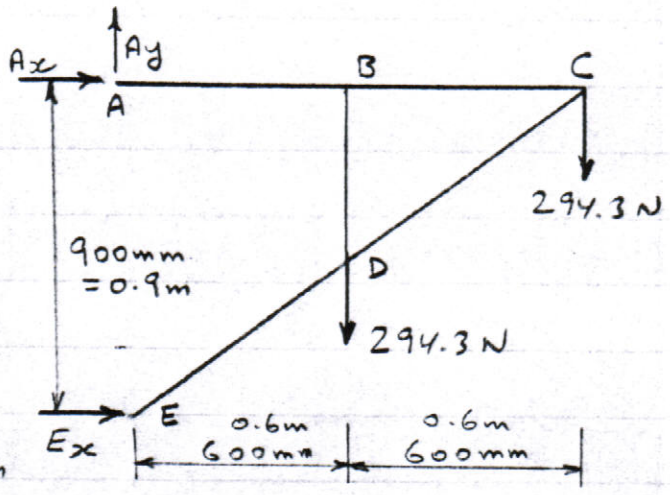
Check:

$\Sigma F_y = 0$; $15\,KN - 10\,KN - 7.07 \times \frac{1}{\sqrt{2}} = 0$

$5 - 5 = 0$ check ✓

3. a)

$30kg \times 9.81 = 294.3N$



$+\circlearrowleft \Sigma M_A = 0$

$E_x \times 0.9m - 294.3N \times 0.6m - 294.3N \times 1.2m = 0$

$\therefore E_x = (176.58 + 353.16) \div 0.9m$

$E_x = \frac{529.74}{0.9} = 588.6N \rightarrow$ ANS.

$\pm \rightarrow \Sigma F_x = 0$

$588.6N + A_x = 0 \; ; \; \therefore A_x = -588.6N = 588.6N \leftarrow$ ANS.

$+\uparrow \Sigma F_y = 0$

$A_y - 294.3N - 294.3N = 0 \; ; \; \therefore A_y = 588.6N \uparrow$ ANS.

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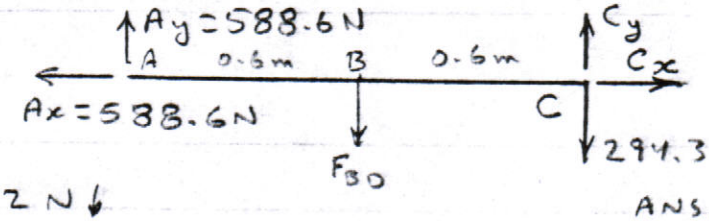
b)



$+\circlearrowleft \Sigma M_C = 0$

$F_{BD} \times 0.6m - 588.6 \times 1.2m = 0$

$\therefore F_{BD} = \frac{588.6 \times 1.2}{0.6} = 1177.2N \downarrow$ ANS.

$\pm \rightarrow \Sigma F_x = 0$

$C_x - 588.6 = 0, \; \therefore C_x = 588.6N \rightarrow$ ANS.

$+\uparrow \Sigma F_y = 0$

$C_y - 294.3N - 1177.2N + 588.6N = 0$

$\therefore C_y = 882.9N \uparrow$ ANS.

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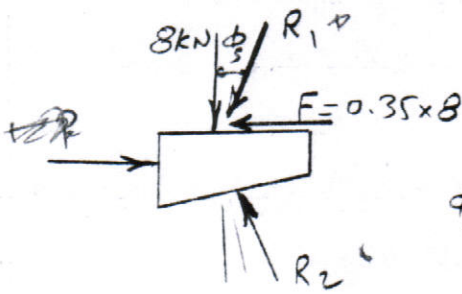
4.

Since $\mu_s = 0.60$ between the board and the ground is greater than $\mu_s = 0.35$ between all wood surfaces, therefore motion will be impending between all wood surfaces in contact and not between the board and the ground. Criterion – 2

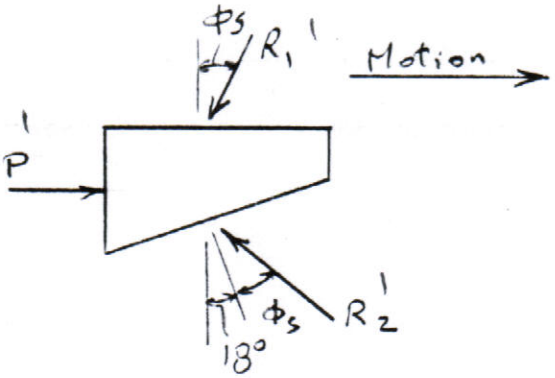
FBD- Top Wedge A



$\phi_s = \tan^{-1}\mu_s$

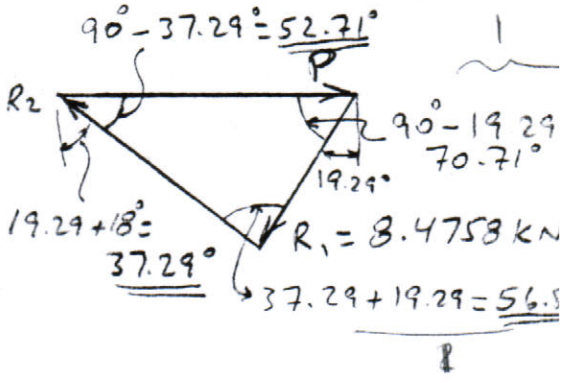
$= \tan^{-1} 0.35$

$= 19.2900°$



$R_1 \cos 19.2900° = 8\,kN$

$\therefore R_1 = \dfrac{8\,kN}{\cos 19.29°} = 8.4758\,kN$



$\dfrac{R_1}{\sin 52.71°} = \dfrac{P}{\sin 56.58°}$

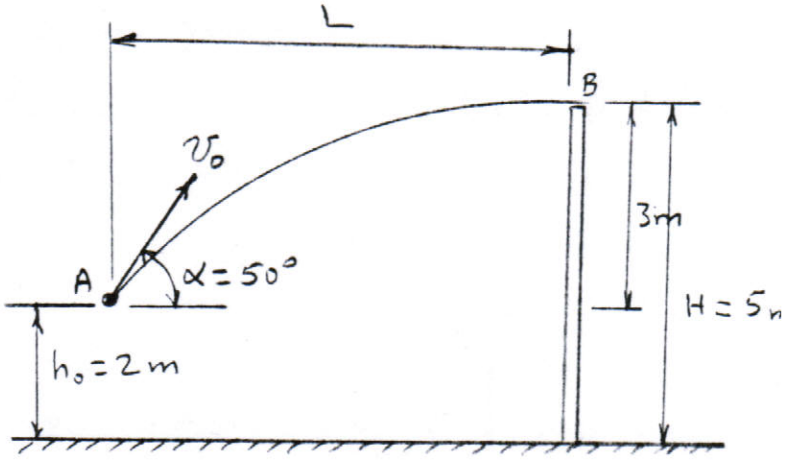
$\dfrac{8.4758}{0.7956} = \dfrac{P}{0.8347}$

$\therefore P = \dfrac{8.4758 \times 0.8347}{0.7956} = 8.892\,kN = \underline{8.89\,kN}$ ANS.

5. a)

$(v_0)_x = v_0 \cos 50°$

$(v_0)_y = v_0 \sin 50°$



For minimum v_0, then B should be the highest point on the projectile. i.e. (v_y) at B = 0

*- Vertical Motion: From A to B.

$(v_y)_B = (v_0)_y + at$, where $(v_y)_B = 0$ & $(v_0)_y$ at A $= v_0 \sin$

and $a = -g = -9.81 m/s$

$\therefore 0 = v_0 \sin 50° - 9.81t$

$\therefore t = \frac{v_0 \sin 50°}{9.81} = 0.078 v_0$ sec.

Now, $y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$

$3m = 0 + v_0 \sin 50° t - \frac{1}{2} \times 9.81 t^2$

Insert $t = 0.078 v_0$

$3 = v_0 \sin 50° \times 0.078 v_0 - \frac{1}{2} \times 9.81 \times (0.078 v_0)^2$

$3 = 0.060 v_0^2 - 0.030 v_0^2$

$0.03 v_0^2 = 3$; $\therefore v_0 = \sqrt{\frac{3}{0.03}} = 10 m/s$ ANS

and $t = 0.078 \times 10 = 0.78$ s.

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b) *- Horizontal Motion

$x = x_0 + (v_0)_x t$ *cos50 5.03m

$L = 0 + 10 \times 0.78 = 7.8$ m Distance from A to the wall. ANS

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END